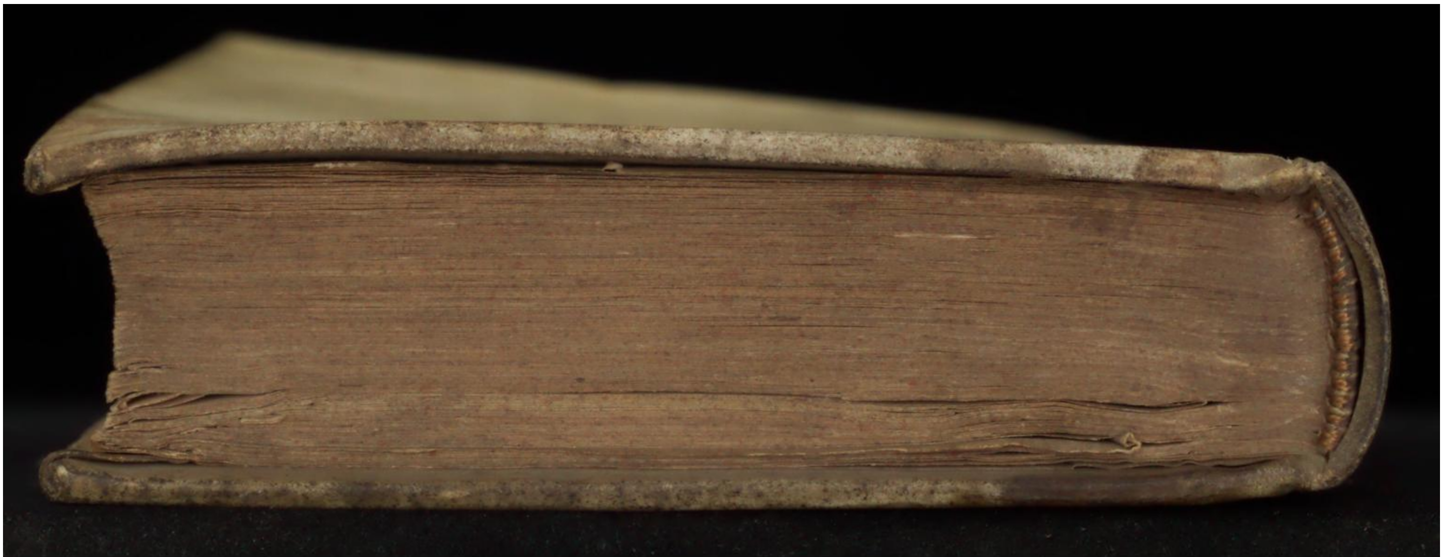
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BERNHARDI NIEUWENTIIT

ANALYSIS INFINITORUM,

Seu

Curvilineorum Proprietates

EX

POLYGONORUM NATURA

DEDUCTÆ.



AMSTELÆDAMI,

Apud JOANNEM WOLTERS,

Anno 1695.

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PRÆFATIO.

QUod expoſitum tibi cernis opuſculum, B: L. domeſtico tantum uſui, privignique mei ſtudiis Mathematicis unicè deſtinatum fuerat, ut poſt elucubrata Analyſeos ordinariæ præcepta ad nova hujus ſeculi inventa & recentiorum Geometrarum ipſi facilior pateret aditus. Quo ſolo forſan nomine hoc, quicquid eſt opellæ, veniam apud æquos rerum æſtimatores merebitur, ſi omni dictionum pompâ viduum, ſtyloque, qualis inter familiares ornatus cujuſcunque expers in uſu eſſe ſolet, negligenter exaratum, compellentibus amicis, in publicum prodeat. Tyroni ſcriptum tyronibus, etſi nonnulla nec provectioribus diſplicitura hic contineri ſperem, hoc ipſo forſitan prodeſſe poterit (magnos enim hic Viros, quorum inventis me plurimum debere gratus agnoſco, eximios facio) quod, quæ fundamentis multum diverſis inædificarunt Geometræ, ex unico tantum ac proprio curvarum principio deducere ſatagat; requirere enim tale quid videbatur, quem diſciplinæ Mathematicæ depoſcunt, naturalis Præceptorum ordo, ac Theorematum etiam ſublimiorum non niſi ex genuinis ſimplicioribuſque principiis eruenda deductio. Quod, quantum incipientium ſtudiis conferat, ignorare non poterunt, qui curvarum

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proprietates ex recepto multorum more explorantes centri gravitatis, truncorum, ungularumque, curvarum curvilineorumque in & evolutionem, ac multijugæ præterea principiorum varietatis tædia devorare neceſſe habuerunt; Sola curvilineorum, tanquam polygona, conſideratio fœcundo ſinu non ea tantum curvarum Symptomata, quæ ab aliis principiis expectari poterant, verum & ipſorum principiorum fontem & originem complexa, quæ hactenus circa curvarum naturam à celeberrimis Mathematicis inventa omnes meritò in ſtuporem rapuerant, admirabili quadam ſerie digeſta orbi litterato expoſuit, inſigniorumque hujus ſeculi Virorum labores ac ſcrutinia demeruit. Hoc tamen ſupereſſe videbatur difficultatis, methodum à principibus hactenus Mathematicis in curvilineorum menſuris adhibitam, concluſionum potius emergentium multitudine, ac cum demonſtrabilibus aliunde veritatibus convenientia, quam principiorum evidentiâ nixam certitudinem ſuam apagogicis magis demonſtrationibus, quam inventionis rationi debere; cujus aſſerti demonſtrationem hic adjicere ſupervacaneum eſt, cum, quæ huc ſpectant, ex *Conſiderationibus* noſtris antehac editis abundè peti poſſint.

Placuit quidem celeberrimo *Walliſio*, non paucis in locis *Arithm. Infin.* de quantitatibus infinitè parvis, non tamen niſi hæſitanter admodum & ſollicitè agere; diſerte que *Schol. Prop.* CLXXXII affirmare, quamvis partem infinitè parvam (ſemel acceptam) nihilo æquivalere; quæ quidem aſſertio eaſdem, quas adverſus *Baroviana* methodi demon-

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monſtrationem in *Conſiderationibus* modo citatis reverenter movimus, difficultates evitare poſſe non videtur.

Tum vel maximè viſum eſt *Cel. Leibnitio*, anno 1689. menſe Febr. in *actis Lipſienſibus*, quædam ex ſuâ *Analyſi infinitorum* depromta diſcurſui de motuum cœleſtium cauſis lemmatum in modum præmittere; (in quem tamen locum non niſi poſt editas *Conſiderationes* incidi) utinam vero placuiſſet Eruditiſſimo D. Authori in re tanti momenti paulo apertius agere, ac quantitatum incomparabilium differentiam nihilo æquipollere demonſtratum dare; cum alioquin fruſtra rejiciantur ex æquationum contextu ea, quæ realis adhuc quantitatis eſſentia gaudent; neque etiam quantitas determinata incomparabili juncta (ſi hæc adhuc quantitas ſit) æqualis ſupponi poteſt eidem quantitati determinatæ, demtâ eâdem incomparabili quantitate: cum ea verè æqualia ſint, quæ differentiam habent nullam, non autem quæ differentiam incomparabilem, (modo quantitatibus adhuc annumeranda maneat.) Num vero quantitatum cum aliis incomparabilium differentiam nihilo ſtrictè æqualem eſſe voluerit Vir Nobil. me dubium reddunt, quæ §. 5. inſinuat D. Author, quo in loco libertatem concedit, *ſi quis adhibere nolit infinitè parvas, quantitates tam exiguas ſumendi, quam ſufficere, judicat, ut ſint incomparabiles & errorem quovis dato minorem producant.* Quod num accuratioris Geometriæ leges exactè ſatis obſervet, hic ulterius diſquirendi locus non eſt, nec niſi poſt explicatam enucleatius D. Autho-

thoris mentem, quod ſuo tempore expectabimus. Ut vero in orbitam redeam, hâc potiſſimum de cauſâ principii, quod curvarum proprietatibus methodo directâ explanandis par eſt ac ſufficit, inventionem pariter ac demonſtrationem difficiliores eſſe reor, quod infiniti quid involvere videtur ipſa curvarum indoles, tale quid ſaltem, quod conceptus noſtros tranſcendit ac ſuperat. Quamobrem & hoc ipſum principium non niſi ex ipſius infiniti naturâ peti poſſe credibile eſt. Cum autem hac quidem lege conditos nos voluerit creator noſter, ut, licet majorem ac minorem quavis conceptâ quantitate nobis repræſentare valeat imaginatio noſtra, non niſi finita tamen ac magnitudine determinata concipere queamus, ad infiniti ipſius notionem veram adæquatamque aſſurgere nequit intellectus humanus: hanc tamen, poſitâ quantitatis cujuſcunque in partes infinitas ſeu omnem datum numerum excedentes diviſibilitate, ex ipſius infiniti attributis veritatem elicere datum eſt, (quam & axiomatis ac fundamenti loco cæteris ſubſtravimus) *quicquid per numerum infinitum multiplicatum nullam quantitatem datam, utut exiguam, adæquare valet, entibus annumerandum non eſt, ac nihilo æquale haberi debet.* Cujus quidem effati certitudinem aliunde ac ex ipſis etiam Veterum aſſumtis (Poſtul. lib. x. *Euclid.* & *Archimed.* de Sphæra & Cylind. Axiom. v.) evincere in proclivi foret, niſi luce ſatis ſuâ enunciatio hæc radiare illis, quibus eam examinandam propoſueram, viſa fuiſſet: & hoc quidem ſolo ac unico principio celebriora hactenus recentiorum Geo-

metra-

metrarum inventa derivari poſſe, confirmationemque ſuam recipere ſequentia, uti ſperamus, palam facient. Accedit maximi hinc momenti præterea veritatem directè ſequi : *Omne nimirum diviſibile, adeoque & omnem quantitatem, vi infinitâ in nihilum eſſe reducibilem, eâdemque vi infinitâ, quantitatem quamcunque ex nihilo produci poſſe*; cum ex eo productum eſſe, in quod diviſione reſolvi poteſt, quidlibet meritò cenſendum ſit. Unde & omni quantitatum generi liquido impreſſus apparet omnipotentiæ ac divinitatis character; nullumque quantum, naturâ ſuâ æternum eſſe, (hoc eſt tale, quod ex nihilo nec produci nec in nihilum redigi poteſt) evidenti ac indubitabili prorſus conſequentiâ evincitur, niſi enim diviſibilia ac quanta quæcunque infinities infinitâ diviſione in nihilum abeant, ac infiniteſimæ pars infiniteſima ſit nihilo æqualis, omnes hactenus Mathematicos non immerito paralogiſmi reos egiſſe videtur Doct. *Cluverus*, periitque in univerſum Geometricarum demonſtrationum ἀκρίβεια; ac magno lapſu corruent plurimæ, ſi non omnes, tum antiquorum, tum & hujus ævi, quocunque etiam ingenii acumine ac felicitate excogitatæ, inventiones. Quo vero, num ex præmiſſo modo Axiomate Methodum indiviſibilium, quæ adhuc deſiderabatur, inculpabilem, primariaque quæ hodiernorum Geometrarum induſtria detexit, legitimè deduxerim, apertius pateat, quem hoc in opuſculo ſervaverim ordinem, paucis exponam.

Ac primo quidem, poſt aſſumtam quantitatis cujuſcunque determinatæ in partes quâlibet datâ

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minores diviſibilitatem, quas cum aliis *infiniteſimas* vocito, Algorithmum hiſce ſpeciatim convenientem ac proprium exhibeo. Unde maximi ſtatim uſus conſectarium ex earum multiplicatione procedit, *infiniteſimam ſcilicet per infiniteſimam multiplicatam nihilo æquivalere*: quæ porro curvis applicata, tandem Lem. 26. & 27. *ipſas omnes ex rectulis infiniteſimis componi, adeoque & curvilinea eſſe infinitorum laterum polygona* (quod a multis antehac aſſumtum fuerat) ex præmiſſis evincunt. Tum poſt hujus Theorematis Lem. 33. demonſtrationem, *in triangulo isosceli, baſin habenti infinite parvam, angulos ad baſin & rectos & æquales eſſe*; *Lem.* 41. & 42. inter eas lineas diſcrimen oſtendo, quæ tanquam infiniteſimæ, adeoque quantitatis adhuc naturam ac eſſentiam ſervantes, aliaſque quæ nihilo æquales in calculo conſiderandæ ſunt. Quod quanti momenti ſit diagnoſticum, illi norint, quibus Analyſis infinitorum familiaris eſt, ac licet ad calculi ſecuritatem omnino requiratur, num antea quiſpiam oſtenderit, ignoro.

Subnexo denique lemmatibus hiſce generali Scholio infiniteſimarum naturam quodammodo illuſtro; unde N. XII. &c. Theorema quoddam prodit, cujus ope earum curvarum tangentes, quarum parametri indeterminatæ ſunt, ad ordinarias curvas reducuntur, hinc & *maximi* & *minimi* determinatio methodo, ni fallor, novâ perficitur.

Cap. I. Tangentium ducendarum methodum exhibeo, *Barovianæque* demonſtrationem, aliarumque inſimul, quæ eodem cum ipſa funda-

damento nituntur: ac poſt particularia quædam in tyronum gratiam ex hiſce deducta compendia, deſignataſque §. 24. iis curvis tangentes, quarum natura non ex applicatarum ad interceptam, ſed ad alterius curvæ ad eundem axem applicitæ ordinatas relatione cognoſcitur, §. 32. ac 33. duplicem propono methodum, ex data qualibet æquatione curvæ (ſive terminis irrationalibus ſive fractis quocunque ſigno copulatis conflata fuerit) tangentem ducendi; quod §. 34. ad ea curvarum genera exporrigo, quarum ſive ad alias ſive ad axem relatio non unicâ tantum, ſed quolibet æquationum numero exprimitur. Donec ultimò §. 35. methodus exoriatur generaliſſima, *applicatis ad eundem axem quotlibet curvis, datâque æquatione communem earum relationem comprehendenti*; (ſive hæc per earum applicatas interceptamve ſive etiam per ipſarum curvarum portiones expreſſa fuerit) *æquationem aliam* (*ſubtangentialem* mihi dictam) *citra ullam calculi moleſtiam inveniendi, quæ ſingularum* (ſi cæteræ ſint cognitæ) *pro arbitrio ſumendarum ſubtangentes unâ operâ determinat*. Quod & §. 37. & ſeq. iis curvis applico, quarum parametri ſunt indeterminatæ; unàque ex generalibus hiſce æquationibus, quarum auxilio innumerarum curvarum tangentes eadem operâ definiuntur, ad particulares quaslibet curvas regreſſionem oſtendo. Hinc §. 41. poſt curvarum quarundam, quæ alioquin anomalæ quodammodo videntur, ad leges æquationum vulgarium reductionem illarum ſubtangentes ex iiſdem principiis inquiro, in quibus indeterminatæ quantitates inter poteſtatum exponen-

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tes reperiuntur, quæque hactenus Geometris aut omnino prætermiſſæ aut parum admodum conſideratæ videntur, cum multorum nihilominus problematum, ſatis alioqui difficilium, conſtructionibus inſerviant; deinde ipſum ſubtangentium problema, in minus provectiorum exercitationem, ex aliis datis inveſtigo; ac quomodo curvæ datæ non recta, ſed curva quælibet deſignata, tangens ducatur, uno aut altero exemplo oſtendo. Tum §. 78, cum ſingulis indeterminatis ſingulæ attribui poſſint infiniteſimæ, earundem plures, ac paucis adhuc huic negotio applicitas ſubtangentium problemati ſolvendo adhibeo; inſimulque generaliorum quorundam, nec ſpecialem curvæ cujuſvis cognitionem depoſcentium, Theorematum inventioni viam ſterno: unde peculiari, ni fallor, methodo obtinetur conſectarii loco §. 87. univerſale *convexitates* & *concavitates* in omnibus curvis *diagnoſticum*; ac definitio puncti, ſi detur, flexus contrarii, quod celebri inter Mathematicos Conchoïdis exemplo illuſtro.

§. 93. &c. de Subtangentium ſubnormaliumque methodo inverſâ paucis ago, quam, licet principia innumeris ejuſmodi problematis ſolvendis ſufficientia addiderim, imperfectam nihilominus reliqui; eò tamen generalem tangentium inverſarum inquiſitionem deduxi, ut inter infinita, in quæ æquatio quælibet ex Cl. *Sluſii* exemplo dividi poteſt, loca, duo tantum determinari neceſſe ſit, quæ datam quandam inter ſe relationem obtinent, talem ſcilicet, qualis inter æquationem

tionem curvæ ipſiuſque Subtangentialem ex legibus antea traditis requiri facillime oſtenditur.

Cap. II. Curvilinea, cum verè polygona ſint, adeoque & polygonorum more menſuranda, §. 1. in trapezia, quæ duo habent latera parallela; deinde in Triangula variis modis, §. 3. &c. diſtinguo; ac poſt præmiſſam indiviſibilium methodum ſcrupulis omnibus liberam, §. 11. Cor. v. fundamentalem Cl. *Wallisii* Arith. Infin. LXIV. Propoſitionem, tum & lemma celeb. *Mercatoris* ac D. *Gregorii* eodem fonte derivo. Quibus §. 13. huic calculi generi magis accommodatum ac generale Theorema ſubnecto; *Omnes ſcilicet terminos, infiniteſimales collectivè ſumtos ipſi termino, unde originem ſumſerunt, quique jugi ipſarum infiniteſimarum coacervatione componitur, tanquam valori abſoluto perpetim æquari.* Hinc poſt varias curvilineorum tranſmutationes, §. 18. Cor. II. methodum exhibeo, ſubnormales omnium ferme curvarum quomodolibet per interceptas applicatas ipſaſque curvas expreſſarum unâ operâ deſignandi; ac Cor. III. Cl. *Wallis*: auctas multataſque ſeries per tangentes inverſas demonſtro. Deinde poſt Celeb. *Newtoni* ſeries in tranſitu ſtrictim ventilatas, §. 24. præter ordinarium infiniteſimarum numerum (quæ Triangulum Celeb. *Leibnitio characteriſtici* nomine vocatum conſtituunt) complures alias, in curvilineorum menſurâ adhibeo; plurimorumque theorematum, omnes in genere curvas ſpectantium, nondumque ſatis diſcuſſorum, originem aperio: ac poſt hinc emergentium, aliquot problematum ſolutiones *involutarum evolutarumque* figurarum

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proprietates, J. Gregorio primum conſideratas eodem cum cæteris fundamento deduco.

Cap. III. Solida tracto, columnas, truncos ungulaſque Cylindrorum ſectione genitas; tum & ea, quæ ex curvilineorum rotationibus oriuntur, corpora conoidica, annularia &c.; methodique indiviſibilium, in quantum ſolidis applicatur, demonſtrationem adjungo. Ac ne, quæ ex Cap. II. peti poſſunt, inutili diſcurſu hic repetere neceſſe habeam, problema infiniteſimarum extra Triangulum characteriſticum aliunde ſeſe ingerentium naturam ac uſum in corporum dimenſione illuſtrans, (quod in planis curvilineis Cap. II, §. 37. generaliter oſtenderam) in Solidis reſolvo; *datâ nimirum Solidi, ex curvilinei circa axem rotatione geniti, aliuſve cujuslibet expreſſione analyticâ, convenientem huic curvilineo applicatam exhibere.*

Cap. IV. Corporum capite præcedenti conſideratorum ſuperficies tracto, eaſque ſpectantem indiviſibilium Geometriam demonſtro.

Cap. V. Poſt trita quædam Baroviana, ac Tyronum captui accommodata §. 6. curvas methodo non admodum uſitatâ expendo; unde varia, & nonnulla etiam nova eas menſurandi oriuntur principia; ac inter alia §. 10. ex quolibet curvilineo infinitas curvas menſurabiles eliciendi methodus, ab *Hugenianâ* evolutione diverſa; quam tamen, tanquam primarium hujus ſeculi inventum, ex hoc, quo cætera derivavimus, etiam fonte manare §. 11. oſtendo.

Cap. VI. Centri gravitatis cum traditis hic principiis ſymphoneſin, Algebraicoſque plurimo-

rum

rum tum curvilineorum, tum ſolidorum, ac ſuperficierum curvarumque, ex prægreſſis capitibus collectos exponentes annexa brevi appendiculâ comprehendo.

Cap. VII. *Maximi* & *minimi* determinationem modo, ni fallor, Mathematicis adhuc intacto, ex infiniti tamen naturâ directè profluenti, commonſtro; *Huddenianæque* methodi, utpote omnium generaliſſimæ, licet principio multum diverſo inventæ, hoc tamen fundamento nixam demonſtrationem adjicio. Quin & particulari hac *extrema* iſthæc definiendi ratio prærogativa gaudere videtur, quod non unum tantum *maximum* & *minimum*, ſed & plura, ſi problematis propoſiti natura ita ferat, eadem opera determinare valeat.

Cap. VIII. Varia calculi in *Analyſi infinitorum* genera, ac imprimis *Leibnitii* Differentialem eodem cum prægreſſis fundamento demonſtrabilem oſtendo; quem & ad eas extendi quantitates, quæ poteſtatum indices indeterminatos habent. Sola *Differentiationum* ſucceſſio, eodem cum cæteris principio deduci renuit. Quæque eam infeſtare videntur difficultates D. Authori in *Conſiderationibus* ex parte propoſui, ac hoc in loco inſtantia una ac altera roboravi; quarum ſolutionem inſimulque legitimam repetitarum differentiationum confirmationem, ne tantum inventum in Geometria intercidat, ſummo deſiderio meritò exoptat orbis Mathematicus.

Cap. IX. Æquationum infiniteſimas continentium, naturam reſpectu earundem, inquiro, ac poly-

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polychreſtum quoddam ex iiſdem conſectarium deduco, quod methodo huic multum promovendæ ejuſque ad innumeras infiniteſimas applicationi inſervit, ac repetitarum differentiationum quoddam velut ſuccedaneum haberi poteſt. Infiniteſimarum enim hinc infiniteſimas, quacunque licet numeri infiniti poteſtate diviſas, (quas alioquin nihilo æquales ac idcirco æquationum, quibus adhærent, terminos merè evanidos eſſe in prioribus eviceramus) non leve tamen in quantitatum aſſignabilium, curvas ſpectantium, rationibus inquirendis pondus habere ex ritè perpenſâ earum indole ac proprietatibus innoteſcit.

Cap. X. Methodum infiniteſimalem inverſam commentariis quibuſdam illuſtro; cumque Cap. II. §. 13. huic aperiat aditum, occaſiones, quibus ab infiniteſimis ad quantitates abſolutas indeterminataſque regreſſio ſecurè datur, ab aliis, quæ errorem forſan inducerent, quibuſdam in caſibus diſtinguo; quædam obiter admonuiſſe contentus, quibus ulterius perficiendis otii magis, quam mihi reliquum eſt, requiritur.

Cap. XI. Miſcellanea quædam, interque ea methodum, quâ ad æquationem ſubtangentialem, in *Conſiderationibus* ad ſolutionem *Bernoulliani* problematis adhibitam, deveni. Tum & ea, quæ cauſticarum, plurimarumque aliarum conſimili modo genitarum curvarum proprietates concernunt, modo à *Tſchirnhauſiano* aliquatenus differenti, generali Theoremate comprehendo.

Schemata, ne in immenſum excreſcerent, pluribus quandoque paragraphis accommodata ſunt; un-

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unde, ne linearum multitudine confuſio inducatur, tyronibus author ſum ut figurarum lineas propoſitioni, quæ præ manibus eſt, inſervientes, miſſis cæteris, ſeligant; novaſque exinde ac ſcopo unicè conuenientes ſibi, (quod facillimo negotio obtinetur) ſi occaſio tulerit, figuras efforment. Signum [ ] Cap. I. §. 17. adhibitum terminorum hinc & inde conſiſtentium tantum ſeparationem, ⊐⊏ æqualitatem denotat.

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ERRATA.

Pag. 3, lin. 20, pro $\frac{e}{w}$ lege $\frac{c}{n}$. p. 6. lin. ultimâ pro cta lege recta. p. 9, l. 4, hinc, lege, hic. p. 17, l. 13, $\frac{dn}{r^e}$, leg. $\frac{d^n}{r^e}$. lin. 23, $d\frac{n}{q}$, leg. [illegible] p. 20, l. 18 $\frac{ly}{\cdot mm}$, l, $+\frac{ly}{mm}$. p. 37, l. 2, gut, l. gt. p. 46, l. 1, l. § 34. p. 47, l. ult., pro $\frac{zz-rz}{t}$ l. $\frac{zz-z}{b}$. p. 56, l. 22, huic, l. hinc. p. 65, l. 18, *gte*, l. $g+e$. p. 67, l. 4, $rx-fy$, l $rx-yfy$. p. 101, l. 9, l. preci-mos. p. 129, lin. 6, $\frac{3}{3}y$, leg. $\frac{3}{2}y$. p. 134, lin. ult. $p+i$, l. $p+1$. p. 139, l. 9, ſen, l. ſeu. p. 19[illegible], l. 25, l. propoſitum. p. 178, l. 19, l. nimirum. p. 181, l. 7. Orei-, leg. orie-. p. 183, l. 18, *kdt*, l. *kr*. p. 249, lin. 21, l. Coroll. III. p. 269, lin. 1[illegible] *incognitam*. p. 270, l. 12, l. inſtar. p. 271. [illegible] penult. bx, l. bx^4.

Bericht voor den Boekbinder.

De Figuren moeten alle achter aangeſtelt werden, na-kander volgende als de getallen daar op ſtaande aanwyſen, en zyn geordineert om buiten het Boek te ſlaan, waarom de witte plaats van 't Papier niet moet werden afgeſneden.

ANALYSIS INFINITORUM,

Seu

Curvilineorum proprietates

Ex

Polygonorum naturâ deductæ.

DEFINITIONES.

1. DAtam quantitatem omnem eam voco, quæ determinabilis eſt, cujuſque magnitudo imaginationis humanæ limites non excedit.

2. Unde, quid per quantitatem quâlibet datâ minorem, aut majorem intelligendum veniat, ipſo nomine apertum eſt.

3. Quantitatem quâlibet datâ minorem, compendii gratia, *infiniteſimam*; majorem, *infinitam* appellare liceat.

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AXIOMATA.

1. QUicquid toties ſumi, hoc eſt per tantum numerum multiplicari non poteſt, ut datam ullam quantitatem, ut ut exiguam, magnitudine ſuâ æquare valeat, quantitas non eſt, ſed in re geometricâ merum *nihil*.

2. Quælibet quantitas data in partes quâlibet datâ minores, tum æquales inter ſe, tum inæquales, eſt diviſibilis.

LEMMATA.

1. OMnis quantitas vel data eſt, vel qualibet datâ minor, aut major.

2. Cumque omnis numerus ſit quantitas, idem de numero intelligendum venit.

3. Quælibet data quantitas per numerum quemlibet eſt diviſibilis.

4. Et quotiens inde ortus, ejuſdem quantitatis pars eſt.

5. Quæ pars præcisè juxta diviſoris denominationem eâ quantitate minor eſt.

Ex. Gr. ſi quantitas *b* dividatur per numerum binarium, quotiens $\frac{b}{2}$ ipsâ *b* duplo

plo minor eſt; ſi per ternarium triplo minor &c.

6. Unde, ſi quantitas per numerum quolibet dato majorem dividatur, erit quotiens pars ejus quantitatis qualibet datâ minor.

Ex. Gr. numerus quolibet dato major vocetur m, (quod & in ſeqq., ni aliud moneatur, obtinebit) item ſint aliæ datæ quantitates b & c, erunt $\frac{b}{m}$ & $\frac{c}{m}$ partes ipſarum b & c qualibet datâ minores; conſequentiæ ratio ex præcedentibus patet.

7. Pars cujuſcunque quantitatis qualibet datâ minor $\frac{b}{m}$, erit etiam omni aliâ datâ quantitate minor.

Demonſtr. Si neges, ſit deſignata quantitas k minor ipsâ $\frac{b}{m}$, erit mk minor ipsâ b, & m minor ipsâ $\frac{b}{k}$; adeoque numerus quolibet dato major m, minor erit numero ſeu quotiente dato, quod repugnat.

8. Si partes, qualibet datâ minores $\frac{b}{m}$ & $\frac{c}{m}$ ad ſe mutuo addantur, vel a ſe mutuo ſubtrahantur, ſumma $\frac{b+c}{m}$, & differentia $\frac{b-c}{m}$ reſpectivè partes erunt qualibet datâ minores ſummæ vel differentiæ totorum, uti patet.

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9. Multiplicetur $\frac{b}{m}$ per datam quandam r, erit factum $\frac{br}{m}$ rectangulum omni datâ minus, aut rectanguli br pars qualibet datâ minor, juxta Lem. 6.

10. Si pars qualibet datâ minor $\frac{b}{m}$ ducatur in ſe ipſam, vel aliam qualibet datâ minorem $\frac{c}{m}$, erit productum $\frac{bb}{mm}$ ſeu $\frac{bc}{mm}$ æquale nihilo ſeu non quantum.

Demonſtr. Multiplicetur enim quotieſcunque libuerit hoc productum, & ſi placeat per numerum omni dato majorem m, emerget $\frac{bb}{m}$ aut $\frac{bc}{m}$, quæ ſingula ſunt qualibet datâ quantitate minora, (Lemm. 7.) adeoque nullam datam quantitatem, ut ut exiguam, æquare valent, quare juxta Axioma 1. $\frac{bb}{mm}$ ſeu $\frac{bc}{mm}$ ſunt nihilo æqualia.

11. Duæ quantitates $\frac{b}{m}$ & $\frac{c}{m}$, qualibet datâ minores ſunt inter ſe, ut quantitates b & c, quarum partes ſunt, ſic $\frac{b}{m}:\frac{c}{m}::b:c$.

12. Si pars qualibet datâ minor $\frac{b}{m}$ per aliam quâlibet datâ minorem $\frac{c}{m}$ dividatur erit quotiens

tiens $\frac{b}{c}$ idem, quod ex diviſione totorum proportionalium reſultaret.

13. Eadem $\frac{b}{m}$ ſi per determinatam r dividatur, erit quotiens $\frac{b}{rm}$ ipſius $\frac{b}{r}$ pars qualibet datâ minor, juxta Lem. 6.

14. Sit determinata r, & fiat, $r : \frac{b}{m} :: \frac{c}{m} : x$ erit x ſeu $\frac{bc}{rmm}$ juxta Lemma 10. nihilo æqualis.

15. Si vero aſſumta ipſius r parte qualibet datâ minori, fiat $\frac{r}{m} : \frac{b}{m} :: \frac{c}{m} : x$, erit $x = \frac{bc}{rm}$, ſeu pars quâlibet datâ minor ipſius $\frac{bc}{r}$, Lemma 6.

16. $\sqrt{\frac{bb}{mm} + \frac{cc}{mm}}$ eſt æqualis parti qualibet datâ minori ipſius $\sqrt{bb+cc}$, ſeu $\frac{1}{m}\sqrt{bb+cc}$, quod ex fundamentis Algebræ notum eſt.

17. Sit autem determinata r, ac detur ex calculo $\sqrt{rr+\frac{bb}{mm}}$, erit hoc æquale ipſi r.

Cum enim $\frac{bb}{mm}$ juxta Lem. 10, ſit æquale nihilo, erit $\sqrt{rr+\frac{bb}{mm}} = \sqrt{rr} = r$.

18. Sint b & c ita ad ſe mutuo relatæ, ut neutrâ earum exiſtente omni datâ majori, ſi alter-

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utra, vel quantitas ſit, vel in totum evaneſcat, etiam altera ſemper neceſſario vel quantitas ſit, vel nihilo æqualis fiat.

Dico, ſi alterutra, verb. gr. *b*, ſit omni datâ minor, etiam *c* fore qualibet datâ minorem.

Demonſtr. ſi neges, erit, juxta Lemm. 1, *c* vel omni datâ major, quod repugnat aſſumtis, vel data, ac tum *b* etiam erit data contra hypotheſin, vel nihilo æqualis, ac tum *b* quoque talis foret, contra ſuppoſitum.

19. Hinc ſi, *Fig. I*, in cujuſvis trianguli *A E G* quolibet latere *A E* ſtatuatur *A C* quavis datâ minor, erunt, ductâ *K C* ipſi *G E* parallelâ, lineolæ, *K C*, *A K*, ſingulæ quavis datâ minores & vice versâ.

Demonſtr. habent enim *A K* & *C K* ad lineolam *A C* relationem lemm. præced. ſuppoſitam.

20. Eadem quoque relatio obtinet, *Fig. II*, inter Curvæ Arcum *A C D*, eique ſubtenſam *A D*, ac ſinum *A E*: unde omnis arcus quovis dato minor ſubtenditur à cordâ quavis datâ minori, habetque ſinum quovis dato minorem, & contra.

21. Quin & expoſitâ curvâ *A C D*, *Fig. III*, notæ proprietatis verſus eandem partem concavâ, eique ſubtensâ *A D*, ſi ducatur ex puncto *D* recta *D G* faciens cum ſubtensâ *A D* angulum

A D G

A D G vel rectum vel recto majorem; tangat autem curvam in puncto *A* linea *A G*, occurrens rectæ *D G* in *G*, ſitque *H G* rectarum *A G* & *A E* differentia ſeu *A G* — *A E*: Dico, ſi Arcus *A C* (vel juxta præcedentem, ſubtenſa *A C*) ſumatur omni dato minor, etiam *H G*, ſeu A *G* — *A E*, fore omni datâ minorem.

Demonſtr. dato enim arcu *A C*, datur *A E*, & ob datam *A G*, datur quoque *A G* — *A E*, ſeu *H G*. Secundo, evaneſcente arcu *A C*, ob linearum *A E* & *A G*, hoc in caſu, cöincidentiam, adeoque & æqualitatem, & ipſa *H G* evaneſcet, quare, per Lemma 18, conſtat propoſitum.

22. In triangulo *A E G*, *Fig. I*, cujus angulus *E* eſt vel rectus vel obtuſus, ducatur *K C* lateri *G E* parallela, erit ut *A E* ad *A C*, ita differentia linearum *A G* & *A E*, ad differentiam *A K* & A *C*.

Demonſtr. nam ob rectarum *K C* & *G E* paralleliſmum erit *A G* : *A E* : : *A K* : *A C*. unde divid. & altern. *A G* — *A E*: *A K* — *A C* : : *A E* : *A C*.

23. Repetito lemmate 21. ſit, *Fig. III*, *A C* arculus omni dato minor, ac per punctum *C* ducatur recta *K L* ipſi *D G* parallela, productâque ſubtensâ *A C* in *E*, erit differentia tangentis A *K* & ſubtenſæ *A C* æqualis nihilo.

Demonſtr. ſit $AE : r$, & ſubtenſa $AC : \frac{b}{m}$, quæ, juxta Lemma 20. qualibet datâ minor eſt, cum arcus AC talis ſit ex hypotheſi; ſit item $HG : \frac{c}{m}$, juxta Lemm. 21. omni datâ minor; tandem differentia rectarum AK & AC ſeu $AK - AC$ dicatur x; erit ex præcedenti $r . \frac{b}{m} : : \frac{c}{m} : x$, unde x ſeu $\frac{bc}{rmm}$, per Lem. 10. aut 14, nihilo æquatur.

24. Hinc, in argumentatione circa quantitates omni datâ minores, ſubtenſa AC æquatur tangenti AK; cum, juxta præcedentem, nulla ſit earum differentia.

25. Tum & tangens AK, ſubtensâ AC, ac Curva AC inter ſe æquales ſunt.

Cum enim Curva AC ſit major ſubtensâ AC, & minor tangenti AK, uti omnibus notum, erit differentia Curvulæ AC & alterutrius rectæ AK vel AC, non major quam differentia rectarum AK & AC, quæ juxta Lemm. 23, nullla eſt.

26. Adeoque omnis Curva ACD ex rectulis quavis datâ minoribus, quæ tanquam tangentes AK, aut ſubtenſæ AC, conſiderari poſſunt, conſtat.

27. Unde pariter clarum eſt, ſi calculum in-

ingrediatur curvæ cujuſvis particula omni datâ minor, ut A C, poſſe ejus loco ſubſtitui vel ipſius ſubtenſam A C, vel tangentem A K hinc curvulæ reſpondentem; utpote quarum omnium differentia nulla eſt.

28. Punctum P & curva A B D, *Fig. IV*, ita ad ſe invicem referantur, ut ductis P A, P D, ad curvam, omnes dëinceps rectæ a puncto P verſus curvam A B D projectæ ab A verſus D perpetuò creſcant, aut ſaltem non decreſcant; ſi ducatur A G, curvam tangens in A, item P B ipſam A G ſecans in S, & curvam in B; Dico primo, A S fore majorem, quam A B.

Demonſtr. Cum P B ſit major rectâ P A, erit angulus P A B major angulo PBA, (I. 18: Eucl.) ſed angulus P B F (I. 32: Eucl.) major P A B; ergo idem P B F, ſeu ipſi ad verticem oppoſitus A B S multo major ipſo P B A, hoc eſt (I. 32, eucl.) A B S multo major ſummâ angulorum B A S & A S B; ergo in triangulo A B S, erit A B S angulorum maximus; quare, (I: 19. Eucl.) A S major ipsâ A B.

29. Secundo, manentibus iiſdem, ſi, *Fig. IV*, ex B ducatur B L ipſi P G æquidiſtans, dico A L fore majorem, quam A S.

Demonſtr. Angulus A F G (I: 32. Eucl.) major Angulo P B F & huic ad verticem

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oppoſito *A B S*, jam autem, ob parallelas *F G* & *B L*, angulus *A B L* eſt æqualis angulo A *F G*; ergo angulus *A B L* major ipſo *A B S*, unde *A L* major, quam *A S*.

30. Retentâ eâdem hypotheſi, ſi *A B* ſit pars curvæ omni datâ minor, dico differentiam ipſarum *A S* & *A B* eſſe nullam.

Demonſtr. *A L* eſt major quam *A S*, ex præced. unde *A S* — *A B* minor eſt, quam *AL*—*AB*, ſed *A L* — *A B* ex Lem. 23. eſt non quantum, adeoque, quod eo minus eſt, *A S* — *A B* quantum non erit, ergo nullum.

31. Hinc corollaria in modum Lemmatum 24, 25, 26, 27, quilibet ſuo marte eliciet, & tum hic, tum inibi, tradita aliis caſibus, ubi curva verſus punctum *P* convexa eſt, accommodabit.

32. Hoc particulare adjungere liceat; ſi curva *A B D* ſit circumferentia circuli, cujus centrum *P*, ſectorem *A P B* eundem eſſe cum triangulo iſoſceli *A P B*, cujus Baſis ſubtenſa *A B*; eſt enim hæc æqualis arcui *A B*, Lem. 25.

33. Si ſuper baſi, *Fig. V*, quavis datâ minori *A B*, ſeu $\frac{h}{m}$, conſtitutum ſit triangulum iſoſceles *A P B*, cujus latus *A P* ſeu *B P* eſt *p*, erunt ſinguli anguli ad baſin *P A B* & *P B A* recti.

Demonſtr. Ductâ enim normali *P D* erit

D B

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DB: $\frac{\frac{1}{2}b}{m}$, factoque radio: r, & ſinu anguli PBD: y, vocatâque PB: p, erit juxta trigonometriæ regulas, $r:p::y:\frac{py}{r}$ ſeu DP. Jam, ex I.47. Eucl: eſt $\frac{ppyy}{rr} + \frac{\frac{1}{4}bb}{mm} = pp$, rejectoq. per Lem. 10. termino $\frac{\frac{1}{4}bb}{mm}$, erit $\frac{ppyy}{rr} = pp$, ſeu $y = r$, hoc eſt, ſinus anguli PBD æqualis radio, unde angulus PBD rectus. Idem eodem modo de angulo PAD demonſtratur.

34. Sit, *Fig. VI*, triangulum PAB rectum in A, habens latus AB quovis dato minus, dicanturq., PB: p, AB: $\frac{b}{m}$, radius: r.

Erit ſinus anguli APB quovis dato minor. Eſt enim $p:r::\frac{b}{m}:\frac{br}{pm}$, ſeu ſinum anguli APB, quod pariter ex Lem. 18. inferri poſſet.

35. Hujus converſum, expoſito angulo APB omni dato minori, ulteriori probatione non indiget.

36. Iiſdem poſitis, hypothenuſa BP lateri AP æqualis eſt. Cum enim ſinus anguli BPA ſit $\frac{br}{pm}$, erit ſinus complementi illius ſeu anguli ABP $\sqrt{rr - \frac{bbrr}{ppmm}}$, & juxta triangulo-

rum

rum vulgarem dimenſionem, $r : p :: \sqrt{rr - \frac{bbrr}{ppmm}}$: AP, ſeu $\frac{p}{r}\sqrt{rr - \frac{bbrr}{ppmm}}$ ſeu p, per Lemm. 10.

37. Angulus A B P eſt æqualis angulo B A P, & uterque rectus; quod, cæteris perceptis, ſatis patet.

38. In dato quovis triangulo A B C, *Fig. VII*, ſi ex angulo B ad oppoſitum latus A C projecta B D abſcindat D C, portionem quavis datâ minorem, ducaturque arculus circularis, ſeu ipſi æqualis recta D E juxta Lemm. 32, faciens B E æqualem B D; erunt D E & E C quavis datâ minores.

Demonſtr. Habent enim reſpectu D C relationem, Lemm. 18, expreſſam.

39. Iiſdem poſitis, ſi angulus A B D ſit rectus; dico triangula A B D & D E C fore ſimilia; juxta enim Lemm. 33. angulorum B D E & B E D uterque rectus eſt; quare D E C (I. 13. Eucl.) quoque rectus eſt, adeoque æqualis ipſi A B D ex hypotheſi;

Secundo, ob angulum B D E rectum, erit E D C, anguli B D A complementum ad rectum, atqui (juxta I. 32. Eucl.) angulus B A D ejuſdem B D A complementum eſt, unde E D C æqualis B A D, hinc & tertius A D B æqualis D C E, quare conſtat propoſitum.

40. Datis (*F. VIII.*) duabus curvis A B D, E F G, quo-

quovis modo mediantibus rectis EP, AP, FP, BP, ad punctum P relatis, ſi A B ſit curvæ B A D pars omni datâ minor, erit & E F pars curvæ E F G omni datâ minor & contra; habent enim relationem Lemm. 18. expoſitam.

41. Sit recta B N, *Fig. IX*, indefinitè verſus N protenſa, ac ductâ ex puncto B ad angulum N B C recto majorem lineâ B C; s, ex cujus extremitate C projecta C A abſcindat a rectâ B N portionem B A quavis datâ minorem: $\frac{b}{m}$; erunt (juxta 38. Lemm.) D B: $\frac{c}{m}$, tum A D: $\frac{d}{m}$, rectum angulum in D facientes ſingulæ quavis datâ minores; facta jam C E ſeu $\frac{f}{m}$ itidem quavis datâ minori, ductiſque E F ſeu h ipſi D B, & H E ſeu k ipſi A B parallelis, quæritur harum ut & linearum F H: l, & F C: g, & H C: n, natura.

I°. quia C D æqualis C B, juxta Lemm. 36, eſt C B: C D :: C E: C F. Hoc eſt $s : s :: \frac{f}{m} : g$. adeoque g ſeu C F $= \frac{f}{m}$ ſeu C E; unde C F infiniteſima. *II^do*. C B: C A :: C E: C H, ſeu $s : s + \frac{d}{m} :: \frac{f}{m} : n$. unde $s n = \frac{s f}{m} + \frac{d f}{mm}$, & per Lemm. 10. $n = \frac{f}{m}$, unde C H æquatur C E ſeu infiniteſimæ. *III^tio*. C B: B D :: C E: E F, $s : \frac{c}{m} : \frac{f}{m} : h$. ergo E F ſeu $h = \frac{cf}{smm}$ ſeu (juxta Lemm. 10.) nihilo. *IV^to*. C B: B A ::

C E:

CE:EH. $s : \frac{b}{m} :: \frac{f}{m} : k$. ergo HE ſeu $k = \frac{bf}{smm}$ ſeu nihilo, Lemm. 10. *Vto*. CD:DA:: CF:FH; $s : \frac{d}{m} :: g$ ſeu $\frac{f}{m} : l$, unde FH ſeu $l = \frac{df}{smm}$ ſeu nihilo.

42. Quare liquet, retentâ eadem Hypotheſi; lineas, *CE*, *CF*, *CH* ſibi mutuo æquari, & *FE*, *EH*, *HF* nihilo æquipollere, adeoque omnes quantitates poſterioribus hiſce multiplicatas tutò ex calculo algebräico, una cum iiſdem, deleri poſſe.

43. Sint quotlibet antecedentia, *c*, *d*, *f*, *&c*, totidemque conſequentia *g*, *h*, *l*, *&c*; item expoſita ratio in ſingulis conſtans, ut *r* ad *s*, ut ſit, $r : s :: c : g$. tum $r : s :: d : h$. nec non $r : s :: f : l$. & ſic in cæteris; dico fore, ut *r* ad *s*, ita omnia antecedentia $c + d + f$ *&c.* ad omnia conſequentia $g + h + l$, *&c.* Hoc ex elementis notum eſt.

44. Iiſdem poſitis, ſint quantitates determinatæ, hoc eſt, quæ eandem continuo ac invariatam ſervent magnitudinem *r*, *s*, *b*, item aliæ indeterminatæ, *x*, *y*, *z*, *n*, ſive tales, quarum magnitudo pro vario reſpectu, aut poſitione, varia eſt, licet eodem litterali ſigno perpetuò exprimantur, quale ſpecimen omnibus notiſſimum in Algebräicâ locorum doctrinâ incognitæ dictæ quantitates exhibent; ſi fuerit in ſingulis, ut *r* ad *s*, ita quælibet *x* ad ſibi relatum *y*, erit ex præ-

cedenti, ut r ad s ita quotlibet x, ad totidem y; ſeu (quod in poſterum hoc modo enunciabitur) ita omnia x, ad omnia y.

45. Quin & aſſumtâ tertiâ determinatâ quantitate b, ſi fuerit $r: s:: x: y$. Cum ſit quoque, $rb: sb:: x: y$. item $r: s:: bx: by$. tum & $rb: s:: bx: y$, nec non, $r: sb:: x: by$. erit quoque in ſingulis hiſce analogiæ generibus eadem, ac in præcedenti, argumentandi methodus, ex Lem. 43. cum in ſingulis prima proportionis ratio ſit conſtans & invariabilis.

46. Quod ſi aſſumantur quantitates indeterminatæ z & n, fiatque, ut ſupra, $r: s:: x: y$. cum ſit quoque $rz: sz:: x: y$. ac etiam $r: s:: xz: yz$. Item $rz: sz:: xn: yn$: erit, velut in duabus præcedentibus, ut, in primâ analogiæ ratione, antecedens ad conſequens, ita, in ſecundâ, omnia antecedentia ad omnia conſequentia; concluſionis fundamentum ex præcedenti & Lem. 43. manifeſtum eſt.

47. Quin & in omnibus Lem. 45 & 46 caſibus, ſi in primâ proportionis ratione tot ſupponantur termini, quot in ſecundâ, erit, ut omnia antecedentia in primâ, ad omnia conſequentia, ita omnia antecedentia in ſecundâ, ad totidem ſeu omnia conſequentia.

Quod probatione ſpeciali non eget, quippe ex præcedentibus ſufficienter cognitum.

48. Sint

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48. Sint duæ indeterminatæ quantitates x & y, quæ ducantur in determinatam r, item aliam s; erit, ut omnia rx ad totidem ry, ita omnia sx ad totidem sy. Patet, cum omnia x ſint ad omnia y, ut omnia x ad omnia y.

49. Hoc addam, licet ex modo monſtratis facili negotio colligatur, ſi fuerit, $r:s::x:y$. ac, aſſumtâ indeterminatâ z, fiat $rz:s::zx:y$. argumentationem Lemm. 43. & 47. non procedere; ſeu licet per ſe analogia in ſingulis conſtet, non ritè colligi, ut omnia rz ad omnia s. ita omnia zx ad omnia y, cum ratio rz ad s, propter ipſius z magnitudinem variabilem, perpetuo non ſit eadem, quod tamen juxta Lemm. 43. requiritur. Quæ obſervatio tyronibus admonitionis loco inſervire poteſt, cum facilis admodum lapſus ſit in hoc argumentandi genere.

Ex Analyſi aſſumo.

50. *Multiplicationem* poteſtatum ab eâdem radice fieri per exponentium additionem; ut ſi ducendum ſit d^2 in d^3, fore productum d^{2+3}, ſeu d^5; ſi d^p in d^q fore d^{p+q}.

51. Unde ſequitur, ſi poteſtates ſint æquales, hoc eſt $p = q$, fore productum d^{2p}; quod ſi denuo per d^p multiplicetur, fiet d^{3p}, atque ita deinceps.

Quare

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Quare ſi poteſtas p alicujus radicis d, ut d^p, veniat quadratè, cubicè, qquadratè aliiſque modis, multiplicanda, oportet poteſtatis datæ exponentem p per numerum dignitatis quæſitæ multiplicare; ſic quadratum ipſius d^p eſt d^{2p}, cubus d^{3p}, qquad: d^{4p}, adeoque ſi d^p ad poteſtatem q evehi debeat, orietur d^{qp}.

52. Hinc jam ordine retrogado conſtat poteſtatum quarumlibet, per alias ejuſdem radicis poteſtates *diviſionem*, exponentium ſubſtractione peragi; ſic d^5 per d^2 diviſum exhibet quotientem d^{5-2}, ſeu d^3; adeoque d^p per d^q diviſum efficit d^{p-q}; item d^n per r^c ſeu $\frac{d^n}{r^c}$ facit $d^n r^{-c}$.

53. Ipſamque radicum omnigenarum ex poteſtatibus quibuſcunque extractionem fieri diviſione exponentium; ſic radix quadrata ex d^2 eſt $d^{\frac{2}{2}}$ ſeu d; cubica ex d^3 eſt $d^{\frac{3}{3}}$ ſeu d; ex d^6 eſt $d^{\frac{6}{3}}$ ſeu d^2.

Radix qquadr: ex rrd^6 eſt $r^{\frac{2}{4}} d^{\frac{6}{4}}$ ſeu $r^{\frac{1}{2}} d^{\frac{3}{2}}$, quod juxta vulgarem notationem hoc modo deſignatur $\sqrt{rd^3}$; tandem radix poteſtatis q ex d^n extracta erit $d^{\frac{n}{q}}$.

Quæ omnia ex diviſionis, extractioniſque radicum notâ methodo manifeſta ſunt.

Scholium Generale.

CUm nullus exiſtat numerus, qui inter progreſſionis Arithmeticæ: 1, 2, 3, 4, &c. terminos locum non inveniat; ſinguliq: hujus progreſſionis termini generentur ex progreſſionis monadicæ reſpectivis aggregatis; numerum quemlibet, ſub ſpecie ſummæ progreſſionis unitatum, conſiderari poſſe hoc ipſo patet. Unde ſequentes, & alicujus in hiſce momenti, numerorum proprietates emergunt; nimirum.

I. Ubi unitatum hæc progreſſio *ſubſiſtit*, ipſarum aggregatum numerum conſtituit *determinabilem*, ac per monadicorum progreſſionem certo termino ceſſantem denotandum; verb: gr. $2 = 1 + 1$, $3 = 1 + 1 + 1$, &c.

II. Ubi vero *nunquam ſubſiſtendo progreditur*, & in multitudinem terminorum omni determinabili majorem aſſurgit hæc progreſſio, aggregatum unitatum numerum conſtituit *omni dato majorem*: *m*, hac de causâ per progreſſionem monadicorum nunquam deſinentem exprimendum; velut $m = 1 + 1 + 1 + 1 + 1$ *& ſic continuo abſque ceſſatione pergendo.*

III. Certo termino ſubſiſtens monadicorum progreſſio, adeoque numerus determinabilis *aliquot progreſſionis unitates*; nunquam deſinens progreſſio ſeu numerus omni determinabili major *omnes hujus progreſſionis unitates continet.*

IV. Numeri omni dato majoris eſſentialis proprietas, ac ratio formalis in perpetuo accretionis actu conſiſtit, ſubſiſtendo enim numerum determinabilem efficeret (No 1.)

V. Quare, ſi quantitas determinata *b* dividatur per numerum omni dato majorem, ſeu continuo creſcentem, *m*, erit fractio $\frac{b}{m}$ quantitas continuo decreſcens; manente enim numeratore, ac denominatore ſine fine aucto, diminuetur ſine fine fractio.

Unde infiniteſimarum natura clarius illuceſcit, patetque ipſas, non in determinato quodam indiviſibili, ut ut

exiguo, ſed in quantitate, cujus ulterior diminutio actu continuo ſine fine peragitur, conſiſtere.

Quibus perceptis Analyſi infinitorum in Geometria ſine lapſûs periculo utemur; cum nihil hic in continui diviſibilitatem, & incommenſurabilium aſymmetriam, quibus ſcopulis alliſa Indiviſibilium Methodus naufragium paſſura videbatur, commiſſum ſit.

VI. Poſſent & plura, ex progreſſionis infinitæ proprietate, circa numeros infinitos demonſtrari: *Verb: gr:* ſi in monadicorum progreſſione ſinguli termini, ſeu unitates, ducantur in determinatam quandam quantitatem $\frac{pk}{r}$, erit, ſi terminorum quotitas aut multitudo ſit $= m$, ſumma hujus progreſſionis $\frac{pk}{r} m$. In omni enim progreſſione æqualium, terminorum aggregatum quodlibet æquatur termino unico in quotitatem terminorum reſpectivam ducto, quod ſatis notum eſt.

VII. Patet hinc m numerum *ſimpliciter* infinitum (hunc eum voco, qui oritur ex progreſſione infinitis quidem, ſed meris unitatibus conſtanti) per quamlibet quantitatem l multiplicatum aut diviſum, exhibere numerum alius generis infinitum lm aut $\frac{m}{l}$, qui ad primum m determinatam habet rationem, ut l ad unitatem, aut ut unitas ad l.

VIII. Quin & hinc apparet, quodvis infinitum pm, eſſe ſummam progreſſionis æqualium p, cujus terminorum quotitas infinita eſt ſeu $= m$; eſt enim,

$pm = p + p + p + p + p$ *& ſic continuo abſque ceſſatione pergendo.* juxta N. 6.

IX. Nec opus eſt, ſuperioribus intellectis hoc effatum adſtruere: omne ſcilicet infinitum in ſuo genere, analyticè efferri, per quantitatem determinabilem in m ſeu numerum infinitum ductam; ſic, pm, km, ym, dm, &c. ſunt ſingulæ, ſed ſui generis, infinitæ quantitates.

X. Ex quibus conſequitur, fore quantitatem cujuslibet

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bet generis infinitam km ad determinatam r, ut infiniteſima $\frac{s}{m}$ ad nihilum geometricum, & e converſo; eſt enim quarta proportionalis $\frac{sr}{kmm}$, quæ in re geometrica pro non quanto eſt habenda juxta Lemm. 10.

XI. Quare & quicquid per infiniteſimam multiplicatum, per infinitum aliquod dividitur, nihilo æquivalet;

XII. Hinc ſi æquationis expoſitæ terminos quoſdam infinitum aliquod multiplicaverit, aliique ſint ex meris determinabilibus quantitatibus compoſiti; omnes termini infinito vacui ex æquatione poterunt, nihilorum ad inſtar, rejici; ſumto enim m pro numero infinito ſit $myy + ly = pxx + mrx + nnm$.

Dico fore $myy = mrx + nnm$. Unde diviſis per m erit $yy = rx + nn$: rejectis terminis pxx & ly, tanquam infinitum non continentibus.

Demonſtr. Cum enim, exiſtente ut ſupra, $ly + myy = pxx + mrx + mnn$, erit, bis factâ diviſione per m, $\frac{yy}{m}$ $\frac{ly}{mm} = \frac{pxx}{mm} + \frac{rx}{m} + \frac{nn}{m}$, unde, per Lemma 10. rejectis rejiciendis, ac ductis omnibus in m, remanebit æquatio, $yy = rx + nn$, q: e: d.

XIII. Eodem modo probabitur ex qualibet æquatione omnes terminos per infinitum aliquod diviſos, cæteris omni infinito ſive multiplicanti ſive dividenti immunibus, tutò poſſe rejici, totâ enim æquatione per m diviſâ, hi ſoli termini juxta Lemma 10. nihili ad inſtar conſiderari poterunt.

XIV. Tum & terminos per infinitum multiplicatos, rurſumque per infinitum diviſos, pro determinabilibus habendos, eſt enim $\frac{pm}{qm} = \frac{p}{q}$.

Hæc autem præſenti inſtituto ſufficiant.

Cap. I.

De Curvarum tangentibus.

1. Sit curva *ADE*, cujus intercepta *AP* = *x*, applicata in dato quolibet angulo *EP*: *y*, ac ducenda ſit recta *TE* curvam tangens in *E*;

Unde ex datâ curvæ æquatione inter applicatam *y*, interceptamque *x* conſtitutâ, quæritur *TE* ſeu *s*, aut hanc tangentem determinans *PT* ſeu *t*, quam axis portionem *PT*, applicatâ *EP*, & tangente *TE* interceptam, ex *Nob. Hugenii* inſtituto *ſubtangentem* vocabimus.

In linea *AQ* ſumtâ *QP* infiniteſimâ qualicunque $\frac{h}{m}$, erectâque *QD* ipſi *PE* parallelâ, ducatur *HD* æqualis & æquidiſtans ipſi *PQ*, quare triangula *TPE*, & *DHE* erunt ſimilia, & $TP : PE :: DH : EH$ ſeu $t : y :: \frac{h}{m} : \frac{hy}{tm} = EH$.

item $TP : TE :: DH : ED$ ſeu $t : s :: \frac{h}{m} : \frac{hs}{tm} = DE$

Unde patet & ipſas *EH* & *DE* eſſe infiniteſimas, cum per numerum omni dato majorem *m* ſint diviſæ, juxta Lemm. 7. Exponatur jam curvæ quælibet æquatio $2rx - xx = yy$ inter *PE* & *AP* conſtituta, hæc eadem inter interceptam *AQ* ſeu $x - \frac{h}{m}$, & applicatam *DQ* ſeu

$y-\frac{hy}{tm}$ ſtatuatur, ſeu quod eodem redit, in expoſitâ æquatione pro x ponatur $x-\frac{h}{m}$, ac loco y ſubſtituatur $y-\frac{hy}{tm}$ (quod in poſterum vocabo *æquationem ad infiniteſimas redigere*) emerget alia æquatio hac formâ:

$$2rx-\frac{2rh}{m}-xx+\frac{2xh}{m}-\frac{hh}{mm}=yy-\frac{2hyy}{tm}+\frac{hhyy}{ttmm}$$

Jam vero: (per Lemma 10.) deletis $\frac{hh}{mm}$ & $\frac{hhyy}{ttmm}$, utpote nihilo æqualibus, tum & terminis, qui datam æquationem componunt, cum ſibi mutuo æquentur, remanebit: $-\frac{2rh}{m}+\frac{2xh}{m}=-\frac{2hyy}{tm}$, & diviſione facta per $\frac{2h}{m}$, ac mutatis ſignis $t=\frac{yy}{r-x}=TP$, quæ invenienda erat.

Quod ſi non QP, uti mox, ſed in lineâ EP aſſumatur EH infiniteſima $=\frac{k}{m}$ erit ob triangulorum TPE, DHE ſimilitudinem.

$EP:TP::EH:DH$, ſeu $y:t::\frac{k}{m}:\frac{tk}{ym}=DH$.

$EP:ET::EH:ED$ ſeu $y:s::\frac{k}{m}:\frac{sk}{ym}:=DE$.

Unde redactâ æquatione modo expoſitâ ad infiniteſimas, ſeu loco x & y ſubſtitutis AQ & DQ, ſive $x-\frac{tk}{ym}$ & $y-\frac{k}{m}$ orietur æquatio nova

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$2rx - \frac{2rtk}{ym} - xx + \frac{2tkx}{ym} - \frac{ttkk}{yymm} = yy - \frac{2ky}{m}$ $+ \frac{kk}{mm}$. & rejectis (juxta Lem. 10.) rejiciendis, iiſque quæ ſe mutuo ex naturâ æquationis tollunt, fiet $-\frac{2rtk}{ym} + \frac{2tkx}{ym} = -\frac{2ky}{m}$, & reductâ æquatione $t = \frac{yy}{r-x}$, eadem cum modo inventâ.

2. Si vero curvæ proprietas hoc flagitet, poterit & ipſa *DE*, ſeu tangentis aut, juxta Lemm. 25, curvæ ipſius portio, infiniteſimæ loco aſſumi;

Sit, verbi gratia, expoſitæ curvæ hæc relatio, ut, vocatâ curvæ portione *AE*: *z*, ſit continuo $zz = rx + yy$, & deſignanda ſit recta curvam tangens in *E*; quapropter aſſumtâ $DE = \frac{l}{m}$: erit, per modo monſtrata

$TE : EP :: DE : EH$, ſeu, $s : y :: \frac{l}{m} : \frac{ly}{sm} = EH$.

$TE : TP : DE : DH$ ſeu, $s : t :: \frac{l}{m} : \frac{lt}{sm} = DH$.

Unde reductâ hac æquatione ad infiniteſimas, hoc eſt, loco rectarum *AP*, *PE* & curvæ *AE* ad æquationem adhibiris rectis $AQ = x - \frac{lt}{sm}$, & $QD = y - \frac{ly}{sm}$, nec non curva $AD = z - \frac{l}{m}$ orietur æquario.

$$zz - \frac{2lz}{m} + \frac{ll}{mm} = rx - \frac{rlt}{sm} + yy - \frac{2lyy}{sm} + \frac{llyy}{ssmm}$$

Quæ, rejectis per Lemm. 10. rejiciendis, ac terminis ſe mutuo tollentibus, nec non diviſione per $\frac{l}{m}$ inſtitutâ remanebit reductâ æquatione,

$$t = \frac{2zs - 2yy}{r}$$

Ex quâ æquatione ſublatâ s aut t, per æquationem $tt + yy = ss$ (quæ triangulo rectangulo EPT ortum ſuum debet, quod ſi obliquangulum foret, aliam æquationem exhiberet.) Tum & loco ipſius z ſubſtituto valore ex æquatione curvæ datâ $zz = rx + yy$ eliciendo, orietur $ttrr + 4tryy = 4ttrx + 4ttyy + 4rxyy$, & cognita erit ipſa t ex ſolis rectis x & y.

3. In tranſitu noto, curvarum æquationes, quæ alioqui ſine curvæ ſuppoſitâ longitudine exprimi poſſe non videntnr, tangentes nihilo minus exhibere ſolis lineis rectis explicabiles; adeoque, ſi ipſius t aut s valor aliunde per lineas rectas ſignificari poſſet, mechanicarum Carteſii plurimas, ſi non omnes, ad totidem Geometricas hac ratione reducibiles exiſtere.

Hoc ſaltem liquet, modo dictas æquationes, per ſolas rectas naturâ ſuâ deſignabiles eſſe. Æquatio enim curvæ inibi diſcuſſæ zz, $= rx + yy$, quæ ipſam curvam z comprehendit, æqua-

æquationem aliam huic curvæ propriam, ad tangentem determinatam producit; $ttrr + 4tryy = 4ttrx + 4ttyy + 4rxyy$, in quâ cum t ſeu PT ſit recta, nullæ niſi rectæ repe-riuntur; unde denuo ſi t poſſet tolli per aliunde conquiſitam æquationem ſolas rectas continen-tem (quod, cum PT ſeu t ſit recta, impoſſibi-le non erit) emerget curvæ æquatio Geometri-ca, ſed de his alibi.

Scholium I. Notetur velim, in expoſito curvilineo APE liberum eſſe quamlibet ex lineis, DH, HE, ED, pro primâ infiniteſimâ adhibere, quoniam cæteræ ex triangulorum TPE, DHE ſimilitudine, & laterum ana-logiâ ſponte fluunt; tum & primam infiniteſimam $\frac{h}{m}$ aut $\frac{k}{m}$, aut $\frac{l}{m}$ ex ſupponentis arbitrio, in omnibus curvæ punctis vel ſemper ſibi conſtantem ejuſdemque perpe-tuo magnitudinis, (ut ſi h aut k aut l ponantur æqua-les determinatæ cuidam r, unde $\frac{r}{m}$ ſibi perpetuo æqua-bitur.) Vel variatâ continuis incrementis decrementiſ-ve quantitate mutabilem aſſumi poſſe, veluti, ſi k æ-qualis y ſupponatur, aut h ipſi t, unde $\frac{y}{m}$ aut $\frac{t}{m}$ item $\frac{l}{m}$ pro variâ ipſius y aut t aut l magnitudine variabitur.

Adeoque & in genere hinc conſequi, pro tangentis inquiſitione, beneplacito cujusvis relinqui, num appli-catam EP, an interceptam AP quin & ipſam cur-vam ADE, in partes infiniteſimas æquales vel inæqua-les, diviſam eſſe ſupponere velit; niſi certam quan-dam ac definitam infiniteſimarum limitationem, ex cur-

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væ rectarumve naturâ aut aliunde ortam, reſpicere neceſſarium ſit.

Hæc præcedentibus ritè perceptis probatione non egent, alioquin ex Axiom: II. immediate profluentia.

Schol. II. Ne vero continuis fractionum tricis implicitus impediatur calculus, aliâ in poſterum infiniteſimarum nomenclaturâ compendium venabimur, ipſam *QP* ſeu *DH* : *e*, *EH* : *a*. *DE* tangentis aut ſi mavis (juxta Lemm. 25.) curvæ portionem. *u*, appellantes, quod & in ſeqq:, niſi ſecus moneatur, perpetim obtinebit, ubi, ſolâ *y* exceptâ quam ordinatæ deſignationi reſervamus, infiniteſimas quaſcunque per vocales, aut Græcas litteras denotabimus.

4. Sit igitur curvæ cujuslibet naturam, per lineas *AP* ſeu *x* & *PE* ſeu *y* exprimens æquatio ita ordinata, ut omnium terminorum ſumma nihilo æquivaleat: Verb: Gr: $x^3 + yxx + lx - y^3 - yyx - ky - s = 0$. oportet ei tangentem deſignare.

5. Reducatur hæc ad lineas *AQ* & *QD*, quæ infiniteſimas involvunt, hoc eſt, ponatur *AQ* ſeu $x - e$ loco *AP* ſeu *x*, item *QD* ſeu $y - a$ loco *y* ſeu *PE*; conſtitutâque juxta curvæ datam proprietatem æquatione, orientur:

Pro x^3 terminus, $x^3 - 3exx + 3eex - e^3$.

$+yxx$ $\quad yxx - 2eyx + yee$.

$\quad -axx + 2aex - aee$.

$+lx$

$+lx$	$lx-le.$
$-y^3$	$-y^3+3ayy-3aay+a^3.$
$-yyx$	$-yyx+2ayx-aax.$
	$+eyy-2aey+aae.$
$-ky$	$-ky+ka.$
$-s$	$-s.$

6. In ortâ jam hac æquatione primum tollendi veniunt omnes termini, ubi nec *a* nec *e* reperitur; quoniam omnium ſumma nihilo æqualis eſt, quippe datam æquationem conſtituentium. Unde & hoc manifeſtum eſt, nullum æquationis curvæ terminum, ipſo *x* & *y* vacuum, in ortâ æquatione reſervari poſſe.

7. Tum ſinguli deleantur termini, ubi ipſorum *a* aut *e* poteſtas aut rectangulum invenitur, quippe, (per ſchol: 2. §. 3. & Lemm. 10.) pro non quantis aut pro nihilo habendi; unde in æquatione ortâ, ii tantum remanebunt termini, qui vel *a* vel *e*, ſed unius dimenſionis, continent; reſtabitque, deletis delendis, $-3exx-2eyx-axx-le+3ayy+2ayx+eyy+ka=o.$ & æquatione in analogiſmum reſoluta:

$3yy+2yx+k-xx:3xx+2yx+l-yy::e:a.$

8. Jam vero, cum ex conſtructione ſimilia ſint triangula *DHE* & *TPE*, eſt *HD*: *EH*:: *TP*: *PE*. ſeu *e*: *a*:: *t*: *y*.

Quare

Quare & ex §. 7, $3yy + 2yx + k - xx : 3xx + 2yx + l - yy :: t : y$. Unde reducta æquatione obtinebitur æquatio ad tangentem: $3y^3 + 2yyx + ky - xxy = 3xxt + 2yxt + lt - yyt$.

Hinc & in æquatione §. 7, remanenti pro *a* ipſum *y*, pro *e* ipſum *t* reponendum eſſe conſtat; ſimulque Eruditiſſimi *Barovii* methodus in aperto poſita eſt.

9. Notari velim, omnes curvarum æquationes duplici terminorum genere componi; quorum primum (ſecluſis ab hac conſideratione quantitatibus l, k, s, quæ pro cognitis habentur) vel ſolis *x* aut *y*, earumque poteſtatibus conſtat: ut y^3, ky, x^3, lx. quos terminos in poſterum *ſimplices* nominabimus: Alterum autem terminorum genus ex ipſis, *x* in *y* ductis conflatum eſt, ut yxx, yyx, & hos propterea *mixtos* dicemus.

10. Tum & obſervare licet, ex ſingulis æquationis Curvæ terminis ſimplicibus, ſingulos tantum huic ſervientes æquationi terminos oriri.

Cum enim poteſtates ipſarum *y* & *x*, per radices, $y - a$, & $x - e$, juxta §. 5. exprimendæ veniant, poſitâ ipſarum *y* aut *x* poteſtate maximâ, (ut in æquationibus vulgo ordinandis fieri ſolet) pro primo poteſtatis termino, ſolus ſecundus remanebit; primo ſiquidem per §. 6,

re-

reliquiſque poſt ſecundum per §. 7, evaneſcentibus.

11. Quinetiam terminum ex ſimplici ſolum y continenti ortum, eſſe eundem æquationis curvæ terminum, per numerum dimenſionum ipſius y multiplicatum.

Sit enim in æquatione curvæ terminus y^c (poſito c pro poteſtatis ipſius y exponente) ac pro y, juxta §. 5, acceptum $y - a$ ſi ad poteſtatem c evehatur, remanebit, per §. 10, ſolus ſecundus $- c a y^{c-1}$, &, juxta §. 8, y ipſi a ſubſtituto, erit terminus ortus $- c y^c$ ex ſimplici y^c.

12. Item, terminum interceptam x concernentem, ex ſimplici ortum, eſſe eundem æquationis datæ terminum per reſpectivum dimenſionum ipſius x numerum multiplicatum; ita tamen ut poſtea unicâ ipſius x dimenſione multetur, ac eidem t ſufficiatur.

Siquidem in poteſtate, cujus exponens b, à radice $x - e$, ſecundus terminus eſt $- b e x^{b-1}$, &, per §. 8, mutato e in t, erit terminus ortus $- b t x^{b-1}$ ex ſimplici x^b.

13. Signa terminorum in utrâque æquatione, tum curvæ, tum ad tangentem, ſunt eadem, cum enim, §. 10, non niſi ſecundi poteſtatum termini æquationem ad tangentem ingrediantur, iſtique, in radicibus binomiis

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$y - a$, $x - e$, ſigno − copulatis, contrario ſigno ac primi afficiantur, tota æquatio comparebit ſub ſignis æquationi ad curvam contrariis; unde mutatis omnibus, (quod ſalvâ æquatione fieri poſſe tyronibus notum eſt) utraque æquatio hoc in caſu, (de cæteris quilibet facile judicabit) iiſdem ſignis in reſpectivis terminis affecta eſſe videbitur.

14. Hinc regula tangentes ducendi oritur, cum curva æquationem habet terminis ſimplicibus conſtantem, Ex: Gr: data ſit curvæ æquatio, $y^4 - sy^3 + ky - x^4 + lxx - mx + n = 0$ quæ ordinetur, ut omnes y â ſiniſtrâ, omnes x à dextra inveniantur, erit $y^4 - sy^3 + ky = x^4 - lxx + mx - n$, unde, juxta §. 11, 12, 13, ſingulis terminis per ipſarum y & x dimenſionum numerum multiplicatis, ac unicâ x in t mutatâ exſurget æquatio ad tangentem.

$$4y^4 - 3sy^3 + 1ky = 4x^3t - 2lxt + mt.$$

15. Jam vero terminos mixtos conſiderantibus, apparet ex ſingulis terminis mixtis binos pro æquatione ad tangentem terminos oriri; ſit enim terminus mixtus $y^c x^b$, & c ac b poteſtatum ipſarum y & x exponentes, oriuntur ex y^c, juxta, §. 7, terminis $y^c - cay^{c-1}$, item ex x^b termini $x^b - bex^{b-1}$, qui in ſe mutuo

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mutuo ducti, deletis ex §. 6. & 7. delendis, dabunt pro $y^c x^b$ duos terminos $- c a y^{c-1} x^b - b e x^{b-1} y^c$ hoc eſt per §. 8. $- c y^c x^b - b t x^{b-1} y^c$.

16. Hos terminos ita eſſe affectos ſtatim videtur, ac ſi, ordinatâ æquatione, ut §. 14, idem terminus $y^c x^b$ ſub contrario ſigno ab utraque æquationis parte reperiretur; & à parte ſiniſtrâ ſola y, à parte dextrâ ſola x conſiderata fuiſſet; legeſque §. 11. & 12. reſpectivè ſubiiſſent.

Quod etiam contrariis ſignis, ſi ab oppoſitis æquationis partibus ſtatui intelligantur, affecti ſint, patet ex §. 15, ubi uterque terminus ſub eodem ſigno oritur, in reducendâ autem æquatione, ut §. 14, termini t continentes ad oppoſitas partes rediguntur, ac illi, á quibus t exulat, unde & ad contraria ſigna; quod ſatis notum eſt.

17. Quare hinc tandem emergit tangentes hujuſcemodi expedita admodum ducendi methodus, ſive ſimplicibus, ſive mixtis terminis æquatio curvæ conſtiterit.

I°. Ordinetur æquatio curvæ, ut omnes termini ſive ſimplices ſive mixti, in quibus y ſeu applicata reperitur, á ſiniſtrâ conſpiciantur, rejectis, per §. 6, quotquot adfuerint, nec y nec x continentibus terminis.

II°. Om-

II°. Omnes termini, quos x intercepta ingreditur, ſtatuantur ad dextram, adeoque omnes termini mixti y & x ſimul continentes, juxta §. 15. & 16, ab utrâque parte æquationis ſub contrariis ſignis videbuntur.

III°. Singuli ad ſiniſtram poſiti in numerum dimenſionum ipſorum y reſpective ducantur, §. 11. & 16.

IV°. Singuli à dextra per numerum ipſorum x reſpective multiplicentur, ac unicâ ipſius x dimenſione multati ducantur in t, §. 12. & 16.

Et habebitur, ex datâ cujuslibet curvæ æquatione inter applicatam & interceptam conſtitutâ, æquatio ad tangentem determinata, Ex: Gr: ex datâ curvæ æquatione, §. 4, $x^3 + yxx + lx - y^3 - yyx - ky - s = o$, erit juxta N°. I. $y^3 + yyx - yxx + ky$ æquationis pars ſiniſtra, & $x^3 + yxx - yyx + lx$ pars dextra, unde & hac facie apparebit æquatio $y^3 + yyx - yxx + ky \square x^3 + yxx - yyx + lx$, &, per N°. III. & IV, ſingulis x & y in reſpectivum dimenſionum numerum ductis, ac à dextrâ x in t mutato, fiet æquatio ad tangentem: $3y^3 + 2yyx - yxx + ky = 3xxt + 2yxt - yyt + lt$, ut ſupra.

18. Quæ autem de figuris verſus axem concavis demonſtravimus, aliis verſus axem convexis quivis applicabit, ac, ſi facto AQ: x, &

QD:

QD: y, fiat *AP*: $x + e$, & *PE*: $y + a$, eandem redituram æquationem, facile animadvertet.

19. Cum autem termini ſimplices ad paraboloïdes, mixti ad hyperboloïdes, commodè referri poſſe videantur; utriuſque tangentem exempli loco per inventum modo canonem definiemus. Sit paraboloïdes, ejuſve complementum cujus æquatio $y^p = r^{p-q} x^q$; erit $t = \frac{p y^p}{q r^{p-q} x^{q-1}}$

$= \frac{px}{q}$, applicatâ fractione ad curvæ æquationem.

Sit hyperboloïdes cujus æquatio $y^p x^q = s^p r^q$, erit hæc per §. 17, ordinata: $y^p x^q \,[\,]\, - y^p x^q$ & $t = \frac{p y^p x^q}{-q y^p x^{q-1}} = -\frac{px}{q}$. Signum autem negativum hic innuere, ipſam t ab alterâ parte ipſius applicatæ reſpectu x ſumendam eſſe, cuivis notum eſt.

20. Nec terminos tantum rationales, integroſque depoſcit hæc methodus, verum & ſeſe ad quoslibet ſurdos fractoſque extendit.

Sit verb: gratia expoſita curvæ æquatio, $\frac{y^6}{r^3} + y\sqrt{rx^3} + \sqrt{C}\,\frac{r^7 x^4}{yy} = x^3 + r^3$. Oportet ei ſubtangentem deſignare.

Quem in finem, juxta Lemm. 50, &c. hoc modo denotetur æquatio.

$y^6 r^{-3} + y r^{\frac{1}{2}} x^{\frac{3}{2}} + r^{\frac{7}{3}} x^{\frac{4}{3}} y^{-\frac{2}{3}} = x^3 + r^3$. erit æquatio ipſam ſubtangentem t manifeſtans

$$6y^6 r^{-3} + y r^{\frac{1}{2}} x^{\frac{3}{2}} + \frac{3}{2} y r^{\frac{1}{2}} x^{\frac{1}{2}} t - \frac{2}{3} r^{\frac{7}{3}} x^{\frac{4}{3}} y^{-\frac{2}{3}} + \frac{4}{3} r^{\frac{7}{3}} x^{\frac{1}{3}} t y^{-\frac{2}{3}} = 3xxt.$$

ſeu reductâ denotatione ordinariâ

$$\frac{6y^6}{r^3} + y\sqrt{rx^3} + \frac{3}{2} yt\sqrt{rx} - \frac{2}{3}\sqrt{C}\,\frac{r^7x^4}{yy} + \frac{4}{3}t\sqrt{C}\,\frac{r^7x}{yy} = 3xxt.$$

Conſequentiæ ratio, quantitatum ſurdarum fractarumque deſignationem Lemm. modo citatis expoſitam intelligenti, obſcura non erit.

21. Hac datâ occaſione ſequentem obſervatiunculam intertexere liceat. Expoſitâ nimirum quotlibet dimenſionum æquatione ita conſtitutâ, ut unum terminum ex merâ ipſius y, alterumque ex eadem ipſius x poteſtate conſtantes habeat, cum terminis mixtis quotlibet intermediis, in qnibus ſumma dimenſionum ipſorum y & x ſemper maximam ipſius y vel x poteſtatem æquat: dico, quovis etiam numero aut ſigno ſinguli afficiantur termini, perpetuo t fore æqualem ipſi x:

detur enim æquatio, $2y^4 + 3xy^3 - 5xxyy + 7x^3y = 6x^4$ erit per ſuperiora $8y^4 + 9xy^3 - 10xxyy + 7x^3y = 24x^3t + 10yyxt - 3y^3t$

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$-21yxxt$, & applicatâ curvæ æquatione, factoque $8y^4 = 24x^4 + 20xxyy - 12xy^3 - 28x^3y$ erit, repoſito hoc valore in locum ipſius $8y^4$, ac diviſione inſtitutâ, $t = x$. demonſtrationem non addo, levi negotio, poteſtatum ſymbolis generalioribus adhibitis, ex præcedentibus eruendam.

22. Sit jam, *Fig. X*, curva *ADE*, cujus pars *AD* vocetur: z, intercepta *AQ*: x, applicata *DQ*: y. hujuſque proprietatem exprimens æquatio ſit $zzy^3x^4 = r^9 + ppy^5zz$. oportet ei tangentem ducere. Quare poſito pro z ipſo $z+u$, pro y ipſo $y+a$, pro x ipſo $x+e$, redigatur ad infiniteſimas, &, deletis juxta ſuperiora delendis, orietur $4zzy^3x^3e + 3zzx^4yya + 2y^3x^4zu = 5ppzzy^4a + 2ppy^5zu$.

Cumque ob triangulorum *TQD* & *DHE* ſimilitudinem, appellatis tangenti *TD*: s, ſubtangenti *TQ*: t, ſit $u = \frac{se}{t}$ item $a = \frac{ye}{t}$, emerget, hiſce valoribus ipſorum u & a loco in æquatione ſurrogatis, ac diviſione per $\frac{e}{t}$ inſtitutâ, æquatio ad tangentem $4zzy^3x^3t + 3zzy^3x^4 + 2zsy^3x^4 = 2ppy^5zs + 5ppy^5zz$ Potuiſſet & regula §. 17, propoſita ad hujus generis curvas extendi, niſi aliam occaſionem inter ſequentia præſtolari maluiſſemus.

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23. Quomodo etiam *Fig. X.* ex datis curvarum tangentibus earum perpendiculares inveniantur, & vice verſâ, abunde conſtat. Siquidem, retentis §. 1, ſymbolis, ſi fiat $QS:l$, eſt ob angulum TDS rectum $TQ:QD::QD:QS$. ſeu $t:y::y:l$.

Sed & quomodo a priori, non ſuppoſitâ tangentis cognitione inveniantur, breviter oſtendam. Ob angulum TDS rectum triangula DHE & DQS ſunt ſimilia, adeoque $DQ:QS::DH:EH$: ſeu $y:l::e:a$. Unde datâ æquatione curvæ, eaque reductâ ad analogiſmum ut in §. 7, ad inventionem normalium pro e ipſum y, pro a ipſum l reponendum eſſe clarum eſt, videantur de hiſce §. 7 & 8.

24. Et hactenus de ducendis tangentibus perpendicularibuſque, ex datâ curvæ æquatione inter applicatam y & interceptam x conſtituta. Sequentibus autem exemplis, quomodo ex data relatione inter duas diverſarum curvarum applicatas, ac alterutrius tangenti, alterius tangens innoteſcat, exhibebimus. Ubi monuero methodum generaliorem in his & cæteris fere cunctis hujus naturæ problematis, in eo conſiſtere, ut tot numero inveniantur æquationes, quot aſſumtæ fuerint infiniteſimæ; ut, his per reductionem ejectis quæſitum fiat manifeſtum.

25. Sint

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25. Sint *Fig. XI.* duæ curvæ, ADE, quam tangut recta TD; item alia curva ANK ad eundem axem poſita; dataque ſit relatio inter utriuſque applicatas DQ & QN; oportet ducere rectam LNK tangentem curvam ANK in N.

Sit $AQ:x$, $QD:y$, $TQ:t$, infiniteſimæ $QP = DH = NI:e$, $EH:a$, erit $EP = y+a$, $AP = x+e$, (quæ ſymbola in ſequentibus, ubi curva iiſdem majuſculis diſtincta reperitur, ni aliud inſinuetur, perpetuo obtinebunt.)

Sit item $QN:z$, infiniteſima $IK:o$ erit: $PK = z+o$; r ſit linea cognita, LQ ſubtangens quæſita $= k$.

Cum ergo hic aſſumtæ ſint tres numero infiniteſimæ, a, e, o, totidem inveniendæ erunt æquationes, quarum prima: $ye = ta$, quia $t:y::e:a$; ſecunda: $ze = ko$, quia $k:z::e:o$; unde ex utraque, $e = \frac{ta}{y} = \frac{ko}{z}$, ſeu $\frac{tza}{ky} = o$, tertiam ipſa curvarum relatio ſuppeditabit.

26. Sit igitur, in exemplum, $yy - zz = rr$, quâ æquatione ad infiniteſimas reductâ, poſitoque pro y ipſo $y+a$, & loco z ipſo $z+o$, orietur $yy + 2ay + aa - zz - 2oz - oo = rr$, &, rejectis juxta §. 6 & 7, rejiciendis, $ay = oz$, unde $o = \frac{ay}{z} = \frac{tza}{ky}$ ex præcedenti, quare $k = \frac{tzz}{yy} = LQ$.

notetur hoc in caſu, quia $\frac{yv}{t} = \frac{tzz}{k}$, lineas QS, & QF, ſi DS & NF ſint reſpectivis curvis normales, in utrâque eſſe æquales.

27. Sit ex datâ curvarum relatione rectangulum DQN æquale quadrato lineæ r, ſeu $yz = rr$. quæ æquatio per infiniteſimas expreſſa dabit $yz + yo + az + ao = rr$, hoc eſt per §.6, & 7, $o = \frac{-az}{y} = \frac{+tza}{ky}$ juxta §. 25, ſeu $k = -t$. ex quo ſigno negativo apparet k & t ad diverſis applicatæ NQ partibus ſtatui oportere, & curvam ANK invertendam. Quod hic, ne figuras multiplicaremus, nos negleximus; & quivis, ubi caſus hoc requirit, in ſequentibus ex calculo facile ſupplebit.

Notetur, ſi y ponatur ad quamlibet parabolam, ut ſit $y^{p} = r^{p-q} x^{q}$, fore z ad quamlibet hyperbolam, cujus æquatio $z^{p} x^{1} = r^{p+q}$.

28. Detur rurſus curvarum relatio $y^{p} = r^{p-q} z^{1}$, erit æquatione ad infiniteſimas reductâ deletiſque delendis §. 6, & 7, $pay^{p-1} = qr^{p-q} z^{q-1} o$, quæ ducta in yz dabunt $pay^{p}z = qr^{p-q} z^{q} yo$, & diviſis per $y^{p} = r^{p-q} z^{q}$, erit: $\frac{paz}{qy} = o = \frac{tza}{ky}$ juxta §. 25, adeoque $k = \frac{qt}{p}$.

29. Hinc, ſi omnis generis Ciſſöidalium tangentes quis deſideret, fiat curvarum relatio,

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$yz = xx$, quæ juxta præcedentia dabit: $yo + za = 2ex$; fiunt autem ex §.25, $a = \frac{ye}{t}$, $o = \frac{ze}{k}$, quibus in locum o & a ſuffectis oritur $\frac{yze}{k} + \frac{yze}{t} = 2ex$, factoque $yz = xx$, & diviſis per xe, fit $\frac{x}{k} + \frac{x}{t} = 2$. ſeu $tx + kx = 2kt$, aut $\frac{tx}{2t - x} = k$. Unde, cum y poſſit poni ad quamvis curvam, omnium Ciſſoidalium tangentes unâ operâ determinantur, quacunque ex curvâ ortum traxerint.

30. Sit *Fig. XII.* curva $ADEF$, ac ſymbola quæ §. 25, hoc tantum diſcrimine, quod *QO* hic dicatur: x; item alia curva $ANKG$, quam tangat recta NR; voceturq: RL: t; ſintq: KM, LN, AO parallelæ, quæritur ipſius RL longitudo, ſi detur æquatio curvæ AEF $r^p - x^p = y^p$, & utriuſque curvæ relatio $y^s = r^{s-q} z^q$.

Cum ſit primò $RL : LN :: KI : IN$. ſeu $t : x :: o : e$. erit $\frac{xo}{t} = e$. ſecundo conſtitutâ curvæ $ADEF$ æquatione inter OP & PE, ſeu inter $x - e$ & $y + a$, fiet $x^{p-1} e = y^{p-1} a$, quæ ad mox inventam æquationem applicita dabit $t y^{p-1} a = x^p o$. tertio, ſi æquatio mutuam curvarum relationem exprimens eodem modo tractetur, emerget, $s y^{s-1} a = q r^{s-q} z^{q-1} o$,

quæ duæ ultimò conquiſitæ æquationes quæſitam exhibebunt $sy^{s-1}x^p = tqr^{s-q}z^{q-1}y^{p-1}$. Quod ſi fiat $q = 1$, & æquatio hæc per datas curvarum æquationes diviſionis ope debitè reducatur, fiet $t = sy^{s-p}r^{p+1-s} - sz$; & facto $p+1 = s$ erit $t = sy - sz$.

NB. ſi s ipſo $p+1$ majus ſupponatur, quantitas sy^{s-p} per r^{s-p-1} dividenda erit, quod ex Lemm. 50, &c. ſatis cognitum eſt.

31. Sit $ADEO$ ſemicirculus, *Fig. XIII.* N°. 1. cujus diameter $AO:r$; curva vero AKG ſit Ciſſöis, quam tangat recta NR, quæritur RL ſeu t.

Sint eadem, quæ §. 25, ſymbola, ac $QO = r - x$ vocetur l, erit $AP = x + e$ & $PO = l - e$; jam primo $RL : LN :: KI : IN$. ſeu $t : l :: o : e$, unde $o = \frac{te}{l}$, & cum ex proprietate circuli ſit $lx = yy$, ſi loco x ipſum $x+e$, loco l ipſum $l-e$, loco y ipſum $y+a$ ſufficiatur, emerget $le - xe = 2ay$. tertiò æquatio Ciſſöidis $xx = zy$ dabit $2ex = yo + za$. tandem æquationibus $lx = yy$ & $zy = xx$ ad x reductis orietur, $lz = xy$. ex quibus t hoc pacto inveſtigatur, quia $o = \frac{te}{l} = \frac{2xe - za}{y}$ erit $tye = 2lxe - lza$, unde, factis ut ſupra $lx = yy$ & $lz = xy$, diviſiſque omnibus per y, fiet $a = \frac{2ye - te}{x} = \frac{le - xe}{2y}$ juxta

mox

mox inventa, quâ æquatione ordinatâ, & ſubſtituto loco xx ipſo zy, ac pro lx ipſo yy emerget $t = \frac{2}{3}y + \frac{1}{2}z = LR$. quæ conſtructionem omnium, niſi fallor, expeditiſſimam admittit, ſumendo nimirum, RL æqualem $\frac{2}{3}DQ + \frac{1}{2}QN$.

32. Hoc etiam fonte manat ſingularis ac utiliſſima methodus tangentes ducendi, quæ nec quantitates ſurdas quomodolibet compoſitas, nec fractiones moratur.

Ex: Gr: ſit *Fig. XI.* curva ADE, cujus intercepta $AQ: x$, applicata $DQ: y$, & æquatio hujus proprietatem exprimens $\sqrt{C\,rrx+y^3} - \sqrt{rr+xx} = y$. & huic tangens ſit deſignanda.

Quem in finem quælibet quantitas ſurda ſtatuatur æqualis alteri cuidam aſſumtæ, fiatque $\sqrt{C\,rrx+y^3} = f$ & $\sqrt{rr+xx} = z$, quævocentur idcirco *æquationes aſſumtæ*, concipianturque totidem curvæ, ACG, ANK, ad eundem axem AQ eſſe deſcriptæ, quarum applicatæ ſint aſſumtis quantitatibus æquales; nimirum $CQ = f$, & $QN = z$; unde æquatio curvæ expoſitæ ADE erit $f - z = y$, quæ vocetur *æquatio ex aſſumtis orta*. Tandem cuilibet indeterminatæ ſua tribuatur infiniteſima, ponendo loco $AQ: x$, ipſam $AP: x + e$; loco $DQ: y$, ipſam $EP: y + a$; loco $CQ: f$, ipſam $GP: f + o$; denique loco $QN: z$, ipſam $PK: z + i$;

exiſtentibus, QP, EH, GM, IK, ſeu e, a, o, i infiniteſimis. Emergetque, juxta ſuperius tradita, æquatio ad infiniteſimas reducta, $f + o - z - i = y + a$, & deletis delendis, ex §. 6, $o - i = a$.

Ex qua ſublatis infiniteſimis invenietur curvæ expoſitæ ſubtangens TQ ſeu t.

Hoc autem duplici modo effectum dari poteſt, quorum uterque examinari quodammodo meretur.

Primus modus in eo conſiſtit, ut, præter æquationem $ta = ye$ expoſitæ curvæ ordinariam, æquationes aſſumtæ omnes ad infiniteſimas redigantur; & ope harum infiniteſimæ ex æquatione ex aſſumtis ortâ, ad infiniteſimas tamen prius reductâ, auferantur.

Sic aſſumta æquatio prima fuit $rrx + y^3 = f^3$ quæ ad infiniteſimas redacta exhibebit $o = \frac{rre + 3yya}{3ff}$.

Secunda aſſumta æquatio, $rr + xx = zz$, dabit ſimili modo tractata $i = \frac{xe}{z}$.

Quibus valoribus in æquatione $o - i = a$ repoſitis, orietur $\frac{rrze - 3ffxe}{3ffz - 3yyz} = a = \frac{ye}{t}$ ex æquatione modo inventâ, unde emergit quæſita ſubtangens $t = \frac{3ffzy - 3y^3z}{rrz - 3ffx}$.

Se-

Secundus hoc idem efficiendi *modus* fit per aſſumtarum curvarum ACG & ANK ſubtangentes RQ & LQ, quæ ſingulæ ſuas æquationes, pro tollendis ex æquatione ex aſſumtis ortâ infiniteſimis, conferunt.

Cum enim ſit in curvâ ADE, $t:y::e:a$, ſeu $ye = ta$; erit eadem ratione, poſitâ $RQ = k$, in curvâ ACG, $RQ:CQ::CM:MG$ ſeu $k:f::e:o$, unde $fe = ko$; nec non in curvâ ANK, poſitâ $LQ = h$, erit, $LQ:QN::NI:IK$ ſeu $h:z::e:i$, unde $ze = hi$.

Hinc invenitur, $a = \frac{ye}{t}$, & $o = \frac{fe}{k}$, nec non $i = \frac{ze}{h}$, quibus jam valoribus loco, i, &, o, & a, in æquatione $o - i = a$ ſubſtitutis, emerget $\frac{fe}{k} - \frac{ze}{h} = \frac{ye}{t}$; adeoque poſt debitam reductionem $t = \frac{ykh}{fh - zk}$.

Quin & circa utrumque hunc modum obſervatu dignum erit, ſecundum, uti primo magis compendioſum, ita & generaliorem eſſe: cum enim primus, per ipſas curvarum proprietates ſuâ naturâ determinatus, non niſi curvæ ADE inſerviat; ſecundus infinitarum curvarum, pro variâ ipſarum y, f, z, ad x relatione caſus comprehendit, levi negotio ad quamlibet curvam particularem determinabilis, ſi loco ſubtangentium

tium h & k, valores juxta primum, quem dedimus, modum inventi, ſufficiantur; hos enim valores ex datâ curvarum, quas reſpiciunt, proprietate etiam datas eſſe, præcedentia intelligenti ſatis manifeſtum erit.

33. *Ex: II.* Aſſumtâ eâdem *Fig. XI.* vocetur AD curva perpetuo c, cujus infiniteſima DE ſit u; dicaturque ipſius tangens TD: s; & manentibus iiſdem, quæ in præcedenti exemplo, ſymbolis, æquatio illius relationem exprimens ſit. $\sqrt{C\frac{r^4c^4}{yy+rx}} + \sqrt{\frac{rrc^4+yyx^4}{cc+xx}} = yy+rc$ oportet ei tangentem deſignare. Cum vero hic fractiones occurrant & termini inſuper irrationales, varii ſunt modi, quibus intentum aſſequi valemus, juxta varias quantitatum aſſumtiones. Sic aſſumto integro termino $\sqrt{C\frac{r^4c^4}{yy+rx}} = ff$, item alio $\sqrt{\frac{rrc^4+yyx^4}{cc+xx}} = zz$, erit æquatio ex aſſumtis orta $ff+zz=yy+rc$; quæ ordinata, ut ſupra, ad curvas reſpectivas, & reducta ad infiniteſimas dabit $2fo+2zi=2ya+ru$. Unde ſublatis per curvarum proprietates infiniteſimis poſitoque $u=\frac{se}{t}$ (eſt enim $t:s::e:u$) erit factâ reductione debitâ $t=\frac{2yykh+rskh}{2ffh+2zzk}$.

Quod ſi aliæ quantitates terminorum irrationa-

nalium fractorumque loco aſſumantur, idem, ſed aliâ ratione, obtinebitur quæſitæ ſubtangentis valor. Sic conſiderari poſſunt primo fractionum denominatores, aſſumique, $ff = yy + rx$, & $zz = cc + xx$. Deinde numeratoribus hac lege aſſumtis, ut per denominatores ſuos diviſionem, ac per ſigna radicalia præfixa extractionem pati capaces ſint, alia æquatio ex aſſumtis orta emerget.

Verb: Gr: ſi fiat $r^4 c^4 = ffr^3 l^3$, & $rrc^4 + yyx^4 = zzrrgg$; quinimo ſi viſum fuerit, & quantitas rationalis yy poſſet aſſumi $= rd$; erit æquatio ex aſſumtis orta $rl + rg = rc + rd$, ſeu, $l + g = c + d$. quem in finem, cum ſola c, (item x & y, ſi adeſſent) ad curvam expoſitam ADE pertineat, ex cæterarum l, g, d, proprietate tot curvæ ACG, ANK, AXZ, ad eundem axem AQ deſcriptæ ſupponantur, ita, ut CQ ſit $= l$, cujus infiniteſima $GM = o$, ſubtangens $QR = k$; tum QN & QX exiſtentibus $= d$ & g, quarum reſpectivæ infiniteſimæ ſint IK & OZ, ſeu, i & λ, ac ſubtangentes LQ & BQ relative h & n; emerget, ſi æquatio ex aſſumtis orta ad reſpectivas reducatur infiniteſimas $o + \lambda = u + i$.

Cum autem, juxta præcedentia, ſit in curvis reſpectivis $o = \frac{le}{k}$, item $i = \frac{de}{h}$, nec non $a = \frac{ye}{t}$, ut & $u = \frac{se}{st}$, ac tandem $\lambda = \frac{ge}{n}$; his in locum infiniteſimarum ſuffectis, & reductione inſtitutâ fiet $t = \frac{skhn}{lhn - dkn + gkh}$.

Quin

Quin & caſus non infrequentes ſolent occurrere, ubi curvæ proprietas inter ipſas x, aut y, (licet ſurdis quibuſdam quantitatibus fractionibuſque implicitas) non conſiſtit; ſed per plurimas indeterminatas, æquationeſque ex illis promanantes, cognita eſt; quæ tamen omnes, ſi modo vulgari tangens inquiratur, ad unicam æquationem reducendæ ſunt; hac autem methodo tractatæ levi operâ abſque reductione intentum produnt.

Sit in exemplum *Fig. XIII.* N°. 2. Conchoïs *ADE*, polo *K* ac normâ *FG* deſcripta, cujus applicata *QD* ſit: y; intercepta *AQ*: x; $AF = DL: r$; $FK: s$; erit $QF = r - x$, quæ vocetur h; item $QK = s + r - x$, quæ dicatur f; KD ſit z, ac ſubtangens TQ: t.

Emergent pro naturâ Conchöidis explicandâ ſequentes æquationes I, $r - x = h$. II, $s + r - x = f$. III, $fr = hz$, eſt enim $QF. h : DL. r :: QK. f : DK. z$. IV, $yy + ff = zz$, ob triangulum *KQD* rectangulum in Q.

Ex hiſce jam æquationibus, abſque reductione ulteriori, quæritur tangens expoſitæ Conchöidis.

Quare, exceptis quantitatibus determinatis r & s, ſupponantur reliquæ, quarum magnitudo pro variâ ſui ad Conchöidem poſitione varia eſt, per reſpectivas infiniteſimas creſcere, *QD*

ſeu

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ſeu y quidem per infiniteſimam a; AQ ſeu x per e; QF ſeu h per u; QK ſeu f per o; & DK ſeu z per i.

Concipianturque ex ſupra inventis æquationibus tot curvæ, quarum applicatæ ſint h, f, z, deſcriptæ eſſe ad axem Conchöidis AF, quot æquationum inibi repertarum multitudo requirit, (quas tamen curvas ex inſtituto omiſimus, ut pateat hanc quidem methodum curvarum iſtarum deſcriptionem pro demonſtratione ſui agnoſcere, in ipſâ autem problematis ſolutione eas abſolute neceſſarias non eſſe) ac redigantur æquationes ad infiniteſimas:

erit ex I, $r - x - e = h + u$, unde $u = -e$.
ex II, $s + r - x - e = f + o$, unde $o = -e$.
ex III, $rf + ro = hz + hi + zu$, unde $ro = hi + zu$.
ex IV. $yy + 2ay + ff + 2fo = zz + 2zi$, unde $ya + fo = zi$.

jam, loco u & o ponendo $-e$, in III. & IV. fiet $\frac{ze - re}{h} = i = \frac{ya - fe}{z}$, quare $a = \frac{zze - rze + hfe}{hy} = \frac{ye}{t}$, eſt enim ex curvâ $TQ.t : QD.y :: DH.e : HE.a$, unde $t = \frac{hyy}{zz - rz + hf}$.

Notetur hinc conſequi compendioſam illam conſtructionem à *Carteſio lib. II. Geom*: inſinuatam, eſt enim Conchöidi in D normalem recta in axe determinans $= \frac{yy}{t} = \frac{zz - rz}{t} + f$.

Obſervari meretur, anxiè inquiri opus non eſſe, num indeterminatæ per infiniteſimas creſcant, an vero decreſcant, cum ipſa operatio ex curvæ naturâ cognitâ, in reliquis hoc ſuapte ſponte patefaciat. Quantum etiam hæc tangentes ducendi ratio uſum, & in calculo algebraïco compendium præſtet, imprimis circa maximi minimique determinationem, aliaque difficiliora matheſeos problemata, rerum peritis ſatis perſpectum eſt.

35. Quapropter, ut calculum hunc, licet præ aliis ſatis expeditum, adhuc quantum fieri poteſt contrahamus, ſequentem ſubjicimus canonem in omnibus, quos hactenus recenſuimus, caſibus utilem. Ubi monuero me, ut alibi, ita & hic, *indeterminatarum* nomine tales quantitates denotare, quæ pro variâ poſitione magnitudinem variant, quales ſunt curvarum omnium interceptæ & applicatæ, quin & ipſæ curvæ ad eas relativæ.

I. Singuli (iis, qui meris quantitatibus determinatis conſtant, rejectis) termini indeterminatas quantitates continentes, per exponentes poteſtatis ſingularum indeterminatarum multiplicentur, unamquamque ſcilicet indeterminatam ſeorſim conſiderando. Unde ſinguli termini æquationis curvæ ſecundum indeterminatarum, quas continent, numerum

in

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in æquatione ad tangentem repetendi erunt, quo facto illi, qui unam indeterminatam continent, ſemel; qui duas, bis; qui tres, ter concurrent ad æquationem tangentis inquiſitioni ſervientem efformandam, ſic terminus ffz^3y propter tres indeterminatas, f, z, y, tres æquationi ad tangentem terminos ſuppeditabit, $2ffz^3y$ ubi ſola f, $3ffz^3y$ ubi ſola z, & ffz^3y ubi ſola y conſideratur, & ſic in cæteris.

II. Ubi indeterminata *applicatam* denotans conſideratur, per omnes aliarum curvarum ſubtangentes, propriam ſubtangentem omittendo, multiplicetur terminus.

III. Ubi indeterminata *curvam* ſignificans conſideratur; unicâ per diviſionem demptâ, eiq; tangenti hujus curvæ ſubſtitutâ, & hic per omnes aliarum curvarum ſubtangentes, propriam ſubtangentem omittendo, multiplicetur terminus.

IV. Ubi indeterminata *axem* ſeu *interceptam* referens conſideratur, unicâ demptâ diviſionis ope per omnes omnium curvarum ſubtangentes, nullam ſubtangentem omittendo; multiplicetur terminus.

V. Circa ſignorum notationem obſervetur, terminos omnes ortos his in caſibus iiſdem ſignis inſigniendos eſſe, quibus termini, unde origi-

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nem duxerunt, affecti ſunt; modo ab eâdem æquationis parte poſiti maneant, & poteſtates indeterminatarum, quæ conſiderantur, ſint affirmativæ ſeu ſigno + notatæ; ſecus contrarium eveniet.

Quo peracto habebitur æquatio, pro cujuslibet curvæ æquationem datam, ſeu per ſe, ſeu per lineas ſibi proprias ingredientis, tangenti inveſtigandâ.

Demonſtrationem, ut ex præcedentibus, ita ex ſequenti exemplo quilibet hauriet.

Scholium. Licet enim hic indeterminatæ continuò per infiniteſimas ſigno + affectas creſcere ſupponantur, quod tamen in multis occaſionibus alio modo contingit, nihil tamen officiet; cum æquationem hanc generalem ad unicam, cujus tangens quæritur, curvam reſtringendo, ſigna in ordinem ſuum ſint reditura. Nec in algebrâ novum eſt, æquationem, cujus omnes termini ſigno + affecti ſunt, nihilo æqualem ſtatuere, juxtaque eam, uti normam, aliarum æquationum, quarum ſigna variant, proprietates inveſtigare, cujus demonſtrationem, cum ex ſequentibus ſatis apparitura ſit, hic pleniori diſcurſu, brevitati ſtudentes non firmabimus.

36. Sit *Fig. XI.* curvæ *ADE*, *ACG*, *ANK*, circa eundem axem *AQ* deſcriptæ, vocenturque curva *AD*: *c*. *DQ* applicata: *y*, item curva *AC*: *d*. *QC* applicata: *f*. tum & *QN*: *z*. & communis omnium curvarum intercepta *AQ*: *x*. ac detur æquatio.

$$ddy^3x + ccf^3z = r^6 + ffz^3y.$$

Quæ-

Quæritur, per canonem modo traditum, curvæ ex hiſce tribus pro arbitrio ſelectæ tangens:

Sit eum in finem curvæ *ADE*, tangens *TD*: *s*, ſubtangens *TQ*: *t*. deinde curvæ *ACG*, tangens *RC*: *h*, ſubtangens *RQ*: *k*. tandem curvæ *ANK*, ſubtangens *LQ*: *n*.

Emerget ſingulis terminis juxta indeterminatarum, quas comprehendunt, numerum ſeparatim tractatis

$ddy^3nkt + 3ddy^3xnk + 2dhy^3xnt$ loco ddy^3x,
$+ ccf^3zkt + 3ccf^3ztn + 2csf^3zkn$ loco ccf^3z,
$+ ffz^3ykn + 3ffz^3ykt + 2ffz^3ytn$ loco ffz^3y,
adeoque, rejectâ quantitate r^6 plane cognitâ, æquatio

$ddy^3nkt + 3ddy^3xnk + 2dhy^3xnt + ccf^3$
$zkt + 3ccf^3ztn + 2csf^3zkn = ffz^3ykn$
$3ffz^3ykt + 2ffz^3ytn$

ad quamlibet ex ſubtangentibus *t*, aut *k*, aut *n*, determinabilis.

Quod ſi hæc ex methodo generali §. 32, & 33, expoſitâ inquiratur: indeterminatæ omnes per reſpectivas infiniteſimas creſcere ſupponantur, hoc eſt, in æquatione ad infiniteſimas redigendâ, quod hic requiritur, loco, x, y, c, f, d, z, reſpectivè ſufficiantur $x + e$, $y + a$, $c + u$, $f + o$, $d + i$, $z + \lambda$. & fiet ſublatis juxta §. 6 & 7. tollendis
$^3dde + 3yyddxa + 2dy^3xi + 2cf^3zu +$

$3ccffzo + ccf^3\lambda = 2fz^3yo + 3ffzzy\lambda + ffz^3a.$

Unde articulus hujus regulæ primus demonſtrationem ſuam petit.

Porro cum ex curvarum naturâ ſic $a = \frac{ye}{t}$, $u = \frac{se}{t}$, $o = \frac{fe}{k}$, $i = \frac{he}{k}$, & $\lambda = \frac{ze}{n}$, hiſce in infiniteſimarum locum ſurrogatis orietur æquatio, ubi ſinguli termini (ſolis illis, in quibus *e* reperitur exceptis) per ejus curvæ ſubtangentem diviſi erunt, ad quam aut cujus applicatam infiniteſima referri debet: iſtique, quos infiniteſimæ *i* & *u* ingrediuntur, per curvarum hiſce infiniteſimis creſcentium tangentes, unicâ curvarum poteſtate diminuti, multiplicati apparebunt.

Unde, æquationem reducendo, ipſâ operatione patebit ſequentium hujus canonis articulorum demonſtratio.

Quam obiter tyronibus indigitaſſe, ne tædium pariam, hic ſuffecerit; exercitatis alioquin ſatis obviam.

37. Sit *Fig.XI.* perſtantibus iiſdem, $fz = yy$, hoc eſt, curva *A D E* ejus naturæ, ut, ductis duabus curvis *A C G* & *A N K* ad eundem axem, ſit perpetuo rectangulum *N Q C* æquale quadrato in *D Q.* oportet uni ex his tangentem deſignare. Quare, adhibitâ §. 35. traditâ methodo, erit, retentis iiſdem ſubtangentium ſymbolis, æqua-

æquatio generalis ad tangentem, $fznt + fztk = 2yykn$, & facto $yy = fz$, ac diviſione inſtitutâ, $nt + tk = 2kn$.

Unde, ex datis duarum quarumlibet curvarum ſubtangentibus, ſubtangens tertiæ manifeſtatur.

Coroll. I. Notetur hanc, ut & conſimiles æquationes ad infinitas curvas determinari poſſe, earumque omnium tangentes hac ratione una opera definiri. Verb: Grat: ſit, f ad triangulum ſeu, $f = x$ erit $k = f = x$, & ſit z ad trianguli complementum ſeu $z = 2r - x$, erit $n = -z = x - 2r$, adeoque y ad circulum, ſeu $yy = 2rx - xx$; cujus ergo ſubtangens $t = \frac{2kn}{n+k}$ $= \frac{2xx - 4rx}{2x - 2r}$ ſeu $\frac{2rx - xx}{r - x}$ ut ſupra.

Coroll. II. Sit yy ad hyperbolam, ſeu $yy = rx + \frac{r}{q}xx$. poteſt ſumi f ad triangulum, & fieri $f = x$; unde, uti mox, $k = f = x$. & remanebit $z = r + \frac{r}{q}x$ ad aliud triangulum, unde $n = \frac{qz}{r} = q + x$; adeoque hyperbolæ ſubtangens $t = \frac{2qx + 2xx}{q + 2x}$.

Coroll. III. Quin & hoc generaliter obtinet; datis tribus curvis, quarum applicatæ ſint in continua proportione, ut ſit $f : y :: y : z$. erit ut ſumma ſubtangentium extremarum, $n + k$, ad alterutrius ſubtangentem k, ita ſubtangens reliquæ, n, ad ſemiſſem ſubtangentis mediæ $\frac{1}{2}t$.

Co-

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Coroll. IV. Sit z ad circulum, & $zz = 2rx - xx$, item f ad hyperbolam æquilateram & $ff = 2rx + xx$, erit $y^4 = 4rrxx \quad x^4$; cujus ergo tangens ex circuli & hyperbolæ tangentibus cognita eſt; neque novit hujus methodi extenſio limitem.

Coroll. V. Hoc tamen inter cætera notetur; cum omnes tangentium methodi in determinatâ alicujus curvæ natura ſubſtiterint, ejuſque parametros determinatas depoſcant, hanc ſolam parametrorum limitationem non reſpicere. Verb. Grat. ſit $z = x$, erit $fx = yy$; quæ æquatio, ſi f foret determinata $= r$, eſſet ad parabolam; ipſâ autem f manente indeterminatâ, erit ad curvam ex infinitis parabolis formatam, quarumque parametri ſunt curvæ pro arbitrio ſumendæ ſe mutuo ſequentes applicatæ; cujuſque æquatio ad tangentem, cum hoc in caſu ſit $n = x$, per modo inventa, erit $tx + tk = 2kx$. quæ æquatio, ſi ad parabolam ordinariam limitari debeat, fiet $z = r$, unde k eſt infinita; quare, per Schol: Gener: Lemm. N. XII. rejectâ tx, erit $tk = 2kx$, & $t = 2x$, uti oportet.

§. 38. Antequam ad alia tranſeam, hoc addo. Sit iiſdem manentibus $zfx + zxx = fyy$, quæ æquatio, præter infinitas alîus generis curvas, etiam eſt ad infinitas hyperbolas, ſeu cur-

vam

vam ex iis formatam. Quæritur ipſius tangens.

Hæc autem juxta præcedentem regulam quæſita dabit æquationem

$$zftnk+zfxnt+zfxkt+2zxtnk+zxxkt$$
$$= 2fyykn+fyynt.$$

quæ, pro variâ ipſarum f & z ad varias curvas determinatione, variarum, imo infinitarum curvarum tangentibus inveniendis ſola inſervire poteſt. Quod præcedentis paragraphi corollaria ſatis faciunt manifeſtum.

39. Quo vero modo generalis hæc æquatio rurſum ad hyperbolam vulgarem limitetur, oſtendiſſe operæ fuerit pretium; cum hic non ſine cautelâ à generali ad particulare regreſſus detur.

Fiat ergo $f = q$, & $z = r$; aut ſi mavis, utraque, cum ad omnes lineas poſſit, ad parallelogrammi rectanguli applicatam determinetur; erunt eo ipſo k & n infinitæ, (cum rectanguli ſubtangens ſit infinita, quod facili negotio demonſtrabile & omnibus notum eſt) dividatur jam tota æquatio per infinitam k, erit $zftn$

$$+\frac{zfxtn}{k}+zfxt+2zxtn+zxxt = 2fyyn$$
$$+\frac{fyynt}{k}.$$

Jam vero, ex Schol: Gener: Lemm. N. XIV, termini $\frac{zfxtn}{k}$ & $\frac{fyynt}{k}$ per infinitam n multipli-

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çati, ac rurſum per infinitam k diviſi fiunt hoc ipſo determinabiles; unde, per ejuſdem Schol. N. XII, cum omnibus cæteris terminis determinabilibus $zfxt$, $zxxt$, rejecti æquationem hanc reliquam faciunt: $zftn + 2zxtn = 2fyyn$, aut repoſitis pro f & z valoribus q & r, ac reductâ æquatione, $qrt + 2rxt = 2qyy$. quæ æquatio eſt ad tangentem hyperbolæ Apollonianæ, cujus æquatio: $qrx + rxx = qyy$.

Poſſet & hæc regreſſio alio modo fieri, dividendo omnes terminos per utramque infinitam kn. unde ex ejuſdem Scholii General: præceptis idem proveniret.

Coroll. Hinc *maximi* ac *minimi* jam plurimis uſitata methodus demonſtrationem ſuam haurit; tum & altiorum quoque graduum curvarum in plano deſcriptiones facillimo forſan modo deducuntur; quæ, cum hujus non ſint loci, uti & plurima alia hinc originem ducentia, latius hoc tractatu non proſequemur.

40. Hoc addam, methodum huic conſequi, quâ ſi determinatæ cujuſdam curvæ, Verb: Gr: Ciſſöidis tangens expetatur, poſſit hac ratione æquatio exhiberi, non modo Ciſſöidis ex circulo aliisve curvis infinitis oriundæ, verum & aliarum inſimul curvarum, ac ſatis primâ fronte à Ciſſöide diſſentiri apparentium tangentes unâ

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unâ operâ determinans. Sit ergo, *Fig. XI*, curva *ANK* Ciſſöis ex circulo *ADE* ortum ducens; ac vocatis radio *r*, *AQ*: *x*. *DQ*: *y*, *QN*: *f*. ipſius æquatio $x^3 = 2rff - xff$. fiatque $2r - x = z$, cui æqualis ſtatuatur curvæ *ACG* applicata *CQ*, erit $x^3 = zff$. ac poſita *x* ad aliam curvam, cujus applicata $QX = d$, erit $dxx = zff$, aut $ddx = zff$, aut $d^3 = zff$ pro libitu; unde tangentem inveniendo per ſuperiora, & Ciſſöidis & infinitarum præterea curvarum tangentes habebuntur.

Potuiſſet & ipſa Ciſſöidis æquatio per aliam indeterminatam dividi, aut multiplicari, Verb: Gr: per *x*; ac foret, $x^4 = 2rxff - xxff$; ubi, factis $2rx - xx = yy$, fieret $xx = fy$. ac, ipſam *x* ad applicatam *d* referendo, $dx = fy$, aut $dd = fy$ unde idem conſequeretur.

Hæc autem mille modis variari poſſe jam ſufficienter probatum autumo.

Scholium. Cunctis fere Mathematicis hactenus in more fuit poſitum curvarum proprietates æquationibus comprehendere; in iiſque quantitates, quæ *parametrorum* nomine venire vulgo ſolent, in omnibus curvæ punctis ejuſdem ac invariatæ magnitudinis conſiderare; uti & pro immutabilibus habere quarumlibet quantitatum æquationem ingredientium, poteſtatum exponentes; cæteris, quæ indeterminatæ audiunt, & quantitatem mutant, reſiduam ſolummodo terminorum æquationiſque partem conſtituentibus; has autem curvas hujus capitis initio diſcuſſimus.

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Alia vero curvarum genera præcedentibus proximè paragraphis expendimus, ubi nimirum & ipſæ parametri indeterminatæ, ac in ſingulis curvarum punctis ex loci, ad quem pertinere ſupponuntur, lege variæ fuerunt; manentibus interim poteſtatum indicibus determinatis ac conſtantibus. Quare & curvarum ordo ad eas impræſentiarum nos deduxiſſe videtur, in quibus poteſtatum indices in ſingulis curvæ punctis immutantur.

Quarum conſideratio, ſi plane nova non ſit, trita ſaltem & multis obvia cenſeri non debet. Ac cum hac in parte Geometria ac imprimis problematum, alias ſatis difficilium, conſtructio maximopere poſſit promoveri; eæque curvæ in Matheſi potiſſimum ad res Phyſicas applicata frequentiſſimè occurrere, quinimo enaſci ſua ſponte videantur; nec earum in plano delineationes uſque adeo ſint cognitæ; operæ forſitan fuerit pretium hiſce curvarum quandam ſpeciem præmittere, quas & ipſa natura quandoque formare deprehenditur, quarumque proprietas expreſſâ æquatione, ad inſtar aliarum, primâ inſtantiâ efferri minime ſolet; quæ tamen curvarum modo recenſitarum originem, ac deſcriptiones in plano, plurimum illuſtrant.

41. Sit, *Fig. XVIII.* N°. 3, recta *AF* determinata = f, cui in dato quolibet angulo *AFG* applicetur recta *FG* itidem determinata = g; eâ conditione, ut (aſſumtis ipſius *AF* parte aut partibus quibusvis aliquotis *AH*, *AB*, &c. quinimo, ſi viſum fuerit, in ipſâ *AF* productâ, ipſius *AF* in datâ quavis ratione multiplicibus *AD*, *AK*, &c.) ſi ad ſingulas aſſumtarum extremitates *H* aut *B*, aut *D*, aut *K* & ſimiles, ſingulæ rectæ *HN*, *BC*, *DE*, *KM*, &c. reſpective applicentur, ipſi *FG* æquidiſtantes,

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hæ legem quandam obſervent ac relationem conſtantem & immutabilem, tum reſpectu ipſuis FG, tum reſpectu aſſumtæ, quam abſcindunt.

Hoc autem toties fieri ſupponatur, donec ipſæ applicatæ proxime HB, BC, &c. non niſi infiniteſimâ parte HB a ſe mutuo diſtent; unde ipſa $ANCGEMQ$, ſeu rectularum, quæ applicatarum extrema connectunt, ſumma in curvam regularem abibit.

Quæritur jam, ex cognitâ applicatarum ſpeciali relatione, ipſius curvæ æquatio Geometrica, ſeu talis, quæ inter hujus curvæ applicatam interceptamque conſtituta eſt, ipſiuſque curvæ tangens.

Sit, ergo curva AGQ ratione modo expoſitâ generata; ac ſumtis $AH = \frac{1}{3}f$, $AB = \frac{4}{5}f$, $AD = \frac{8}{7}f$, $AK = 2f$, *& ſic pro arbitrio pergendo.* ſpecialis applicatarum inter ſe relatio ſit ea, quæ ſequitur, nimirum, $HN = \frac{1}{9}g$, $BC = \frac{16}{25}g$, $DE = \frac{64}{49}g$, $KM = 4g$, ac ſic in cæteris. Unde partium ipſius AF aliquotarum, ipſiuſque multiplicium exponentes numeros vocando generali applicatione z, hæc emerget curvæ ex præmiſſâ hypotheſi proprietas; ſcilicet,

ſi

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ſi AB aut AD aſſumta fuerit $= zf$, erit ipſam reſpiciens applicata BC aut $DE = zzg$. ex quibus hoc pacto curvæ æquatio Geometrica innoteſcit: ſit: $AD = zf = x$, erit, $z = \frac{x}{f}$; tum & $AD = zzg = y$, erit $zz = \frac{y}{g} = \frac{xx}{ff}$, & $\frac{ff}{g}y = xx$. unde patet curvam AGQ eſſe parabolam, cujus latus rectum $= \frac{ff}{g}$.

Subtangens autem hujus ſeu t eſt ex § 35. $= \frac{ffy}{2gx}$, & loco x & y valores reponendo $= \frac{zf}{2}$.

Ne autem, ut in § proximè prægreſſo, applicatarum abſciſſarumque relationes numeris continuo exprimere neceſſe habeamus, in ſequentibus numerorum loco ſolam z adhibebimus, ipſâque quoslibet tum partium aliquotarum, tum multiplicium ipſius AF exponentes numeros ſignificabimus, tum & curvæ, quæ præ manibus eſt, ſubtangentem t, uti ſæpius, dicemus.

42. Sit, ſtante §. 41. Hypotheſi, $AD = zf = x$, ac $DE = \frac{g}{z} = y$, erit $z = \frac{x}{f} = \frac{g}{y}$, & $xy = fg$; unde curva AEQ eſt hyperbola, ac ſubtangens $t = -x$.

43. Manentibus iiſdem, ac $AD = zf = x$, ſit $DE = y = \sqrt{zfg - zzgg}$, ſeu, $zfg - zzgg = yy$; unde in ultimâ hac æquatione poſito $\frac{x}{f}$ loco z, erit $xg - \frac{xx}{ff}gg = yy$, æquatio ad Ellipſin; & facto $g = f$, ad circulum $fx - xx = yy$; ſub-

ſubtangens ex præcedentibus nota eſt.

43. Sit, poſitis p & q poteſtatum determinatis indicibus, ac cæteris ut ſupra, $z^p f = x = AD$, ac $z^q g = DE = y$, eſt $z^p = \frac{x}{f}$; & $z^q = \frac{y}{g}$, unde poſt reductam æquationem $g^p x^q = f^q y^p$, locus ad paraboliformem quamlibet. Quod ſi q ſigno negativo afficiatur, fiatque $\frac{g}{z^q} = y$, erit æquatio $y^p x^q = g^p f^q$ ad hyperboliformem.

45. Sit $zf + zzf + z^3 f = x = AD$, ac $zg + zzg + z^3 g = y = DE$, erit $z + zz + z^3 = \frac{x}{f} = \frac{y}{g}$, & $gx = fy$ ad lineam rectam.

46. Sit $zzf + z^3 f = x = AD$, & $z^q g = DE = y$, emerget, ipſâ z per æquationum comparationem eliſâ, æquatio ad x & y; cujus ut & præcedentium omnium ſubtangentes ex §. 35. cognitæ ſunt. Poſſent & hic præcedentium converſæ demonſtrari. Sed, cum operæ vix futurum ſit pretium, his ſuperſedebimus.

47. Sit poſt hæc præmiſſa, jam in *Fig. XVIII.* No. 1, $AF = f$, & $FG = g$, item $AN = r$, voceturq; $AQ = fz$, & $QD = \frac{g^z}{r^{z-1}}$, unde ſi intercepta fiat $x = 1f, 2f, 3f$, &c. aut $\frac{1}{2}f, \frac{1}{3}f, \frac{1}{4}f$, &c. erit applicata reſpectiva $y = g, \frac{g^2}{r}, \frac{g^3}{rr}$ &c. aut $\surd rg, \surd C\,rrg, \surd\surd r^3 g$.

quod

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quod poteſtatum deſignationem ex Lemm. 50. &c. intelligenti, ſatis patet.

Quæritur jam ex datâ hac proprietate hujus curvæ *NDO* æquatio, quapropter fiat ut ſupra $zf = x$, erit $\frac{x}{f} = z$; tum facto $\frac{g^z}{r^{z-1}} = y$, ac ipſo $\frac{x}{f}$ in locum ipſius poteſtatis z repoſito fiet $g^{\frac{x}{f}} = r^{\frac{x}{f}-1} y$, quæ debitè reducta dabit $g^x = r^{x-f} y^f$.

Jam autem, ut ex hac æquatione ſubtangens inveniatur, ſit $TQ : t$, $QD : y$, $QP = DH : e$, $HE : a$, tum $AQ : x$, ponaturque loco x ipſa $x+e$, ac loco y ipſa $y+a$, in æquatione inventâ.

Erit $g^{x+e} = r^{x-f+e} y^f + f r^{x-f+e} y^{f-1} a$; quæ diviſa per æquationem datam $g^x = r^{x-f} y^f$, exhibebit $g^e = r^e + f r^e y^{-1} a$; & facto $e = \frac{h}{m}$ (poſito m loco numeri infiniti) cum ſit $t : y :: e : a$. erit $a = \frac{hy}{tm}$, quæ in locum ipſorum e & a ſubſtituta dabit, $g^{\frac{h}{m}} = r^{\frac{h}{m}} + \frac{f r^{\frac{h}{m}} h}{tm}$, unde æquatio ad

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ad tangentem determinata $t\,m\,g^{\frac{h}{m}} - t\,m\,r^{\frac{h}{m}} = f\,h\,r^{\frac{h}{m}}$.

& facto $h = t$, aut $e = \frac{t}{m}$ erit $m\,g^{\frac{t}{m}} - m\,r^{\frac{t}{m}} = f\,r^{\frac{t}{m}}$, quæ ſubtangentem *T Q* ipſius curvæ determinat.

Coroll. I. Obſervetur hujus curvæ ſubtangentem t eſſe in omnibus curvæ punctis ejuſdem magnitudinis, æquatio enim ipſius valorem manifeſtans, ſolâ t, quæ quæritur, exceptâ ex quantitatibus, g, r, f, immutabilibus, ac ſibi per totam curvæ longitudinem conſtantibus componitur.

Coroll. II. Tum & hoc notari meretur, inventam modo æquationem $g^{x} = r^{x-f} \cdot y^{f}$ eſſe curvæ Logarithmicæ propriam; ac hujus ope ex facili inveniri, ex dato quidem numero logarithmum, & è converſo.

Si enim fiat *A N* ſeu $r =$ *unitati*, *A F* ſeu, $f = 100000$, item *F G* ſeu $g = 10$, emerget $10^{x} = 1^{x-100000} \cdot y^{100000}$.

Et, cum unitatis ad quamlibet evectæ poteſtatem non mutetur valor, ſed maneat eadem ac ſibi perpetuo conſtans unitas, adeoque ſit $1^{x-100000} =$ *unitati*, inventa modo æquatio

eadem

eadem erit cum hac $10^{x} = 1y^{100000}$, ad ſpecialem illam pertinens Logarithmicam, quæ jam ob egregium, quem in Geometria practica præſtat, uſum inclaruit; quomodo vero ex dato numero y inveniatur ipſius logarithmus x, & contrarium, hinc ſatis apparet, ac uberior ejus dilucidatio hujus non eſt loci, cum hanc æquationem ex more in algebra uſitato tractanti ultro ſeſe offerant.

48. Sit, retentis ſuperioribus, in eâdem *Fig. XVIII.* No. 1, æquatio curvæ $y^{x} = r^{x-ſgſ}$ oportet ei tangentem deſignare, unde ipſi x ſuffecto $x+e$, tum ipſi y ſubſtituto $y+a$, uti paragr. præced. emerget æquatio ad infiniteſimas redacta $\overline{y+a}^{x+e} = r^{x+e-ſgſ}$, ſeu multiplicatione inſtituta, $y^{x+e} + xy^{x+e-1}a + ey^{x+e-1}a = r^{x+e-ſgſ}$, deletiſque ex §. 7, delendis $y^{x+e} + xy^{x+e-1}a = r^{x+e-ſgſ}$; quæ diviſa per æquationem curvæ exhibet $y^{e} + xy^{e-1}a = r^{e}$, & loco a repoſito $\frac{ye}{t}$ ac æquatione in t ductâ, $ty^{e} + xy^{e}e = tr^{e}$. quæſita æquatio ad tangentem;

Poſſit autem, ex Scholio 1. §. 3, poſitâ m infinita, loco e ipſa $\frac{b}{m}$ ſurrogari, & habebitur alia æquationis forma.

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49. Quoniam vero hæ curvæ vulgaris non ſunt ordinis, ac circa Logarithmicam, §. 47, ex ejus deſcriptione æquationem deduxerimus, hic è converſo ex æquatione datâ deſcriptionem eruemus. Sit ergo æquatio modo propoſita $y^x = r^{x-f} g^f$, unde extractâ radice poteſtatis f, emerget, $y^{\frac{x}{f}} = r^{\frac{x}{f}-1} g$; & ponendo $\frac{x}{f} = z$ ſeu numero, (cum poteſtas lineæ per eandem alterius lineæ poteſtatem diviſa numerum generet, eſt enim $f : x ::$ *unitas* : $\frac{x}{f}$ quare, $\frac{x}{f}$ eſt numerus) erit æquatio $y^z = r^{z-1} g$, ſeu, $y = r^{\frac{z-1}{z}} g^{\frac{1}{z}}$, & $x = fz$.

Hinc ſi *AQ* ſeu $x = fz$ ſint $= 1f, 2f, 3f$, &c. erunt *QD* reſpective ſeu $y = g, \sqrt{rg}, \sqrt{C\,rrg}$ &c. unde curvam hanc deſcribendi modus innoteſcit.

50. Sit jam expoſita curvæ relatio $g^x = r^{x-y} f^y$ ac quæritur recta eam contingens, quare, æquatione ad infiniteſimas redactâ, eſt $g^{x+e} = r^{x+e-y-a} f^{y+a}$, quæ diviſa per æquationem curvæ exhibet $g^e = r^{e-a} f^a$; & cum ſit $a = \frac{ye}{t}$, hoc repoſito in æquatione erit $g^{te} = r^{te-ye} f^{ye}$, & extractâ radice poteſtatis e, fiet $g^t = r^{t-y} f^y$.

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Cum autem hujus curvæ deſcriptio in mox prægreſſarum morem difficulter ſuccedat, aliam ineundo viam eam deſcribemus, ac ope logarithmicæ vulgaris.

Sit ergo, ſumendo p determinatam, n vero indeterminatam, $g^x = r^{x-p} n^p$; quæ æquatio eſt ad logarithmicam PDK ordinariam, uti ex §. 47. conſtat, hæc, *Fig. XVIII.* N. 2. parametris $AP, = r$, $AC = p$ & $CK = g$, juxta eundem §. 47, deſcribatur, eritque abſciſſa $AB = x$, & BD quælibet $= n$.

Jam, cum ex mox aſſumtâ hypotheſi conſequatur etiam eſſe $r^{x-y} f^y = r^{x-p} n^p$, erit utrobique dividendo per r^{x-y}, juxta Lemm. 50. &c. $f^y = r^{y-p} n^p$, quæ æquatio itidem eſt, ſed ad aliam logarithmicam IFL; hæc rurſum, juxta §. 47, parametris $HI = r$, $CH = p$, $CL = f$ deſcripta cenſeatur ad eundem cum primâ axem OL, & factâ perpetuo $GF = BD = n$, erit ſemper $GH = y$.

Quæ GH ſeu y ſi applicetur denuo reſpective ad AB ſeu x, hoc eſt ſi ſit perpetuo $BM = GH$, donec curva NMO deſignata ſit; erit hæc eadem cum illâ, cujus tangentem modo inveſtigavimus. Quod ut a poſteriori probemus eſt

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ſt $g^{x} = r^{x-p} n^{p}$ ſeu, $g^{x} r^{p-x} = n^{p}$.
um $f^{y} = r^{y-p} n^{p}$, hoc eſt diviſione per r^{y-p}
actâ, $f^{y} r^{p-y} = n^{p} = g^{x} r^{p-x}$; quæ æqua-
io diviſa per r^{p-x} dabit quæſitam $r^{x-y} f^{y} = g^{x}$.

Schol. Poſſent & his plura hujus generis exempla adji-
i, curvarumque, in quarum æquationibus ipſæ indeter-
inatæ inter poteſtatum indices reperiuntur, contem-
latio ulterius extendi; ipſæque curvæ in loca & claſſes
ias diſtribui; adeoque & illæ examinari, in quibus po-
ſtatum indices & ſuis rurſum affecti ſint poteſtatibus,
ique rurſum aliis, &c. tum & eæ hujus generis curvæ,
arum æquationes multiplici terminorum numero con-
ant.

Illas enim, quarum inter duos tantum terminos conſi-
it æquatio, terminis abſolute cognitis hiſce non annu-
eratis, propoſitionibus modo prægreſſis conſideravimus,
leoque & primam earum claſſem diſcuſſimus.

Reliquarum autem curvarum examinationem, inte-
um forſan tractatum depoſcentem, hic prætermitte-
us; cum ſola earum tangentes ducendi methodus huc
ntum directè ſpectet, quæ, modo traditis rite perpen-
s, ſatis, ni fallimur, elucidata comperietur.

Approximationes enim, quarum auxilio ſubtangentium
antitas præterpropter in terminis uſitatis exhiberi po-
ſt, hic non tractamus.

51. Sit *Fig. XIV.* duæ curvæ *ABCD* &
FGH (ut ad alias tranſeamus) ita relatæ, ut,
a puncto *O* projiciatur recta *OBF*, curvas ſe-
ans in *B* & *F*, lineæ *OB* & *OF* relationem

 quan-

quandam aſſignatam ad ſe mutuo habeant; &, ex *O* elevatâ rectâ *OK* ipſi *OF* normali, ducatur *BS* tangens curvam *ABC* in B; quæritur in *OK* punctum *K*, ubi *F*K tangens curvam *EFG* in F rectam K*O* ſecat. Sumtâ in curvâ *A*B*C* infiniteſimâ B*C*, ductâque *OCG* erit, juxta Lemm. 40, F *G* ſuæ curvæ infiniteſima: hinc ductis centro *O* arcubus B*L* & F*I*, erunt juxta Lemm. 38 & 39, *CL*, BL, & *GI*, F*I*, infiniteſimæ; & triangula, *SO*B & BL*C*, item K*O*F & F*IG*ſimilia Fiant ergo: *O*B: *y*, *O*F. *z*, *OS*: *t*, *O*K: *k*, infiniteſimæ BL: *e*, *CL*: *a*, F*I*: *i*, *IG*: *o*. Unde *OC*: *y*+*a* & *OG*: *z*+*o*. quare poſitis quatuor infiniteſimis totidem opus erit æquationibus, quarum prima (per: 39 Lem.) $t:y::e:a$, unde $e = \frac{ta}{y}$; ſecunda $k:z::i:o$. Unde $i = \frac{ko}{z}$; tertia ex ſectorum ſeu triangulorum O B L & O FI analogiâ ortum ducit, nam: $y:e::z:i$. unde $e = \frac{yi}{z} = \frac{ta}{y}$, quare $\frac{tza}{yy} = i = \frac{ko}{z}$, ac tandem $\frac{atzz}{kyy} = o$; quæ tres æquationes in cunctis hujus naturæ problematis obtinent, quartam ſuggerit ipſa curvarum aſſignata relatio.

52. Sit ea Verb: Gr: ut O B ſit ad O F perpetuo ut *r* ad *s*; erit adeo $r:s::y:z$, ſeu $sy = rz$, & ad infiniteſimas: $sa = ro$; unde $o = \frac{sa}{r} = \frac{atzz}{kyy}$ ex præcedenti; unde $k = \frac{tz}{y}$; quia $rz = sy$. quod monſtrat hoc caſu F K & B *S* fore parallelas.

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53. Sit jam rectangulum O B in O F ſemper æquale quadrato definito rr; erit, $zy = rr$; & $zy - za + yo + ao = rr$, ſeu juxta: §. 5 & 6, $o = -\frac{az}{y} = \frac{atzz}{kyy}$ ex §, 51, unde $k = \frac{-tz}{y}$; quod ſignum oſtendit K à contrariâ ipſius *S* parte ſtatuendum, uti abundè notum.

54. Quod ſi detur B F ſemper determinatæ rectæ r æqualis, erit $z - y = r$, adeoque $o = a = \frac{atzz}{kyy}$; unde $k = \frac{tzz}{yy}$; & hinc omnigenarum Conchoidum, prout *A* B *D* vel recta, vel curva quælibet aſſumta fuerit, tangentes profluunt.

55. Fiat $zz - yy = rr$, erit $zo = ay$, & $o = \frac{ay}{z} = \frac{atzz}{kyy}$, & $k = \frac{tz^3}{y^3}$. & ſic in cæteris poteſtatibus; ſi fuerit $z^p - y^p = r^p$ erit $ky^{p+1} = tz^{p+1}$. Quomodo autem, ubi plura data ſint puncta, tangentes ducendæ veniant, hinc levi operâ deducitur.

56. Ad tangentes curvarum provehimur, cum curvæ ad alias curvas variis modis referuntur. Sint *Fig. XV.* tres curvæ L*CD*K, *GEFH*, *MN*OP ita relatæ, ut ducta quâcunque rectâ *CEN* ad poſitione datam parallelâ, curvasq, ut vides, ſecanti, ſit ſemper *C E* ad *C N*, ut p ad q; dataque B *C* aut *AC* ſubtangente alterutrius, quæritur alterius tangens. Aſſumtâ *CD*, curvæ L*CD* portione infiniteſimâ: u, ductâque *DO* ipſi *CN* parallela, jungantur, *NR*, *EI*, ipſi *CD* ſeu u æquales & æquidiſtantes, fiantque *CN*: z, *CE*: y, F*I*: a, *R*O: o, B*C*: t, *AC*: k, erit F*D*: $y + a$, *OD*: $z + o$.

Jam

Jam $BC : EC :: EI : FI$. hoc eſt $t : y :: u : a$. unde $u = \frac{at}{y}$;

Item $AC : CN :: NR : RO$. ſeu $k : z :: u : o$. quare $u = \frac{ko}{z} = \frac{at}{y}$, ergo $\frac{kyo}{tz} = a$. Sed ex relatione, $p : q :: y : z$, fit $qy = pz$, & (juxta methodum inſinuatam §. 6 & 7,) $qa = po$, unde $a = \frac{po}{q} = \frac{kyo}{tz}$, & $tpz = kqy$; unde $t = k$, quia $pz = qy$. quare puncta B & A hoc in caſu coincidunt.

57. Sint curvæ $LCDK$ & $GEFH$ ita relatæ, ut recta EC ad curvam LC perpetuo datam obtineat rationem, ut p ad q. quæritur BC ſeu tangentis curvæ $GEFH$ determinatrix. Factis omnibus, ut in præcedenti, ſit LC curva : c, erit $LD : c+u$, & $t : y :: u : a$, ergo $u = \frac{at}{y}$; ſed ex relatione $p : q :: y : c$. unde $qy = pc$, & $qa = pu$, quare $u = \frac{qa}{p} = \frac{ta}{y}$, & hinc $qy = pt = pc$; ergo $t = c$. ſeu BC æqualis curvæ LC.

58. Si vero, ut unico calculo plures caſus comprehendantur, relatio data ſit $p^k : q^s :: y^k : c^s$, erit $p^k c^s = q^s y^k$, & pro y, ac c, poſitis $y+a$, ac $c+u$, emerget juxta §. 5 & 6. $s p^k c^{s-1} u$

=

: $kq^s y^{k-1} a$, & applicatâ æquatione ad yc,
it $sp^k c^s yu = kq^s y^k ca$; quæ ſi hinc inde di-
vidatur per $p^k c^s = q^s y^k$, fiet $\frac{kca}{sy} = u = \frac{at}{y}$ &
$\frac{kc}{s} = t$.

Quod ſi curva *LCDK* ad circulum deter-
minetur, erit *GEFH* ex innumeris Cycloidi-
bus una; cujus adeoque tangens, & omnium
curvarum ſimilem ortum habentium hic oſtendi-
tur.

59. Quin & methodi hujus elucidandæ gra-
tia, Cycloidis *ALMO* tangentem *Fig. XVI.*
ex aliâ ad ſemicirculum hypotheſi inveſtigabi-
mus, quod primo in quibuſcunque hujuſcemo-
di curvis univerſaliter aggrediemur, ſint *ABG*
& *ALO* curvæ ad eundem axem ita relatæ, ut
ductâ *LBE* axi normali, habeat *LB* curvis in-
terceptæ ad curvam *AB* rationem ſemper ean-
dem, ut p ad q.

Sit $LE{:}z$, $BE{:}y$. unde $BL{:}z-y$, $KE{:}t$,
$IE{:}h$, KB & IL reſpectivas curvas tangant,
ſitque BF tangenti KB normalis, $EF{:}l$. tum
$BF{:}r$. & curva $AB{:}c$.

Infiniteſimæ $ED{:}BP{:}LN{:}e$, $CP{:}a$, $BC{:}u$,
$MN{:}i$,

Erit primo $t:y::e:a$, unde $a = \frac{ye}{t}$; tum

 h:

$h:z::e:i$, quare $i = \frac{ze}{h}$; tertio $r:y::u:e$, & hinc $u = \frac{re}{y}$; jam vero ex curvarum relatione $p:q::z-y:c$, ſeu $qz - qy = pc$; & quoque juxta ſæpius inſinuata $qi - qa = pu$; quapropter ſi loco, i,a,u, ſufficiantur modo inventi valores, ordinatâ æquatione fiet $qtzy - qhyy = prht$; factoque $yy = tl$, (quia $t:y::y:l$) erit $h = \frac{qyz}{pr + ql}$.

60. Quod ſi curva A L O ſit Cyclöis primaria, & *A* B *C* huic reſpondens ſemicirculus, erit $p = q$, & $hr + hl = yz$.

61. Quomodo autem generaliori modo poſſint inquiri ſimilium curvarum tangentes, poſita relatione $p^{k\,s} : q^{s} :: \overline{z-y}^{k\,s} : c$ ex præcedentibus quivis colliget.

62. Sit Fig. *XVII.* curva *ACD*, rectæque *A Q*, *D Q* poſitione datæ; item alia curva *A* F *D* ad priorem ita relata, ut ductis duabus rectis *E M* ad *D Q*, & *E* B ad *Q A* parallelis, ſemper habeat intercepta *AM* ad curvam *A*B rationem datam, ut *p* ad *q*; lineæ FEK & *CBH* tangant curvas, dicaturque BH: *h*, K*M*:*t*, *AM*: *x*, *ME*: *y*, curva *A*B:*c*, infiniteſimæ BR: NL: *i*, CR: FG: *a*, MP: EG:*e*, BC: *u*; erit FP: $y + a$, AP: $x + e$, curva AC: $c + u$; jam datâ tangente H B: *h*, quæritur K M: *t*, quæ tangentem curvæ A F D determinat; eſt igitur $t:y::e:a$ unde $a = \frac{ye}{t}$; ſecundo $h:y::u:a$, ſeu $\frac{yu}{h}, = a$

$\frac{ye}{t}$, & hinc $u = \frac{he}{t}$; ſed relatio curvæ $p : q :: x : c$. ſeu, $qx = pc$, quare etiam $\frac{qe}{p} = u = \frac{he}{t}$, unde $t = \frac{ph}{q}$.

63. Quod ſi curva *ABCD* ſit quadrans circu-, cujus radius r, peripheria d; ſtatuaturque ratio p ad q eadem, quæ r ad d; erit $t = \frac{rh}{d}$.

64. Nec opus hic adjicere comprehenſivam magis relationis hypotheſin; quæ ſi ſit: $p^s : q^s :: x^s : c^s$ erit $t = \frac{sxh}{kc}$; unde ſublatâ c per æquationem datam ipſa t per lineas rectas exprimetur.

65. Sint *Fig. XIV*, duæ curvæ *ABCD* & *EFGH*, ita relatæ, ut a puncto quopiam projectâ quavis rectâ *OBF*, ſemper O B ad curvam EF rationem habeat conſtantem, ut p ad q; tangant SB & K*F* curvas reſpectivas, ſitq: KO normalis OB, ducatur OG, ut vides, ac vocetur OB:z, K*F*:f, O*F*:y, KO:h, SO:t, curva E*F*:c; infiniteſimæ vero *FG*:u, *FI*:o, *BL*:e, *CL*:a, quæritur SO ſeu t; primo (per Lemm. 38 & 39,) $t : z :: e : a$, & $e = \frac{ta}{z}$; ſecundo ob parallelas *FI* & BL $z : e :: y : o$, unde $e = \frac{zo}{y} = \frac{ta}{z}$, hinc $o = \frac{tya}{zz} = \frac{hu}{f}$, quia $f : h :: u : o$; adeoque $\frac{tfya}{hzz} = u$; jam vero ex relatione $p : q :: z : c$; quæ proportio ad infiniteſimas redacta dat $u = \frac{qa}{p} = \frac{tfya}{hzz}$, unde $tfpy = qhzz$, ſeu (quia $qz = pc$.) $t = \frac{hcz}{fy}$.

 66. Et

66. Et factâ ſuppoſitione generaliori, $p^k : q^s :: z^k : c^s$, erit hæc redacta ad infiniteſimas, $szu = kca$, quæ comparata, cum æquatione omnibus hiſce curvis communi, (vide prægreſſ.) $tfya = hzzu$, dabit $t = \frac{khcz}{syf}$; unde mediante æquatione relationem exprimente c poteſt tolli, & t per rectas exprimi.

67. Quod ſi *E F G* ad circulum determinetur, erit *A* B *C D* ex Spiralibus quædam, ipſaque *O K* ex naturâ circuli infinita, adeoque $f = h$; ut & ob eandem rationem $y = r$ radio; quare hoc caſu $t = \frac{kcz}{sr}$.

Aliarum Spiralium tangentes, cum poſt hæc nihil difficultatis habeant, tranſilio.

68. Quia autem occurrunt quandoque curvæ, quæ tum ad determinatum aliquod punctum, ſimul & ad lineam velut axem referuntur; unde ſuppoſitæ infiniteſimæ quodammodo intricari videntur, præmittam his in caſibus generale lemmaticum.

Sint in figuris appoſitis *XIX*, N°. 1. & *XIX*. N°. 2. curvæ *AHR*, mediantibus rectis B *A* & B *H*, ad punctum B; & ope parallelarum *A D* & H *L* ad axem B *D* relatæ; ſit B *E* ipſi B *A* normalis, & *A M* tangat curvam; vocenturque *A*B: *z*, B*D*: *x*,

MD:

MD: h, *MA*: k, BE: t, *AD*: y, *AE*: s, infiniteſimæ *A* H: u, *AI*: i, H*I*: o, *AK* in fig. Nº. 1, ſit e, & *K*H: a, aſt Nº. 2. *AK*: a, *K*H: e. emergent, quæ ſequuntur, æquationes,

$$a = \frac{ki}{h} = \frac{ko}{y} = \frac{se}{t} = \frac{sa}{z}.$$

Unde duarum quarumlibet infiniteſimarum ratio in aperto eſt; hoc eſt: Iº. $a : e :: z : t$. IIº. $a : i :: z : \frac{sh}{k}$, quo facto $= p$, erit $a : i :: z : p$. IIIº. $a : o :: z : \frac{sy}{k}$, quo facto $= l$, erit $a : o :: z : l$. IVº: $e : i :: t : p$. Vº. $e : o :: t : l$. VIº. $i : o :: h : y :: p : l$.

Quo vero p & l inveniantur, ductâ *EG*. parallelâ *D*B, & (*Fig. XIX.* Nº. 2,) productâ *DA* in *G*, erit $EG = p$ & $GA = l$.

Quin & propter triangulorum *DA*B & *FE*B ſimilitudinem, eſt $z : y :: t : \frac{ty}{z}$ *EF*; cui ſi addatur *G*F: x, erit *GE* ſeu $p = \frac{ty}{z} + x$, & ob eandem rationem erit *GA* (F*ig. XIX.* Nº. 1,) ſeu $l = y - \frac{tx}{z}$, & (in F*ig. XIX.* Nº. 2,) $l = \frac{tx}{z} - y$.

69. Sit F*ig. XX*, curva *IC*B*A*, & recta *IN*, in quâ deſignatum punctum *N*. recta *NA* ſit poſitione data; item alia curva I H *G* F ita ad priorem relata, ut, a puncto *N* ductâ quâlibet rectâ *NG*B; ac deinde ex puncto *G* lineâ *GR* ipſi *NA* parallelâ, habeat intercepta I *R* ad curvam

I B rationem ſemper eandem, ut p ad q; ſit LN ipſi N B normalis, rectequæ B L & GM reſpectivas curvas tangant; quæritur $MN : t$; data ſint LB: f, LNh, GN: z, BN: y, IR: x, RN: l. curva IB: c, GR: b, PS: n, tum infiniteſimæ, BC: u, BD: o, HE: a, EG: e, HK: i, erit $z : e :: y : o$, item $f : h :: u : o$, unde $o = \frac{ey}{z} = \frac{hu}{f}$; & hinc $u = \frac{fye}{hz}$.

Jam ex curvarum relatione $p : q :: x : c$, unde $\frac{qi}{p} = u = \frac{fye}{hz}$, quare $\frac{pfye}{qhz} = i$; jam vero juxta præcedentem $e : i :: t : \frac{tb}{z} + l$, unde $i = \frac{tbe + lze}{tz} = \frac{pfye}{qhz}$, & tandem $t = \frac{qhlz}{pfy - qhb}$.

70. Si jam IBAN ſit quadrans circuli, & ratio p ad q ſit ratio radii r, ad peripheriæ quadrantem d, erit IHGF Quadratrix veterum, ac p & y & r æqualia, item $q = d$ & $h = f$ ob infinitam puncti L diſtantiam, unde $t = \frac{dlz}{rr - db}$. & ſi quæratur linea PN invenietur $= \frac{dzz}{rr}$.

71. Si relatio data fuerit $p^{ks} : q :: x^{ks} : c$. tangens ex anterioribus eluceſcet.

Schol. Et huc quidem uſque ea potiſſimum quæſtio ventilata fuit, quâ recta inquirebatur in dato puncto curvam datam tangens. Unde Verb: Gr: *Fig.* X, ex data curvæ *ADE* proprietate, ipſiſque A *Q*, *QD*, ſeu x & y datis (dato enim puncto *D*, & hæc data ſunt) invenire neceſſarium fuit, trianguli rectilinei *TQD* proprietatem, aut rationem

nem, quam inter ſeſe habent *TQ* & *QD*, ſeu intercepta & applicata ejuſdem trianguli. Cum autem multis quidem modis hæc quæſtio poſſit variari, petique, ut, dato in triangulo *TDQ* puncto *D*, inveniatur curva *ADE*, ipſam rectam *TE* in dato puncto contingens: unde tum curvæ parameter, item *AQ* ſeu *x*, poſt generalem curvæ naturam cognitam inveſtiganda forent. Quin & plurimis modis diverſa, circa rectas curvaſque ſemutuo tangentes, quæri poſſent, pro eorum diverſitate, quæ tanquam data ſupponuntur. Hæc vero licet ex facili, præcedentibus perceptis, conſequantur, in tyronum gratiam unum aut alterum exemplum addemus.

72. Sit *Fig. X*, recta *TD* in infinitum verſus *E* protenſa, ejuſq: poſitio reſpectu lineæ *TS* data, in eâque aſſignatum punctum *D*, oportet curvam *ADE* hanc rectam in *D* tangentem deſignare. Ductâ ex *D* rectâ *DQ* ipſi *TQ* normali, aut ad quemlibet datum angulum, ſit ex trianguli *TQD* proprietate (factâ *TQ*: t, & *QD*: y) perpetuo $p:q::t:y$, unde æquatio trianguli $pt = qy$, vocatisque ut ſupra, $QP = DH: e$, & $EH: a$, erit hæc reducta ad infiniteſimas $qe = pa$, quæ in omnibus hiſce caſibus obtinet.

Jam quæratur parabola *ADE* ipſam rectam *TD* in puncto *D* tangens, cujus æquatio $rx = yy$, facto $AQ: x$; dabit hæc ad infiniteſimas redacta $re = 2ay$; unde $a = \frac{re}{2y} = \frac{qe}{p}$, unde cum p & q item t & y in triangulo data ſint (trianguli enim proprietas & punctum

D.

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D. datum eſt) invenietur parameter parabolæ $r = \frac{2qy}{p} = \frac{yy}{x}$ ex naturâ parabolæ, unde $x = \frac{py}{2q} = \frac{t}{2}$. quare ex cognitâ *A Q* ſeu *x* & latere recto parabolæ ea poterit deſcribi.

73. Expetatur jam *A D E* circulus, cujus æquatio $2rx - xx = yy$, quæ rectam *T E* in *D* contingat. ea reducta ad infiniteſimas erit: $\frac{re - xe}{y} = a = \frac{qe}{p}$; unde circuli ſemidiameter $r = \frac{qy + px}{p} = \frac{yy + xx}{2x}$ ex æquatione circuli; ex quibus tum *x*, tum *r* in quantitatibus cognitis dabuntur, uti notum eſt.

74. Quod ſi hyperbola dictam *T E* tangens in *D* deſideretur, cujus æquatio, $rxx + rhx = hyy$, poſito latere ipſius recto $= r$, transverſo $= h$; erit hac ad infiniteſimas redactâ, $\frac{2rxe + rhe}{2hy} = a = \frac{qe}{p} = \frac{ye}{t}$ ex æquatione trianguli, unde $r = \frac{2hyy}{th + 2tx} = \frac{hyy}{xx + hx}$ ex quibus *r* & *x* innoteſcent. Cum autem ad ipſam *h* determinandam æquatio non ſuppetat, ea manebit hoc in caſu non omnino limitata.

Unde, licet hyperbola *A D E* in puncto ſui *D* unicam tantum admittat tangentem rectam, è contrario recta data *T E* in puncto ſui *D* innumeras habet hyperbolas tangentes, prout hæc

aut

aut illa ex ipſis x, r, & q (quarum non niſi duæ per æquationes inventas definiri poſſunt) arbitraria remanſerit.

Poſſet jam eadem hæc quæſtio rurſus mutatâ facie proponi, ac ex datâ tum curvæ, tum trianguli proprietate punctum contactus mutui inveſtigari; quæ tamen, ut & alia huc ſpectantia, cum prioribus intellectis nullam fere difficultatem (unico caſu excepto, ubi cæteris datis curvæ natura quæritur) habeant, hic prætermittimus.

Cum autem non inter rectas tantum & curvas, verum & inter duas curvas, aut plures, contactus mutuus poſſit celebrari, licet hoc problema per communem ac omnes curvas eodem ſui puncto tangentem rectam exiguo ſatis labore efficiatur; rem forſan tyronibus utilem præſtitiſſe videbor, ſi quo pacto ex traditæ methodi generali lege effectum dari poſſit, paucis demonſtravero; cæteris ſpeculationibus, quæ hinc originem ducere poſſunt, in locum commodiorem relegatis.]

75. Sit ergo *Fig. XXI.* No. 1, curva *ADE* data, & in eâ punctum *D*. Oportet ei ducere curvam *TDE* tangentem.

Reſumantur ſymbola ordinaria, fiatque $AQ: x$, $QD: y$, infiniteſimæ, $DH = QE: e$, $EH: a$, $DE: u$, ſubtangens $TQ: t$. quapropter, ſi curva expoſita *ADE* ſit circulus, cujus æquatio $2rx - xx = yy$, ac debeat duci ex parabolis quædam *TDE*, circulum parte ſui convexâ tangens; ſit ipſius æquatio $tt = ky$; erit redactâ utrâque ad infiniteſimas $\frac{re - xe}{y} = a = \frac{2te}{k}$, & æquatio ad tangentem determinata $kr - kx = 2ty$

quæ

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quæ cum alia æquatione $tt = ky$ combinata, dabit poſt debitam reductionem, $t = \frac{2yy}{r-x}$ & hac cognitâ latus rectum parabolæ $k = \frac{2ty}{r-x}$.

76. Quod ſi expetatur: parabola parte ſui concavâ circulum tangens, *Fig. XXI*, No. 2, & ejus æquatio $kt = yy$; quæ cum æquatione circuli redacta ad infiniteſimas erit: $2re - 2xe = ke$; unde latus rectum $k = 2r - 2x$ ejuſque intercepta TQ ſeu ſubtangens quæſita $t = \frac{yy}{2r-2x}$

77. Manente ADE circulo, & ſuppoſitâ TDE ex hyperbolis aliquâ, ſit hujus æquatio, $ktt + kht = hyy$; emerget ex utriuſque curvæ æquatione ad infiniteſimas redacta, $re - xe = ya$; tum $2kte + khe = 2hya$; unde $2hr - 2hx = 2kt + kh$; quæ æquatio cum hyperbolæ æquatione debitè tractata dabit, tum t ſubtangenrem, ſeu hyperbolæ interceptam; tum h aut k alterum ex hyperbolæ lateribus, altero manente indeterminato. Unde patet circulum innumeris hyperbolis uno in loco tangi poſſe. Sed hæc ſufficiant.

Schol. Plures quidem in præcedentibus infiniteſimas adhibuimus; ſed in ſingulis curvis, non niſi eas, quæ *Fig. XXIII*, No. 1, triangulum DEH conſtituunt; cum autem hiſce non cöerceatur hæc methodus, ſed & aliæ ac plures in calculum deduci queant, quæ eandem cum modo recenſitis curvam ſpectant; nimirum ductâ præter

tan-

angentem *TD* inſuper rectà *BE*, curvam *ADE* tangenti
ı *E*, tum *TC* ipſi *BE* perpendiculari, erit juxta lemma-
a præmiſſa triangulum *BTC* pariter ex infiniteſimis com-
oſitum; quin & *Fig. XXII.* ductis *DR* ac *ES*, tangen-
ibus *TD* ac *BE* normalibus, erit his intercepta axis por-
io *RS*, aliæque quam plurimæ *Hλ*, *λE*, &c. ex infini-
eſimarum genere; quarum conſiderationem, cum tangen-
ium doctrinam multum promoveat, quin & curvarum
nixtilineorumque menſuras immane quantum adaugeat,
is annectere methodique hujus univerſalitatem, ac majo-
em longe, quam prima fronte promittere videtur, exten-
onem demonſtraſſe forſan non infructuoſum fuerit; inſi-
nulque non ubique obvium theorematum fontem hoc rivo
caturientem oſtendiſſe; modo, quod hoc in labyrintho fi-
um eſt Ariadnes, inter infiniteſimas, ac quantitates reje-
taneas ex legibus lemm. 10. &c. exactè diſtinguatur.

78. Sint *Fig. XXII.* ut ſupra, $TQ:t$, $AQ:x$,
$QD:y$; Infiniteſimæ $DH:e$, $HE:a$, $DE:u$, *DR*
angenti *DT* normalis; *BE* ſit ea linea, quæ
angat curvam in *E*. puncto ipſi *D* proximo, &
uic perpendicularis *ES*. erunt, juxta lemm.
8, $BT:i$. tum $RS:o$, pariter infiniteſimæ.

Detur jam curva *OML* ejus naturæ, ut, ductis
DM, *EL*, &c. ipſi *AO* parallelis, ſit continuò
M æqualis *TQ*, & *GL* æqualis *BP*; erit, factâ
XM ipſi *GI* æquali & æquidiſtanti, lineola *XL*,
eu differentia linearum *GL* & *IM* æqualis
$BT + QP$ ſeu $e+i$, & $MX = a$. Sit inſuper alia
urva *ZKV* illius proprietatis, ut ejus applicata
QK ipſam *QR* ſeu l, & *PV* ipſam *PS* ſeu $l-e+o$
conſtanter adæquet; erit *KN* parallela $QP:e$,
& *NV* ſemper æqualis $RS - QP$ ſeu, $o-e$, tan-
gat autem *MY* curvam *OML*, ſitque $IY:h$. quæ-
ritur $QC:k$, tangentem curvæ *ZKV* determinans.

Huic ſcopo tot quærendas eſſe æquationes, quot aſſumtæ fuerint infiniteſimæ, ex anterioribus manifeſtum eſt. Harum I. $ye = at$. II. $le = ay$. Quarum ratio jam ſæpius inſinuata. III. $BP : PE :: PE : PS$, ſeu, $t+e+i : y+a :: y+a : l+o-e$, unde rejectis juxta §. 6 & 7, rejiciendis $li+le+to-te = 2ay$; & quia $le = ay$, reſtat $li+to\ te = ay$. IV. $CQ : QK :: NK : NV$. ſeu $k : l :: e : o-e$, unde $ko\ ke = le$. V. $YI : IM :: MX : XL$. $h : t :: a : e+i$, unde $at = he+hi = ye$ ex primâ, ergo ex tertiâ & quartâ, erit $o = \frac{ay+te-li}{t} = \frac{ke+le}{k}$; unde, facto $le = ay$, orietur $e = \frac{ki}{k-t} = \frac{hi}{y-h}$ ex quintâ; quare tandem $k = \frac{ht}{2h-y}$.

79. Notetur, ut habeatur ratio infiniteſimarum QP & BT ſeu e & i, ex præcedenti eſſe $h : y-h :: e : i$; tum etiam $k : k-t :: e : i$. Item ſi ratio QP ad RS ſeu e ad o. expetatur, eam hoc analogiſmo exprimi, $k : k+l :: e : o$; juxta IV. æquationem præcedentis.

80. Iiſdem poſitis jam loco l, applicetur eadem t ad e, ſeu TQ in QK, & PB in PV, donec per puncta K, V, &c. deſcribi poſſit curva ZKV. quæritur QC ſeu k, quæ tangentem CK patefacit, Cum ſit $QK : t$, & $PV : t+e+i$, erit

eſt differentia $NV: e+i$; ideoque $CQ: QK::$ $IN: NV.$ $k: t:: e: e+i$, unde $\frac{te}{k} = e+i$. ſed in curva OML eſt $IY: IM:: MX: XL$, ſeu $h: t:: a: e+i$; quare $\frac{ta}{h} = e+i$; & hinc $\frac{te}{k} = \frac{ta}{h}$, & $e = \frac{ka}{h} = \frac{ta}{y}$, ac tandem $k = \frac{ht}{y}$.

81. Hinc facile patet, ſi, relatione curvæ IMO pro lubitu aſſumtâ, inveniatur juxta canones prægreſſos linea YI ſeu h; ac curva QKV ejus naturæ ſtatuatur, ut QK quælibet perpetuo æquetur ipſi IM ſeu g; fore ſemper QC ſeu $k = \frac{ht}{y}$.

82. Notetur hujus ope methodum §. 35, propoſitam ampliari, ac etiam *Fig. XXIII.* N°. 1. ad curvas PLI applicatam EP pro axe habentes extendi; cum loco citato omnes ad eundem axem AQ deſcriptæ eſſe fuerint ſuppoſitæ. Sit, verb: gr: $QA: z$; $HL: f$; $DQ: y$; ac mutua curvarum relatio $fz = yy$; erit factâ $QZ = HL = f$, vocatiſque ſubtangentibus $QY: n$, $QW: k$, $HF: h$, juxta §. 37, $nt + tk = 2kn$; ſed ex præcedenti $k = \frac{ht}{y}$; unde emerget æquatio pro ſubtangentibus hoc in caſu determinandis $nty + tth = 2htn$.

83. Sit *Fig. XXIII.* N°. 1. curva ADE cum ſuperiori eadem, eademque ſymbola; r ſit li-

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nea determinata; fiatque $y : t :: a : e :: r : z$, quæ z applicetur in HS, ad $EH : a$, idque perpetim, donec per omnium applicatarum extremitates deſcribi queat curva RSS. Quæritur $HK : d$, unde ducta KS curvam RSS contingat. eſt $MS : a$, ſitque $MR : \theta$; erit I^{o}. $HK : HS :: MS : MR$. $d : z :: a : \theta$, & hinc $d\theta = za = re$, ex hypotheſi.

II^{do}. Poſitâ curvâ ILP ejuſmodi, ut HL ſit conſtanter æqualis ipſi QT ſeu t; ductâque FL eam tangenti ſit $FH : h$, erit, juxta §. 78, $FH : HL :: GL : GI$. ſeu $h : t :: a : e + i$, unde $he + hi = at = ye$.

III^{tio}. Redactâ æquatione datâ ad infiniteſimas, erit $PE : PB :: r : ER$, ſeu $y + a : t + e + i :: r : z + \theta$; unde remotis per §. 6 & 7, rejectaneis, $za + y\theta = re + ri$; & quia $za = re$, eſt $y\theta = ri$; unde $\theta = \frac{ri}{y} = \frac{re}{d}$ adeoque $e = \frac{di}{y} = \frac{hi}{y-h}$, ac ultimo quæſita $d = \frac{hy}{y-h}$.

84. Repetitis omnibus, ſit $t : y :: e : a :: r : f$; applicetur continuo f ad e, ſeu QZ ad QP, ut hac ratione conſtituatur curva ZX; cujus ducenda ſit tangens WZ. ſit $QW : b$, infiniteſima $VZ : \lambda$, erit $QW : QZ :: VX : VZ$. $b : f :: e : \lambda$. unde $fe = b\lambda$.

II^{do}. Sit curva $A\Lambda$ talis naturæ, ut $Q\Lambda$ perpetuo ipſi QT ſeu t æqualis ſit; & $QY : n$, unde ducta $Y\Lambda$ curvam $A\Lambda$ tangit; erit $QY : Q\Lambda :: \Lambda N : N\Delta$, ſeu $n : t :: e : e + i$. ergo $te = ne + ni$;

&

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x redacto analogiſmo dato ad infiniteſimas fiet, $P:PE::r:PX$. ſeu $t+e+i:y+a::r:f-\lambda$; quare $fe+fi-t\lambda = ra$; &, quia $ra = fe$, fit $fi = t\lambda$. unde $\lambda = \frac{fi}{t} = \frac{fe}{b}$, ideoque $e = \frac{bi}{t} = \frac{ni}{t-n}$, denique $b = \frac{tn}{t-n}$.

85. Sint *Fig.* XXIII. N°. 1, curvæ *ADE*, *ILP*, cum præcedentibus eadem, eademque ſymbola; curva vero *RSS*, habeat applicatam H*S* tangenti *DT* ſeu s perpetuo æqualem. quæritur HK tangentem KS definiens.

Quoniam $DE = u$, & infiniteſima; erit ductâ *DO* ipſi *BE*, quæ curvam in *E* tangit, normali, $EO = ED = u$. juxta lemm. 42; ac ducta *CT* parallela *DO*, hoc eſt ad eandem *BE* perpendiculari, erit $EC = ET = s+u$ ex lemm. 36; ſit porro CB: λ infiniteſima ex lem. 18, erit $BE = s+u+\lambda = ER$; quare ob parallelas & æquales *E*H, *MS*, fiet *MR* ſeu $RE - \mathrm{H}S = u+\lambda$: tandem HK vocetur c.

Eſt HK: HS:: *MS*:*MR* ſeu $c:s::a:u+\lambda$, quare $\lambda = \frac{sa-cu}{c}$; jam vero, ob angulos *CTE* & *TCE* ex lemm. 33. rectos & æquales, erunt triangula *BCT* & *TQD* ſimilia; quare $TD : TQ :: BT : CB$. ſeu $s:t::i:\lambda$, unde $\lambda = \frac{ti}{s} = \frac{sa-cu}{c}$, & hinc $\frac{ssa-cti}{sc} = u = \frac{sa}{y}$

(nam

(nam TD, $s:DQ$, $y::DE$, $u:EH$, a) quare $a = \frac{ceyi}{ssy - ssc} = \frac{ye}{t}$ (quoniam TQ, $t:QD$, $y::DH$, $e:EH$, a) ergo $\frac{ctti}{ssy - ssc} = e = \frac{hi}{y-h}$ (juxta §. 79.) unde $c = \frac{ssyh}{tty + ssh - tth}$, & facto $ss - tt = yy$, eſt $c = \frac{ssh}{tt + yh} = HK$. Exempla non addo particulares caſus concernentia.

86. Cum vero omnia hæc in curvis verſus axem concavis oſtenſa ſint, eæque, quæ verſus axem convexæ ſunt, *Fig. XXIII.* N°. 2. aliquid in infiniteſimarum poſitione a prioribus diverſum obtineant, è re forſan erit prægreſſum problema hoc in caſu ſolvere, unde cætera innoteſcent. Poſitis ergo omnibus, ut in præcedenti, quæ tum linearum, tum infiniteſimarum pariter ac curvarum ADE, ILP, RSS naturam ac relationem mutuam ſpectant: notetur hoc in caſu, GI eſſe $= e - i$, cum enim ſit $TQ = HL = t$, ac $BT = i$, item $QP: e$, erit $BP = t + e - i = EI$, unde $GI = e - i$.

Secundo, quia $DT = HS = s$, & $CT = \lambda$, $DE = Do = u$, erit $CE = BE = ER = s + u - \lambda$; quare $MR = u - \lambda$, hoc in caſu.

Ut ergo HK ſeu c determinetur eſt HK, $c:HS$, $s::MS$, $a:MR$, $u - \lambda$. Quare $\frac{cu - sa}{c} = \lambda, = \frac{ti}{s}$ ob an-

angulum EBC rectum, tum in triangulis DEH & TDQ ſimilibus eſt. $s:y::u:a$ unde $u = \frac{sa}{y}$, item $t:y::e:a$. unde $a = \frac{ye}{t}$.

Jam in curva ILP eſt $FH:HL::GL:GI$, ſeu $b:t::a:e - i$, unde $he - hi = ta - ye$; ex quibus tandem æquationibus, eliſis infiniteſimis, orictur, ut ſupra, $c = \frac{ssh}{tt + yb}$.

Schol Atque hac qnidem occaſione in ſpeculationem quandam circa curvarum flexus & curvaturas inſperatò dilabimur; quæ tum à veteribus, tum etiam à recentioribus Mathematicis inter primarias licet habita, nunquam tamen, quantum mihi conſtat, hoc fonte derivata, hac ſaltem ratione tractata fuit: quâ levi ſatis negotio, & magis forſan directè, quam ex radicum æqualitate, ex cujuslibet curvæ æquatione cognitâ dignoſcitur curvæ ſpecialis indoles, ipſiuſque verſus axem aut quamlibet expoſitam rectam concavitas aut convexitas, tum & punctum, ſi detur, flexûs contrarii.

87. Sit *Fig.* XXIII. N°. 1 & 2. curva ADE; patet ex iis, quæ lemm. 23, &c. demonſtrata ſunt, omnes curvas ex tangentium TD, BE portionibus infiniteſimis DE, $E\varphi$ componi, quæ angulos in E efformant, unde clarum ſatis eſt, ſi modo dicti à tangentibus efformati anguli $DE\varphi$ concavum ſuum verſus axem AQ convertant, curvam verſus AQ fore concavam, *Fig.* XXIII. N°. 1. ſi vero apicem ſuum axi AQ obvertant recenſiti anguli, uti *Fig.* XXIII.

N°. 2, curvarum verſus eandem *A Q* fore convexam; adeoque & hos angulos curvaturarum ſolas normas exiſtere.

Quo poſito, figuras obiter inſpicienti apparet, ſi recta *TDE* ſupponatur tangere curvam aut ei cöincidere in rectulâ infiniteſimâ *DE*, proximam huic infiniteſimam *E*Ω verſus axem in *B* prolongatam, ab eodem abſcindere infiniteſimam *BT*; hac quidem differentiâ in utrâque curvâ, ut, I^{to}. Vocatis ſubtangenti primâ *TQ*: t, tum *QP*: e, & *BT*: i. ſi curva *ADE*, *Fig.* XXIII, N°. 1, ſit verſus *AQ* concava, ſit ſubtangens ſecunda $BP = t + e + i$; aſt *Fig.* XXIII. N°. 2, ubi curva verſus *AQ* eſt convexa, $BP = t + e - i$. II^{do} Tum & tangenti *TD* vocatâ s, ac $DE = OE$: u, item *BC*: ϑ, patet in curvâ verſus axem concavâ, *Fig.* XXIII. N°. 1, tangentem ſecundam *BE* fore $= s + u + \vartheta$; in altero vero caſu, ubi curva verſus axem convexa exiſtit, $BE = s + u - \vartheta$.

Ex quibus duobus criteriis manifeſtus conſequitur character curvarum quarumlibet curvaturam, reſpectu axis, quem reſpiciunt, dignoſcendi, modo perſpectum habeamus, num ſubtangens ſecunda *BP* per $t + e + i$, num vero per $t + e - i$ in expoſitâ curvâ ſit denotanda.

88. Quod

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88. Quod ut in curvâ qualibet geometricâ indagemus, inſimulque convexitatis aut concavitatis curvarum notas diagnoſticas ad quantitates poſitivas, cum §. præced: non niſi per infiniteſimas explicatæ fuerint, referamus: concipiatur in utrâque figurâ nova deſcripta eſſe curva PIL hujus naturæ, ut EI ipſi BP, tum HL ipſi TQ, ſingulæque porro curvæ PLI applicatæ reſpectivis curvæ ADE ſubtangentibus jugiter adæquentur; ductâque recta FLI ipſam in L tangenti vocetur ſubtangens FH: h; ac cum ex naturâ curvæ ſit $HL: = t$, poſitâ infiniteſimâ $GL = EH = a$, erit infiniteſima GI in *Fig.* XXIII. N°. 1, $= e+i$, in *Fig.* XXIII. N°. 2. $= e-i$; eſt enim in utrâque ipſa GI ſubtangentium BP & TQ differentia, uti patet.

Jam autem cum ſit perpetuo $FH: HL :: GL: GI$, ſeu vocando GI generali appellatione $= o$, (quæ tum per $e+i$, tum per $e-i$ pro rerum exigentiâ poterit explicari) $h : t :: a : o$. erit $ta = ho$; ſed eſt $ta = ye$ ex curva ADE; (cum ſit $TQ : QD :: DH : EH$. ſeu $t : y :: e : a$.) ergo tandem in utroque caſu perpetim invenietur, $ye = ho$.

89. Quapropter, ut ad rem propius accedamus, cum in *Fig.* XXIII. N°. 1, ubi curva *ADE* verſus axem AQ concava eſt, ſit *GI* ſeu $o = e + i$, erit $ye = ho = he + hi$, & æquatione in analogiſmum reſolutâ $y - h : h :: i : e$. unde patet hoc in caſu *h* eſſe minorem ipſâ *y*.

Si vero, uti *Fig. XX*III. N°. 2, curva *ADE* verſus AQ convexa eſt, erit *G*I ſeu $o = e - i$, adeoque $ye = ho = he - hi$, ſeu $hi = he - ye$, & æquatioue ad proportionem redactâ $h - y : h :: i : e$. unde patet rurſum hoc in caſu contrarium evenire, ac *h* eſſe majorem ipſâ *y*.

90. Unde tandem, vocatâ curvâ *PL*I *curvâ ſubtangentiali* ex proprietate ſuperius memoratâ, hic emergit in curvis quibuslibet curvaturarum character.

Si ad curvæ cujuſvis *ADE* ordinatam *EP* applicetur curva ſubtangentialis *PL*I. Hujuſque ſubtangens F*H* ſeu *h* fuerit minor curvæ expoſitæ applicatâ *y*; erit curva *ADE* verſus axem ſuum AQ concava.

Si vero fuerit ſubtangens *h* major ipſâ *y*, contrarium obtinebit, ac curva *ADE* verſus axem ſuum AQ erit convexa.

91. Quod ſi *Fig.*XXIII.N°.1. TQ & *BP* non in *HL* & *E*I. ad ordinatam *EP* ſeu *y*, ſed in QΛ & *P*Δ ad interceptam AQ ſeu *x* applicentur; erit hoc in caſu *A*$\Lambda\Delta$ curva ſubtangentialis, in quâ

Q

$Q\Lambda = t$, $\Lambda N = e$, $N\Delta = o$, vocetur etiam YQ ſubtangens hujus $= k$, erit $k : t :: e : o$, & $te = ko$.

Unde ſi ſit $o = e + i$, ac curva ADE verſus axem concava, erit $te = ke + ki$ & $te - ke = ki$, quare k minor ipſa t.

Quod ſi $o = e - i$, eſt $te = ke - ki$, & $ki = ke - te$, adeoque k major ipſa t.

Et ſequens oritur curvarum flexuræ diagnoſticum in caſu, quo curva ſubtangentialis $A\Lambda\Delta$ ad curvæ interceptam AQ applicata eſt,

Si fuerit ſubtangens k minor ipſâ curvæ expoſitæ ſubtangenti t, erit curva ADE verſus axem ſuum concava.

Si vero k fuerit major ipſâ t, erit curva ADE verſus axem ſuum convexa. NB. Potuiſſet & hoc ex §. 80 & 81 elici, cum ibi demonſtretur eſſe $k : t :: h : y$.

Coroll. I. Patet ex duabus præcedentibus, ſi ſubtangens t curvæ ADE, cujus flexus quæritur, explicetur per interceptam AQ ſeu x. commodiſſime adhiberi præcedentem; ſi vero per applicatam y deſignetur ſubtangens t, inquiſitioni curvaturæ potius inſervire §. 90.

Coroll. II. Quo vero methodi hujus uſus per exempla pateat, ſit curva, cujus intercepta x, applicata y, ſubtangens t, ejuſque æquatio $2rx - xx = yy$ ad Circulum. Quæritur ejus curvatura verſus interceptam AQ.

Erit

Erit ex §. 35, æquatio ad tangentem $yy = rt - xt$, quæ reducta ad x fiet $2rx - xx = rt - xt$; ſed per y deſignata fiet $rrtt - yytt = y^4$. quarum prima *Fig.* XXIII. N°. 1, eſt æquatio ſubtangentialis ſeu curvæ $A\Delta\Delta$ ad axem AQ; ſecunda erit æquatio ſubtangentialis ſeu curvæ PLI ad applicatam EP poſitæ.

Reſumendo ergo primam $2rx - xx = rt - xt$ erit ex §. 17, YQ ſeu $k = \frac{rt - xt}{2r - 2x + t}$; quæ inquiratur num ſit major aut minor quam t, juxta præcedentem, quare ſi ſit k ſeu $\frac{rt - xt}{2r - 2x + t}$ major aut minor ipſa t; ſequetur tandem (eas quantitates inſtar æqualium tractando, hoc enim fieri poſſe cuivis notum eſt) fore etiam nihilum majus aut minus ipſo rr; ac cum conſtet nihilum eſſe minus ipſo rr, nec nunquam ei æquari poſſe; etiam conſequitur k ipſâ t ſemper minorem eſſe, adeoque juxta §. 91, curvam ADE fore verſus axem ſuum concavam.

Coroll. III. Idem ſequenti modo colligi poteſt ex æquatione ſubtangentiali ad y reducta, quæ eſt $rrtt - yytt = y^4$; poſita nimirum t ad y in curva PLI applicari, cujus ſubtangens $h = \frac{rrtt - yytt}{2y^3 + tty}$; quæ inquiratur num ſit major aut minor quam y, juxta §. 90, invenietur ipſa

ipſa h minor ipſâ y; adeoque per §. 90, curva expoſita erit verſus axem $A\,Q$ concava.

Coroll. IV. Sit curva, cujus æquatio, $ry = xx$ erit æquatio curvæ ſubtangentialis ad y appoſitæ $ry = 4tt$, cujus curvæ ſubtangens h invenietur major ipſâ y, adeoque curva expoſita verſus axem erit convexa, ex §. 90.

Idem circa k & t comperietur, ſi ſubtangentialis curva ad x fuerit applicita.

92. Et hæc quidem circa concavitatis aut convexitatis curvarum dignotionem, quando h aut k ipſâ y aut t minor aut major exiſtit; ubi autem ea emergit quantitatum, h & y, tum k & t inter ſe relatio, ut non tantum ſe invicem utrinque excedere, verum & ſibi mutuo æquari poſſint; ſequetur in caſu hoc æqualitatis curvam nec convexam nec concavam eſſe (ſecus enim una alteram excederet juxta §. 90 & 91,) adeoque & hac ratione punctum flexionis contrariæ in qualibet curva determinari; eo enim loco curva verſus axem nec convexa nec concava eſt.

Sit in exemplum *Fig.* XIII. No. 2, Conchois Nicomedis polo K, normâ FG deſcripta, vocatâque $FK: S$, tum $AF =$ L$D: r$, $QF: x$, $QD: y$, item TQ ſubtangenti: t, erit æquatio proprietatem ipſius exprimens $xxyy = rrxx + 2srrx - x^4 - 2sx^3 - ssxx$.

Unde

Unde juxta regulas §. 17 & 35, traditas, ac ope æquationis curvæ eliſâ y, emerget ſubtangentis TQ valorem manifeſtans æquatio, poſt debitam per $2x + 2s$ depreſſionem, ea, quæ ſequitur:

$$rrxx + srrx - x^4 - sx^3 = -srrt - tx^3,$$

quæ, poſitâ perpetim TQ ſeu t in QC, ſubtangentialis curvæ ACG naturam prodet. Cujus rurſum ſubtangens QR ſeu k ex regulis modo citatis hâc ratione determinabitur,

$$\frac{-srrt - x^3 t}{2rrx + srr - 4x^3 - 3sxx + 3txx} = k.$$

qui ipſius k valor, ſi ſupponatur major aut minor aut æqualis ipſi t, colligetur etiam eſſe $\frac{-2srr + 3x^3 - 2rrx + 3sxx}{3xx}$ eſſe majorem, minorem, aut æqualem ipſi t, hoc eſt invento ipſius valori, $\frac{rrxx + srrx - x^4 - sx^3}{-srr - x^3}$ quibus denuo quantitatibus ex æquationum lege comparatis, ac per $s + x$ diviſis, emerget fore $2srr$ majorem, minorem, aut æqualem, ipſis $3sxx + x^3$; quæ omnia, cum pro variâ

ipſius

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ipſius x determinatione ſint poſſibilia, apertè conſequitur:

I. Si $2srr$ ſit minor ipſis $3sxx+x^3$, fore k minorem t; adeoque curvam verſus axem AF concavam.

II. Si contrarium eveniat, fore convexam.

III. Si ſibi mutuo æquentur, ſeu ſi ſit, $2srr = 3sxx+x^3$, curvam in hac ipſius x magnitudine fore nec concavam nec convexam; verum hac æquatione ipſum curvæ flexus contrarii punctum definiri.

Schol. Placuit hoc potius Conchöidis exemplo præ aliis uti, cum inter problemata illuſtria à præſtantiſſimis hujus ævi Mathematicis numeretur; tum & canonumſuperius traditorum in inquirendis ſubtangentibus præcepta ſequi; ut, quod ante circa ſignorum notationem inſinuavimus, apertius pateat. Ipſam enim t continuo negativè valore expreſſam eſſe, cum a contrariis ipſius x partibus reſpectu applicatæ QD inveniatur, nemo non videt, ac nihilo ſecius ſigna, ex canonum legibus oriunda, ſervando ad æquationem legitimâ ſignorum qualitate affectam deveniri.

Poſſent & in modum §. 37 & 38, hæc infinitis curvis ſimul & ſemel unâ opera applicari, ac licet ſubtangens t per quaslibet aliarum curvarum applicatas deſignaretur, idem effici; unde ipſam t ad expoſitæ curvæ abſciſſam ordinatamve x aut y, quod hiſce exemplis perpetuo factum fuit, primo referre neceſſarium non erit: verum hæc, cum ſatis leviter ex ante dictis eruantur, compendii gratiâ, privato cujuslibet ſtudio committimus.

§. 93.

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93. Neſcio an operæ futurum ſit pretium ſpeculationem quandam, quam per otium ulterius perficere mihi animus fuerat, hic adjungere circa *methodum tangentium inverſam*; qua ſatisfit huic problemati: *Dato valore lineæ cujuslibet ſubtangentis* (*ſeu*, Fig. XXII. *ipſius* TQ *inter applicatam* D *Q tangentemque* D T *interceptæ*) *in terminis æquationis proximis, invenire curvæ huic valori convenientis æquationem.*

Quos autem terminos æquationis *proximos* appellem, exemplo potius manifeſtabitur: ſit Fig. XXII. TQ: t, AQ: x, QD. y, TD: s, curva *AD*: z, cujus æquatio $y^3+fyy = x^3+rxx$, erit juxta §. 17 aut 35, ſubtangens TQ ſeu $t = \frac{2y^3+2fyy}{3xx+2rx}$; hos terminos ſubtangentis valorem exprimentes *proximos* dico, ſiquidem ex æquatione curvæ proxime dimanantes; ſi vero hic ſubtangentis valor per applicationem æquationis curvæ immutetur, Exemp: Grat: ponendo $3y^3 = 3x^3+3rxx-3fyy$ adeoque $t = \frac{3x^3+3rxx-fyy}{3xx+2rx}$; hos voco terminos *remotos.* Idem in cæteris caſibus judicium fiet.

Jam vero ne ſubtangentis valore dato, dubium circa hanc terminorum diſtinctionem oboriatur, ſequens exſurgit ex ſuperioribus terminorum *proximorum* diagnoſis; patetque.

I. Subtangentem t ſemper unius eſſe dimenſionis, ac per fractionem exprimi.

II. Fractionis hujus numeratorem nullos terminos ingredi, in quibus applicata y, aut tangens s (quæ tangens unicam dimenſionem nunquam tranſcendit) non reperiatur.

III. Si termini æquationis fuerit *ſimplices*, nullam interceptam x in numeratore, nullam aut y neque z in denominatore conſpici.

IV. Si *mixtos* continuerit æquatio curvæ, tam x quam y, item z & s reperiri poſſe in utrâque fractionis parte; hac tamen lege, ut, fractione denuo ad æquationem reductâ, quæ nihilo ponatur æqualis, ac omnibus t in x, ac s in z mutatis, nullus reperiatur terminus mixtus, qui juxta indeterminatarum ſuarum, quas continet, numerum repetitus non appareat; numeris præfixis ſingularum indeterminatarum poteſtatum exponentes, ipſorumve rationes, perpetuo exprimentibus.

V. Unde ſequitur terminorum, in quibus eædem indeterminatæ concurrunt, ſigna, poſt peractam per numeros præfixos legitimam diviſionem, fore eadem.

94. Ut ergo ad rem propius accedamus, hoc pacto ſolvitur recenſitum modo problema.

I. Mutato t in x, ac s, ſi occurrat, in z curvam denotantem, omnes termini in eandem partem rejiciantur; ac curioſè diſpiciatur, num *ſimplices* ſint termini, num vero intercurrant *mixti*.

II. Si omnes ſint *ſimplices*, ſinguli per numerum dimenſionum indeterminatarum ſuarum dividantur & habebitur æquatio curvæ.

III. Si *mixti* adſint, & legem diagnoſtici IV. & V. præc. §. obſervent: ſinguli eaſdem indeterminatas continentes per reſpectivum dimenſionum numerum, ad quem indeterminatæ evectæ ſunt, hac lege dividantur, ut ex ſingulis idem reſultet terminus, qui juxta indeterminatarum ſuarum numerum repetitus apparebit.

IV. Quorum terminorum hac ratione ſæpius occurrentium unico tantum ſervato, ſimplicibuſque juxta N°. 2. tractatis, emerget quæſita curvæ æquatio.

Ex I. Detur $t = \frac{y^3 + fyy + rry}{xx + fx + rr}$; juxta N°. 1. mutato t in x, ac rejectis omnibus ad eandem partem erit, $x^3 + fxx + rrx - y^3 - fyy - rry$; cum autem hic omnes ſint ſimplices, juxta N°.

Jo. 2. erit $\frac{1}{3}x^3 + \frac{1}{2}fxx + rrx - \frac{1}{3}y^3 - \frac{1}{2}fyy - rry$ = o, æquatio curvæ deſiderata.

Exemp. II. Detur $t = \frac{3x^3 + 2ryy - 2xyy - xxy}{3xx + 2yx + yy}$

rit omnibus juxta N°. 1. peractis

$x^3 + 2yxx + yyx - 3y^3 - 2ryy + 2xyy + xxy$

bi cum mixti adſint, $2yxx$ & xxy, item yx & $2xyy$ ſinguli ſecundum duarum indeterminatarum numerum bis repetiti, ſi mixtorum indeterminatas continentium alter per imenſiones ipſius x, alter per dimenſiones ipus y juxta N°. III. dividatur, orientur termi $yxx + yyx$ ſecundum *N. IV*; *ſimplicibuſue* per ſuarum indeterminatarum exponentes ivisis, naſcetur æquatio curvæ requiſita

$^3 + yxx + yyx - y^3 - ryy = 0$.

Exemp. III. Ne autem æquationes, quæ urvas involvunt, neglexiſſe videar, ſit, poſitâ loco curvæ,

$$= \frac{6ry^3zz + 4ry^3zs + rry^4 - yxxz^3 - 3yxxzzs}{2yxz^3},$$

ui oportet æquationem curvæ congruam exhiere. quare juxta N°. 1, t in x, ac s in z ver, omnibuſque terminis ad eandem partem reactis erit

$2yz^3xx + yz^3xx + 3yz^3xx - 6ry^3zz$
$-4ry^3zz - rry^4$

bi terminus yz^3xx tres continens indeterminas ter, ac ry^3zz duas habens indeterminatas,

bis repetitur, ſolo rry^4 exiſtente *ſimplici*, cumque etiam pateat terminos *mixtos* per indeterminatarum reſpectivum dimenſionum numerum diviſos eundem quotientem relinquere, fractionem ipſam t exprimentem in terminis æquationis *proximis* datam eſſe jam certi ſumus, juxta §. 93. enumerata criteria: adeoque *ſimplici* ex more tractato, & mixtorum iiſdem indeterminatis conſtantium unico retento, exoritur æquatio curvæ $yz^3xx - 2ry^3zz - \frac{1}{4}rry^4, = 0$ aut additâ qualibet determinatâ quantitate, $yz^3xx - 2ry^3zz - \frac{1}{4}rry^4 + p^6 = 0$.

NB. Determinatam quantitatem ſemper inventæ æquationi addi poſſe, ex tangentium methodo directâ patet, cum in ſubtangenti inquirendâ continuo excidat; quin & aliquando neceſſarium videtur.

Exemp. IV. Detur $t = \frac{-xy}{2x+y}$ erit $2xx + xy + xy$ adeoque $xx + xy = 0$, æquatio quæſita, ſeu additâ determinatâ $xx + xy = pp$, cum primo inventa æquatio, non niſi falſam radicem admittat.

95. No-

95. Notetur, quod in ſequentibus uſum aliquem præſtiturum eſt, ex methodo modo traditâ ſequi, quo pacto, *Fig. XXII.* ex dati *intervalli* cujusvis *QR* (quam lineam & *ſubnormalem* in ſeqq· vocabo) valore analytico curvæ *ADE* æquatio conveniens indagari poſſit, poſitâ enim $QR = l$, eſt perpetuo ſubtangens ſeu $t = \frac{yy}{l}$, uti notum; quæ ſubtangentis expreſſio ad terminos proximas reducta, juxta §. 94. requiſitam æquationem prodet.

Schol. Præcedentium demonſtratio ex §. 14, 17, 35. non difficulter colligetur; cum, quæ inibi circa methodum tangentium directam præcipiantur, hic ordine retrogrado ſint tradita: ac ſolus jam terminos *remotos* ſubtangentis valorem exprimentes ad *proximos* reducendi modus hac in parte deſiderari videtur, quod effectum dabitur, ſi cognita ſit methodus æquationem datam in duo loca, vel duas potius æquationes, reſolvendi, quæ datam inter ſe legem & conditionem obſervant, talem ſcilicet, qualis inter æquationem curvæ, eamque, quæ ſubtangentis valorem exprimit, ex ſupra traditis requiritur. Quod, quam variis modis tentari queat, æquationum conſtructiones palam faciunt; hoc enim caſu æquationem datam quamcunque in infiniti generis loca dirimi, in eaque quotlibet fere quantitates, ex aſſumtæ hypotheſeos inſtituto, ſeu æquationum, tum ipſius curvæ naturam, tum ejus ſubtangentem exprimentium relatione mutua, determinationem patientes invehi poſſe, poſt editum Celeberr: *Sluſii Meſolabum* nemo ignorat.

Cumque jam termini ſubtangentium proximi ex methodis §. 14, 17, 35, oriundi ex facili dignoſcantur; ubi nimirum curvarum æquationes per unius pluriumve cur-

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varum interceptas applicataſque, quin & ipſas reſpectivas curvas exprimuntur; reſtaret hanc regulam extendendi modus ad ejuſmodi curvas, quarum indeterminatæ inter poteſtatum exponentes reperiuntur, uti §. 47. & ſeqq: tum & ad alias, quarum æquationes non, niſi mediantibus infiniteſimis, exprimi valent; quas inter ſequentia magis commode tractabimus: licet utrumque hoc non tantum ſed & omnia curvarum genera ad methodum §. 35 expoſitam referri valeant. Hanc autem ſubtangentium ſubnormaliumque varietatem exactius diſcutere, ac ad unum pluresve, omnibus tamen inverſæ methodi caſibus ſufficientes, canones reducere, noſtri nunc eſt nec inſtituti, nec otii: non enim huc ſpectantes methodos omnes, quæ animo adhucdum obverſantur, explorare permiſit temporis anguſtia. Interim, ne curioſioribus inquirendi occaſionem præſcindam, paucula quædam tentamina ſubjicio; inſimulque quomodo ex ſingulis immenſum exemplorum ac theorematum agmen, citra ullam calculi moleſtiam, educi queat, exhibito levi ſpecimine oſtendam.

96. Quem in finem, quamcunque terminorum ſummam tum determinatorum, tum *ſimplicium x* continentium, quibuslibet ſignis affectorum, ut $x^3 - bxx$ &c. $+ r^3$ voco z; ac per eandem z, aut ſolam, aut in quotlibet x ductam, ut zx, zxx &c. ſi per majuſculam quamlibet vocalem A multiplicetur, ut (Az) (Azx) &c. denoto omnem ipſius z terminorum ſeriem, cum ſinguli termini per dimenſionum ipſius x numerum ſunt multiplicati: ſic $(Az) = 3x^3 + 2bxx$ &c. & $(Azx) = 4x^4 + 3bx^3$ &c. Ubi vero z

vel

vel zx &c. per eandem A dividitur, ut $\left(\frac{z}{A}\right)$ $\left(\frac{zx}{A}\right)$ &c. deſignatur eadem terminorum ipſius z ſeries, cum ſinguli termini per reſpectivum dimenſionum ipſius x numerum ſunt diviſi, ſic $\left(\frac{z}{1}\right) = \frac{1}{3}x^3 + \frac{1}{2}bxx$ &c. & $\left(\frac{zx}{A}\right) = \frac{1}{4}x^4 + \frac{1}{3}bx^3$ &c

Sic & ſumma terminorum *ſimplicium* y continentium vocata f, ſi per majuſculam quandam conſonantem Verb: Gr: B multiplicetur, deſigno per (Bf) aut (Bfy) hanc ſummam ſimiliter multiplicatam; ſin vero per B dividatur, erit $\left(\frac{f}{B}\right)$ aut $\left(\frac{fy}{B}\right)$ ſumma eadem ſimiliter diviſa.

NB. Has quantitates parentheſi includo, cum non niſi junctim in æquationis diviſione aut multiplicatione &c. adhiberi poſſint, uti ex earum naturâ cognitâ ſatis innoteſcit.

Tandem quantitates determinatas, dimenſionibus æquandis quandoque inſervientes, appello m, ſubnormalem: l, ac ſubtangentem: t, dico.

97. Sit $\frac{z}{f} = l = \frac{yy}{t}$, erit $zt = fyy$; unde æquatio juxta §. 94, Ex. I. huic ſubtangenti conveniens eſt $\left(\frac{zx}{A}\right) = \left(\frac{fyy}{B}.\right)$

Corol. I. Jam facto $f = m$, erit $l = \frac{z}{m}$, & $B = 2$; unde æquatio $\left(\frac{zx}{A}\right) = \frac{myy}{2}$, patetque ſubnormalium valores per fractionem $\frac{z}{m}$ expreſſos omnes eſſe ad terminos *proximos*, adeoque & ad curvam congruam reducibiles.

Coroll. II: Quod ſi fiat $f = y^p$, erit $l = \frac{z}{y^p}$ & juxta §. 94. *Exemp. I.* reducibilis ad hanc æquationem $p+2 \left(\frac{zx}{A}\right) = y^{p+2}$; cujus ope l reducta ad x fiet $= z$ diviſæ per $\sqrt{p+2,}\left(\frac{zx}{A}\right)^{\frac{p}{p+2}}$, & facta $\frac{p}{p+2} = \frac{q}{k}$ fiet $p+2 = \frac{2k}{k-q}$; rurſumque poſita $q = 1$, erit $p+2 = \frac{2k}{k-1}$, hoc vocetur h, erit $l = z$ diviſæ per $\sqrt[\frac{1}{k}]{h\left(\frac{zx}{A}\right)}$ & æquatio curvæ reſpondentis $= h\left(\frac{zx}{A}\right) = y^h$.

Cum autem z hic per quantitatem irrationalem ſit diviſa, notum eſt, ſi poteſtas ſigni radicalis $\frac{1}{k}$ fiat negativa, ipſam z per eandem quantitatem multiplicatum iri: quo in caſu erit $h = \frac{2k}{k+1}$, & æquatio $h\left(\frac{zx}{A}\right) = y^h$, mutato juxta

ı ſtatim inſinuata ipſius *h* valore.

Coroll. III. Unde ſolvitur hoc problema: ata quantitate irrationali ex terminis, tum deerminatis, tum ſimplicibus *x* continentibus, uolibet radicali communi ſigno copulatis contanti, duas alias quantitates *z* invenire; ut, harum altera per dictum irrationalem dividaur, altera vero per eandem multiplicetur, in troque caſu emergat ſubnormalis ad terminos roximos, adeoque ad curvam convenientem, educibilis; ipſiuſque curvæ æquationem deterninare.

Sit $\sqrt[k]{xx+rx+rr}$ expoſita quantitas irratioalis; multiplicentur termini ipſam componenes per dimenſiones reſpectivas ipſius *x* (unde erminus determinatus excidet, cum *x* in eo limenſiones habeat nullas) & fiet $2xx+rx$; um ſinguli per *x* in duplum exponentis ſigni adicalis ductam dividantur, & emergit quotiens $\frac{x+r}{2k}$;

Qui ſi multiplicetur per exponentis ejuſdem ıc unitatis differentiam emerget $z = k-1$ in $\frac{x+r}{2k}$, quod primo caſui inſervit, cum *z* dividitur per quantitatem expoſitam irrationaem.

Si vero idem quotiens per exponentis dicti ac unitatis ſummam multiplicetur, orietur $z = \overline{k+1}$ in $\frac{2x+r}{2k}$, quod in ſecundo caſu requiritur, ubi z multiplicatur.

Demonſtrationem ex præmiſſis quilibet eruet, eamque proinde cum cæteris hinc deducendis (ſi z & f, ad terminos quoſvis mixtos extendantur, ipſique ſignorum exponentes q & k, quam pati poſſint, varietatem admittant) brevitatis ergo prætereo, æquatio curvæ mox inſinuata eſt.

98. Reaſſumatur *Fig. XI.* ſervatiſque §. 36. ſymbolis, ac §. 37. hypotheſi $fz = yy$, invenitur æquatio ſubtangentialis $nt + kt = 2kn$, unde $l = \frac{yy}{t} = \frac{fz}{t} = \frac{nfz + kfz}{2kn} = \frac{fz}{2k} + \frac{fz}{2n} = \frac{yy}{2k} + \frac{yy}{2n}$.

Coroll. I. Unde ſequitur, duabus curvis ad eundem axem deſcriptis, ſi correſpondentium applicatarum rectangulum fz per utriuſque curvæ ſubtangentem, n & k ſucceſſivè dividatur, erit quotientum ſummæ ſemiſſis æqualis ſubnormali l, ad tertiam quandam curvam pertinenti, cujus ordinatæ quadratum yy æquivalet fz ſeu expoſito applicatarum in reliquis curvis rectangulo; $l = \frac{fz}{2k} + \frac{fz}{2n}$.

Coroll. II. Hinc etiam patet problema Cor: §. præcedent: infinitis modis ſolvendi methodus; poſitâ enim z æquali cuidam quantitati irrationali,

nali, ſive uno ſive pluribus vinculis radicalibus ligato, erit $\frac{nf+kf}{2kn}$ ipſam multiplicans quantitas; quæ, cum f ſit arbitraria, infinitis modis variari poteſt.

Schol. Quoniam vero curvæ non tantum tres, uti modo, ſed pro arbitrio multæ aſſumi, interque ſingulas infiniti generis relationes ſtatui poſſint, quin & ipſæ curvæ iis admiſceri; quantus ſe pandat reducibilium ſubnormalium tangentiumque campus, quarum curvæ abſque ulla calculi moleſtia ſtatim ſe produnt, nemo non videt, ſufferit hic digitum intendiſſe. Quin & vario, præter modo tradita, principiorum genere oriundas methodos, ſubnormales ad curvas congruas reducendi his adjungere in proclivi eſſet, niſi peculiarem ſibi tractatum potius depoſceret materiæ dignitas: quare è multis unicam, ex infiniteſimarum calculo originem trahentem annecto, & hoc ex anterioribus demonſtrabile præmitto lemmaticum.

99. Sit indeterminata x, creſcens aut decreſcens per infiniteſimam e; y per a; h per i, &c. erit expoſita quæcunque æquatio ad infiniteſimas redacta (in quâ indeterminatæ ſingulæ in proprias ſibi infiniteſimas, determinatasque ductæ ſint) ſequenti modo ad æquationem ordinariam reducibilis: ſi, nimirum, mutatâ infiniteſimâ in propriam ſibi indeterminatam, emergens terminus dividatur perpetuo per exponentem dimenſionum indeterminatæ

Exemp: Gr. detur, $rhi + hhi + \frac{6h^3 i}{r} = yya + rya$, erit hæc reducibilis ad hanc æquationem $\frac{1}{2}rhh + \frac{1}{3}h^3 + \frac{3}{2}\frac{h^4}{r} = \frac{1}{3}y^3 + \frac{1}{2}ryy$, & ſic in cæteris.

100. Jam cum juxta §. 23. in omnibus curvis ſit $ya = le$, fiat $l = \frac{hz}{k}$; poſitis h, k, z, pro quantitatibus vel ex arbitrio, vel ex progreſſu calculi determinandis; erit $\frac{hze}{k} = ya$ adeoque $e = \frac{kya}{hz}$; tum aſſumtâ h pro quotlibet terminis ſimplicibus x continentibus, ſignoque radicali cujuslibet poteſtatis q copulatis, qui ab aſymmetriâ liberati litterâ g deſignentur, erit $h = g^{\frac{1}{q}}$, & $h^q = g$; quæ æquatio ad infiniteſimas redacta juxta methodos ſuperiores, erit $qh^{q-1}i = \frac{(Ag)e}{x}$ & ponendo $e = \frac{kya}{hz}$ fiet tandem, $qh^qzxi = (Ag)kya$. quæ ultima vocetur *æquatio reducenda*; cujus reductio poſſibilis exiſtit, ſi in alterâ parte exceptis determinatis, nulla indeterminata niſi h, in alterâ vero nulla niſi y reperiretur, juxta præcedentem; quot autem modis hoc effici poſſit ſequentia conſectaria manifeſtabunt. Quotlibet determinatæ denotentur per m, & ſingulæ per r.

Coroll. I. Sit $h^q = mx + m$, unde $(Ag) = mx$; & $k = r$, erit æquatio reducenda $qh^qzi = mrya$; quæ reducibilis eſt, I. ſi z per quotlibet terminos, vel h continentes, vel determinatos, exprimatur. II. Si z quotlibet terminos ſimplices x continentes, communi vincu-

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o non adſtrictos comprehendat; ſiquidem, cum ex hypotheſi fuerit $h^q = mx + m$, ſit $x = \frac{h^q - m}{m}$ qui valor in omnibus ipſam z componentibus terminis repoſitus efficiet terminum $qh^q zi$, qualem primo loco deſideravimus.

Unde patet ſi fuerit $\overline{l = x^p + rx^n}$ &c. in $\overline{\sqrt{rx+rr}}^{\frac{1}{q}}$ hoc eſt, ſi ſubnormalis valor exprimatur per quodlibet ſignum radicale (ſub quô x ad plures unicâ dimenſiones non aſſurgit) in quotlibet numero terminos ſimplices x continentes multiplicatum; erit ad curvam, ex præcedenti, reducibilis; cujus æquatio levi negotio ſe pandet.

Coroll. II. Si vero fiat $h = \sqrt{Cx^3 + rxx}$, erit $q = 3$ & $(Ag) = 3x^3 + 2rxx$; poſitoque $(Ag) = zx$ erit $3xx + 2rx = z$; unde factâ k determinatâ erit $kl = \frac{3xx + 2rx}{\sqrt{Cx^3 + rxx}}$; & æquatio reducenda $qh^q i = mya$, ſeu $3h^3 i = mya$: quæ reducta dabit æquationem curvæ $\frac{3}{4}h^4 = \frac{1}{2}myy$, ex æquatione reducendâ, juxta §. 99, reſultantem.

Hinc ſequitur, poſito $(Ag) = zx$ ſeu $\left(\frac{Ag}{x}\right) = z$; ſi h ſit æqualis cuilibet quantitati irrationali x continenti; ac z eam multiplicans ſit æqualis ſummæ terminorum ſub ſigno radicali copulatorum

torum

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rum in exponentes ipſarum x reſpective ductorum, ac unicâ x per diviſionem multatorum: l ſeu $\frac{bz}{m}$ ſemper fore reducibilem ad curvam, cujus æquatio $\frac{q}{q+1} h^{q+1} = \frac{1}{2} myy$.

Coroll. III. Quod ſiquis alterius generis poriſmata deſideret, multiplicet æquationem reducendam $qh^q zxi = (Ag) kya$ utrinque per $y^f h^b$, emerget: $qh^{q+b} y^f zxi = (Ag) k h^b y^{f+1} a$. Quæ, facto $qy^f zx = (Ag) h^b$, relinquet $h^{q+} i = my^{f+1} a$; unde æquatio curvæ, omnibus his caſibus congruens $\frac{1}{q+b+1} h^{q+b+1} = \frac{1}{f+2} my^{f+2}$. Ex: Gr: $h = \sqrt{xx+rr}$, fiat $f = 0$ & $b = 2$; erit $q = 2$, unde $(Ag) = 2xx$, & $2zx = xxhh$; quare $z = x^3 + rrx$, ac kl ſeu $ml = zh = x^3 + rrx\sqrt{xx+rr}$, & æquatio curvæ correſpondentis $\frac{1}{3} h^3 = \frac{1}{2} myy$.

Schol. Cum autem hic ex data h ſemper ipſa z determinetur; quod primario requiritur tamen, hoc eſt, ut, datis tum h tum etiam z, inveniatur æquatio curvæ, cujus ſubnormalis l analyticè effertur per $\frac{hz}{k}$; hoc eſt, ut dato quolibet intervalli valore curva ipſi conveniens reperiatur, unde problema hocce generaliter ex parte ſolutum eſt. Quem in finem, cum exponentes b & f adhuc aptæ ſint determinationem recipere, poterit earum ope curvæ æquatio indagari, quæ aliquando erit geometrica, aliquando ex earum genere, quas tractavimus §. 47. & ſeqq:; aliquando alîus generis. Accedit, quod, ſi termini mixti conſiderentur, cum

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rum hactenus non niſi ſimplices adhibuerimus, tum ſi in æquationem reducendam aliæ ex calculo poſtea determinandæ quantitates ingerantur, variis modis intentum poſſimus aſſequi; inſimulque experiri, quam curvæ ſpeciem æquatio requiſita exprimat. Hæc autem ob cauſas ſupra memoratas in aliam occaſionem reſervamus, iis interim ſua in rem Mathematicam merita gratulaturi, quibus hæc ad finem perducendi vel otium vel facultas ſuppetit.

101. Sit *F. XXIV.* curva *ILO* ad axem *KP* ope applicatarum *TL*, *BI*, pertinens, eamque tangat in *L* recta *NLI*; oportet deſcribere aliam curvam *ADE*, ejus naturæ, ut, ſi ducatur eam tangens *TD*, ſit ſemper *QT* ad applicatam *QD*, ut *KT*, ad *TL*. In quem finem ductâ *KM* parallela *TL*, jungatur *LS* parallela *KT*; ſumtâque *TB* infiniteſimâ, compleatur parallelogrammum *BIMK*; hinc productis *ST* & *MB* verſus *D* & *E*, donec ſe mutuo ſecent in *E*; ſumatur *DE* infiniteſima, ac demittantur normales *DQ*, *EP*, erunt *D* & *E* puncta in curva *ADE* quæſita, quam tangit *DT*. vocentur jam $KT: x$. $TL: f$. $ST: g$. $NT: b$. $TQ: t$. $QD: y$. $TD: s$; infiniteſimæ, $TB: i$. $CB: \theta$. $TC: \lambda$. $GI: \pi$. $SR: \mu$: $MR: \nu$. $DE: u$. $DH: QP: e$. erit ergo $SR : SE :: CB : CE$. ſeu $\mu : g + s + u :: \theta : s - \lambda + u$, unde rejectis per Lem. 10. rejiciendis $= \frac{s\mu}{g+s} = \frac{yi}{s}$ (nam $TB, i : CB, \theta :: TD, s : DQ, y$.) adeoque $\mu = \frac{gyi + syi}{ss}$.

Rurſus quoniam $MB, g + \nu + \lambda : KB, x + i :: SM, \pi : SR, \mu$. erit juxta Lemm. 10,

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$\pi = \frac{g\mu}{x} = \frac{fi}{b}$ (nam *NT*:*TL*::*LG*:*GI*. ſeu $b:f::i:\pi$.) unde $\mu = \frac{fxi}{gb} = \frac{gyi+syi}{ss}$ ſeu $ssfx = ggyb + gbsy$; in quâ æquatione poſitis, tg loco sx, & poſtea yx loco ft, (cum ſit $g:x::s:t$, & $f:y:x::t$.) emerget $sx = gb + bx$, unde tandem $s = \frac{gb}{x-b}$; quare, datis x, b, & g magnitudine & poſitione, datur in producta *ST* ſeu g, linea *TD* ſeu s, & punctum *D* in curvâ *ADE* quæſitâ: unde ſequens oritur analogiſmus $x-b:b::g:s$. & propter triangulorum *SKT* & *DQT* ſimilitudinem $x-b:b::g:s::f:y::x:t$.

Exempla non addo; cum *KT* + *QT* ſeu $t+x$ aut *KQ* vocatâ n, æquatio inter n & y ex facili inveniatur, tum ſi curvæ, *Fig. XXV*. ab eâdem axis parte ſint poſitæ, erit s hoc in caſu $= \frac{gb}{b-x}$ & $QT - KT = n = t - x$; uti ſchema inſpicienti ſatis apparet: omnes enim hujus problematis caſus percurrere, majoris foret operæ, quam utilitatis.

CA-

CAPUT II.

De Planorum Curvilineorum dimenſionibus.

1. EX lemm. 26 & 27, curva *ABCDE*F *Fig. XXVI.* rectulis conſtat, angulos in *B*, *C*, *D*, *E*, &c. facientibus; unde ſpatium *A*F*O* (cujus angulum *AO*F hic rectum ſupponimus) polygonum eſt; quod polygonum ſi lineis ipſi F*O* parallelis *EP*, *DQ*, *CR* &c. parteſque axis infiniteſimas æquales aut inæquales *PQ*, *QR*, *RS* &c. intercipientibus, dividatnr trapezia *DQPE*, *CRDQ* & ſimilia; erit omnium trapeziorum ſumma ex Geometriæ legibus æquale ſpatio infinitè polygono ſeu curvilineo *ABCDE*F*OA*.

2. Idem conſtat; ſi planum in trapezia *EGID* ope linearum ipſi *A*O æquidiſtantium dividatur. Quibus principiis omnis fere curvilineorum menſura hactenus nota abſolvitur.

3. Ad inveniendum ergo trapezii *D*Q*PE* valorem, ſit *D*Q: *y*. *DH* ſeu Q*P*: *e* infiniteſima, ſit & *EH*: *a*; quare *EP*: *y*+*a*, unde trapezium *D*Q*PE* æquatur facto ex *D*Q+*PE* in ½ Q*P*. ſeu $\frac{2ye+ae}{2}$; & deleto (juxta lemm. 10.) infiniteſimarum rectangulo *ae*, erit trapezium

pezium $DQPE = ye =$ rectangulum $DQPH$.

4. Hinc, poſita y ad locum curvæ cujuslibet, erunt omnia ye æqualia ſpatio curvilineo quæſito: Unde directè hoc theorema: ſi axis figuræ AO dividatur in partes infinininteſimas æquales aut inæquales AS, SR &c. & ad ſingula diviſionum puncta eleventur applicatæ SB, RC, QD. quarum ſingulæ parte infiniteſimâ DM, CN, &c. proximam excedunt; erunt omnia rectangula ex ſingulis applicatis in ſingulas infiniteſimas ſimul ſumta, ſeu omnia ye, æqualia curvilineo $AOFA$.

5. Si vero fuiſſet $EP:y$, unde $DQ:y-a$, cæteris ut ſupra, foret idem trapezium $DQPE$ æquale rectangulo $ZQPE$ æquale rectangulo $ZQPE$ ſeu ye, juxta modo tradita. Quæ & in figuris verſus axem AO convexis obtinent.

6. Quin & hinc ſequitur; *Fig. XXVII.* ſi fuerint duo ſpatia AOF, AOL, in eâdem altitudine AO poſita, factiſque applicatis, $QD:y$, $QR:z$, & in utrâque QP infiniteſimâ ſemper æquali: e; primum ſpatium æquari omn. ye, & ſecundum omn. ze: quare primum ad ſecundum, ut omn. ye ad omn. ze; & (diviſis rectangulis per æqualem altitudinem e) ut omnia y ad omnia z; hoc eſt ut ſumma applicatarum in primâ ad ſummam applicatarum in ſecundâ figurâ.

7. Huic,

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7. Hinc, ſi y ad z, conſtantem in ſingulis rationem habeat, ut r ad s; erit quoque, juxta lemm. 43, ut r ad s ita omn: y ad omn: z.

8. Quibus adde, ſi ad hæc duo ſpatia aſſumatur tertium, cujus applicata *QH*: h; & fiat perpetuo $r : s :: y : z \mp h$; erit quoque $r : s ::$ omn. y: omn. $z \mp h$; hoc eſt ut r ad s, ita figura prima ad ſummam vel differentiam ſecundæ & tertiæ: idque tam in integris *AOF*, *AOL*, *AOM*, quam partibus *AQD*, *AQR*, *AQH*; modo eadem ſervetur altitudo, & infiniteſimarum conſtans æqualitas.

9. Ac idem evenire patet, quotcunque figurarum applicatæ quibuslibet ſignis copulatæ aſſumantur; modo, quæ mox inſinuavimus, ritè obſerventur.

10. Sit jam *Fig. XXVIII.* curva *QADEF*; non ad axem, ſed ad punctum *Q* relata. Aſſumtâ *DH*, infiniteſimâ, ductisque *QD*, *QE*, jungatur *DH*, ut *QD* ſit æqualis *HQ*; dicatur *QD*: y. infiniteſima *DH*, e, *HE*, a; et triangulum *QDE* æquabitur facto ex *QE* in *DH*, ſeu $\frac{ye+ae}{2}$ juxta lemm. 33, & juxta lemm. 10. $= \frac{ye}{2}$ adeoque totum ſpatium *QAF*, quod æquatur omnibus triangulis *QDE*, *QEF* &c. æquale erit omnibus $\frac{ye}{2}$,

ſeu omnibus triangulis QDH; unde ſimilia, ac ſupra, deduci poſſent.

11. Sit *Fig.* XXIX. figura AFO eadem quæ §. 1; ſymbola, quæ in §. 3; tangat TD curvam in D, voceturque $TQ: t$; erit propter triangulorum TQD, DHE ſimilitudinem, $TQ, t: QD, y :: DH, e: EH, a$. unde $ye = ta$: quare ob parallelas EG, DI, exiſtente $GI = a$, ſi linea TQ, in IM, normalis ipſi FI, ſtatuatur; erit rectangulum $GIMX = ta$: idque ſi fiat toties, donec per puncta MLK, deſcribi poſſit curva $KLMNO$; conſtabit ſpatium KFO totidem ta, quot rectangulis ye figura AFO conſtat: quare, quia ſemper $ye = ta$, erunt omnia $ye =$ totidem ta, hoc eſt ſpatium AFO æquale ſpatio OFK.

Coroll. I. Sit AFO paraboliformis, & AQ vocatâ x, ejus æquatio $r^p - q x^q = y^p$; erit, juxta §. 19, Cap. I, TQ ſeu $t = \frac{px}{q}$; unde ſpatium $FOK =$ omn. $\frac{pxa}{q}$; jam vero quia $RD = AQ = x$, & $BR = a$, erit trilineum SAF æquale omnib: $BRDE$ ſeu omn. xa, ex §. 4, ſunt autem omn: $\frac{pxa}{q}$ ad omnia xa ut p ad q; ſed omnia $\frac{pxa}{q} =$ omn: ye; ergo omn. ye ſeu data parabola AOF, ad complementum SAF ſeu omn. xa, ut p ad q: hoc eſt parabola AFO ad rectan-

rectangulum circumſcript. *SAFO*, ut p ad $p + q$; ſeu ut exponens poteſtatis applicatarum, ad exponentium applicatarum & partium axis ſummam.

Coroll. II. Sit *Fig. XXX.* hyperbolöides quodlibet *CEDV*, ad aſymtotos *KA*, *AO* relatum, cujus æquatio, retentis ſymbolis ſuperioribus $y^p x^q = s^p r^q$; erit ob tangentem *DT* linea *QT* $\frac{px}{q}$: jam vero, cum ſit $ye = ta$, erit poſito perpetim *QT* ſeu t in *IM*, rectangulum *DQPH* ſeu ye, ſemper æquale *MIGL* rectangulo ſeu ta; adeoque, ſi id fiat continuo, erit figura *KAOBC* verſus *KC* interminata, ſeu omnia ye, æqualis figuræ *KNSH*, ſeu omn. $ta =$ omn. $\frac{pxa}{q}$. atqui (vocatis brevitatis cauſâ figurâ *KAOBC*: f, & rectangulo inſcripto *NBOA*: h,) erit figura *KNBC*: $f - h =$ omn. xa, (cum *ID* ſit $= x$, & $GI = a$) unde omnia $\frac{pxa}{q}$ ſeu omnia ye ſeu f, ad omnia xa ſeu $f - h$ ut p ad q; & $p : p - q :: f : h$; hoc eſt figura *KAOBC* ad rectangulum inſcriptum *NAOB*, ut exponens poteſtatis applicatarum p, ad differentiam exponentium applicatarum & partium axis $p - q$. unde ſequitur, poſitâ q minori quam p, curvam verſus *K* interminatam eſſe menſurabilem; ſi $p = q$, infinitam: ſi q major quam p, pluſquam infinitam: item hyperbolöidea omn. (ſolâ Apollonianâ exceptâ) ab unâ par-

parte *K* terminabiles, ab altera *V* pluſquam interminabiles exiſtere; cum, quæ illic fuit ratio p ad $p-q$, hic ſit q ad $q-p$, quare conſtat. ſed hæc in tyronum gratiam.

Coroll. III. In tranſitu noto (ſi ſolam è cono ſectam hyperbolen reliquam facias) *Fig. XXXI.* omnia ſpatia *CBDE*, *EFGH*, inter aſymptoton & hyperboliformem quamlibet conſtituta, ac utrinque applicatis *CB*, *DE*, ſeu *EF*, *GH*, ad eandem aſymtoton terminata, menſuram admittere; quod coroll: præcedentis non difficile conſectarium eſt. Unde rurſum infinita curvarum genera quadrantur; factis enim *HG*: y, *AG*: x, *EF*: s, *AF*: r; ſit huic curvæ conveniens æquatio $y^p x^q = s^p r^q$; jam autem productâ *AE* ſeu b verſus *N*, huic ad perpendiculum excitetur recta *NH*; & vocatis *AN*: z, *HN*: f, ſi ad has ipſas æquatio ab aſymtotis reducatur, prodibit $\overline{sz - rf}^p$ in $\overline{rz + sf}^q = b^{p+q} s^p r^q$, quæ facto $r = s$ fiet $\overline{z-f}^p$ in $\overline{z+f}^q = b^{p+q}$.

Trilineum jam *NEH* eſſe quadrabile nemo non videt, cum juxta mox propoſita quadrilineum *EFGH* menſuram patiatur, modo p & q diverſæ ſint magnitudinis.

Coroll. IV. Abundè quidem hinc innoteſcit, *Fig. XXXII.* & *XXXIII.* datâ cujuſvis generis pa-

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parabolâ, ejuſve trilineo, aut hyperbolâ, & exiſtente applicatâ $QD:y$, interceptâ $QA:x$; rum $QR = OF:b$, axe $AO:c$, fore omnia QR ſeu b ad totidem ſeu omnia QD ſeu y, ut $p \mp q$ ad p. Si vero quæratur, quam habeant rationem omnia b ad quamlibet poteſtatem k evectæ, ad totidem y in eâdem poteſtate conſtituta, ſeu omnia b^k ad omnia y^k, ſequenti modo inveſtigari poteſt; ac primo, in *F. XXXII.* ſeu paraboloïdibus: propter æquationem $r^{p-q}x^q = y^p$, eſt $y^k = r^{\frac{pk-qk}{p}} x^{\frac{qk}{p}}$; fiat jam $y^k = r^{k-1}z$, erit valore hoc in locum ipſius y^k repoſito, ac diviſione legitimè perfactâ $r^{p-qk} = z^p$, quæ æquatio denuo eſt ad parabolam; ſit ea AGO, ubi manente $AQ:x$; & $AO:c$, ſit $QP:z$; & $OG:d$; jam vero ex Coroll. I, ut $p+qk$ ad p, ita omnia GO ſeu d ad omn: PQ ſeu z; hoc eſt ita omnia b^k ad omnia y^k. cum enim mox fuerit ſuppoſitum $y^k = r^{k-1}z$, erit eandem ob cauſam $b^k = r^{k-1}d$. unde conſtat ratiocinium.

Idem ſi fiat in hyperboloïde *LAOFDL*, *Fig. XXXIII.* cujus hic æquatio, retentis ſymbolis $y^p x^q = b^p c^q$, fiatque $y^k = b^{k-1}z$, invenietur ratio omnium y^k ad omnia b^k, ut p ad $p-qk$. Poſſet &, reducendo hyperboliformem hanc æquationem ad paraboliformem, idem

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unicâ operâ obtineri; cum enim, juxta hanc reductionem, fiat $y^p = b^p c^q x^{-q}$, erit interceptæ exponens hic negativus, adeoque quæſita ratio juxta parabolæ leges ut p ad $p - qk$; & ſequitur utiliſſimus hicce canon utrique caſui congruens: *In indagandâ ratione omnium y^k ad omnia b^k, in hyperbolöide pariter ac paraboliformi, primò, ſi opus ſit, æquatio eò reducatur, ut x & y ab oppoſitis æquationis partibus conſpiciantur, hoc eſt, ut paraboliformem repræſentet; eritque, ut dignitatis applicatarum, ac dignitatis interceptarum per quæſitam poteſtatem multiplicatæ ſumma: $p \mp qk$; ad dignitatem applicatarum p, ita omnia b^k ad omnia y^k.*

Eſſe autem interceptæ in parabolöide poteſtatem $= q$ in hyperbolöide $= -q$ poſt reductionem ſatis apparet. Notetur ipſum k ad quoſvis, tum fractos, tum negativos, exponentes extendi poſſe. Exempla his caſibus inſervientia tranſilio.

Coroll. V. Deducitur hinc celebratiſſima *Cl. Walliſii Arith: Infinit:* prop. LXIV, cum ſit p ad $p \mp qk$, ut unitas ad unitatem $\mp \frac{qk}{p}$; ſi enim curvarum æquationes ad quæſita y^k reducantur, emerget pro parabolâ ſeu ſerie directâ $y^k = r^{\frac{pk-qk}{p}} x^{\frac{qk}{p}}$; pro hyperbola verò ſeu ſerie inverſâ $y^k = b^{\frac{pk}{p}} c^{\frac{qk}{p}} x^{-\frac{qk}{p}}$; quare, ſi po-

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…ſtas ipſi x ſeu interceptæ adhærens ſumatur …ro ſeriëi indice, erunt, juxta modo inventa …mnia y^k ad omnia b^k ut unitas ad interceptæ …oteſtatem (cum ipſam y in requiſitis dimenſio…ibus conſtitutam repræſentat) ſeu ſeriëi indi…em unitate auctum.

Coroll. VI. Et hactenus quidem cognita ra…io omnium y^k ſeu (ducendo *Fig. XXXII.* …trumque in infinitefimam $QN: e$) omnium …$^k e$ ad omnia $b^k e$; ut vero valor eorum abſo…utus alio modo inveniatur, erat, ex modo præ…greſſo Coroll. IV, ut $p \mp qk$ ad p, ita omnia …e ad omnia $d\,e$, hoc eſt ita paraboloïdes AGO, …d rectangulum $AOGK$ ſeu $d\,c$, quare $\frac{p\,d\,c}{p \mp qk}$ $=$ omnia $z\,e =$ omnia $\frac{yk}{r^{k-1}}e$; ſed ex eadem Coroll. IV, eſt $b^k = r^{k-1}d$; unde valore …er hanc æquationem invento ipſi d ſuffecto, …emerget omnium $y^k e$ valor abſolutus $= \frac{p\,c\,b^k}{p \mp qk}$ $= \frac{1\,c\,b^k}{1 + \frac{qk}{p}}$.

Coroll. VII. Quod ſi (facto indice $\frac{qk}{p} = n$) in …invento valore omnium $y^k e = \frac{1\,c\,b^k}{1 \mp n}$, lo…co b^k, ſurrogetur æquivalens quantitas per

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curvæ utriuſque æquationem reperta; hoc eſt, in parabola $r^{k-n}c^n$; in hyperbola $b^k c^n c^{-n}$; exprimentur omnia $y^k e$ in parabolâ per $\frac{c^{1+n}}{1+n}$ in r^{k-n}; aſt in hyperbolâ per $\frac{c^{1-n}}{1-n}$ in $b^k c^n$.

Unde videtur naſci lemma a *Clar: Mercatore* prop. XVI. Logarithmotechn: ſuæ demonſtratum: cuique ſoli exercitationem ſuam ſuperſtruxit ingenioſiſſ: *D: Gregorius*, cum enim ſint omnia $y^k e$ = omni $r^{k-n}x^n e$ in parabolâ, & in hyperbol: $b^k c^n x^{-n} e$; manifeſtum eſt (rejectis utrinque quantitatibus determinatis, utpote nullam mutationem inferentibus) fore perpetuo omnia $x^{\mp n} e = \frac{c^{1 \mp n}}{1 \mp n}$; facta ſcilicet maximâ $x = c$. ex quo ſequitur hoc generale.

Coroll. VIII. Si linea recta, $AO:c$, in partes innumerabiles ſeu infiniteſimas e dividatur; erit ſumma quarumvis dignitatum ab innumeris rectis, $QA:x$, ab extremitate A propoſitæ rectæ continuò incipientibus genitarum (quarumque ſingulæ in infiniteſimam propriam $QN:e$, ipſarum terminos reſpectivos claudentem multiplicatæ ſunt) ſeu omnium $x^{\mp n} e$, æqualia rectæ expoſitæ, $AO:c$, ſeu maximæ x poteſtati aſſumtis poteſtatibus ipſarum x proxime ſuperiori, ac diviſæ per ſuum exponentem = $\frac{c^{1 \mp n}}{1 \mp n}$ ratio ex

 mox inventis apparet. NB: Quare hic, à udat: Authoribus prætermiſſam, parentheſi inuſam conditionem addiderim, demonſtratiois ſeries oſtendit, conferatur utraque.

Fuſius paullo hæc proſequi operæ duxi preum; ut quomodo ſumma omnium yke, tum er Applicatam $OF:b$, ſeu figuræ baſin, uti *oroll. V*; tum per interceptam $AO:c$, ſeu fiuræ axem aut altitudinem exprimenda veniat, yrones addiſcant.

Poſſent & alia hinc erui, ac ex valore $\frac{1cbk}{1+n}$ ope æquationis curvæ reſpectivæ ipſa c tolli, une aliûs denuo generis canon enaſcetur.

12. Quod ſi præcedentis *Coroll*: effatum modo ad rem præſentem magis accommodato efferatur, ſic ſonabit: Sint quotlibet legem quamunque ſervantes indeterminatæ quantitates x, y, ſ, z, &c. creſcentes aut decreſcentes per reſpecivas ac proprias ſibi infiniteſimas e, a, θ, μ &c; c quælibet cujuſcunque indeterminatæ poteſtas n propriam infiniteſimam ducta aſſumatur, ut $x^p e$, $y^p a$, $ſ^p \theta$, $z^p \mu$ &c. (quales terminos ſemer, ubi infiniteſimam vel propriam vel alienam omitem habent, in poſterum vel *proprias* vel *lienas infiniteſimales* vocabimus) dico: quamlibet *propriam infiniteſimalem* in ſuo genere, infiniies (ſeu toties, quoties ipſa infiniteſima, Verb: Gr:

Gr: *e* aut *a*; indeterminatam propriam, quam componit, Verb: Gr: *x* aut *y*, ingreditur) ſumtam (quarum infiniteſimalium infinitam ſummam ſimpliciter voce *omnium* denotamus in ſeqq:) æquari indeterminatæ maximæ poteſtati proxime ſuperiori ac diviſæ per ſuum exponentem: Sic omnia $xxe = \frac{1}{3}x^3$ omnia $z^3\mu = \frac{1}{4}z^4$ &c.

Demonſtratio eadem eſt, ac præcedentis corollarii.

13. Hinc ſequens profluit maximæ utilitatis in calculo infiniteſimali theorema, nulli, quantum mihi conſtat, adhuc animadverſum.

Sit expoſita quælibet quantitas aut æquatio, indeterminatas quaſcunque, quæ per infiniteſimas creſcunt aut decreſcunt, continens; eaque redigatur ad infiniteſimas more jam ſæpius oſtenſo; dico omnes terminos infiniteſimales infinities, juxta leges §. præced. ſumtos ex eodem expoſitæ quantitatis termino oriundos (quinam autem illi ſint, ſequens docebit ſcholium.) Ipſi termino unde originem ſumſerunt, tanquam valori abſoluto, perpetim æquari; & è converſo. Si hîc valor abſolutus indeterminatæ ac maximus; in infiniteſimalium vero infinitâ ſerie continuo per infiniteſimarum applicationem mutatus intelligatur.

Demonſtratio duobus in caſibus abſolvitur:

I, cum

I, cum terminus Verb: Gr: ry^p præter determinatas r, quæ nullam mutationem inferunt, unicam tantum indeterminatarum, uti y, continet. II, cum terminum Verb: Gr: ry^pf^q plures unâ indeterminatæ y & f, in ſe mutuo ductæ, ingrediuntur. In I. caſu ſit quantitas expoſita ry^p; erit ea ad infiniteſimas reducta $ry^p + pry^{p-1}a$. Jam eſſe omnia $pry^{p-1}a = ry^p$ ex §. præcedenti patet. In II. caſu, ſit expoſita quantitas $x^py^qz^b$; erit eadem ad infiniteſimas redacta, $x^py^qz^b + py^qz^bx^{p-1}e + qx^pz^by^{q-1}i + bx^py^qz^{b-1}u$.

Ubi tres termini infiniteſimales ex uno expoſito oriuntur; quos infiniteſimales brevitatis ergo, litterâ M; expoſitum vero terminum littera N deſignabimus.

Eſſe autem omnia M æqualia abſoluto valori ipſius N, hoc modo evincitur; fiat $N = f^d$, erit utrâque ad infiniteſimas redactâ $N + M = f^d + df^{d-1}o$; & deletis æqualibus, infiniteſimalibuſque infinities ſumtis, omnia $M =$ omnia $df^{d-1}o$; ſed omnia $df^{d-1}o$ juxta præcedentem $= f^d = N$; ergo omnia $M = N$; *Q: e: d.*

Schol.

Schol. Potuiſſet generalis regula terminum quemlibet ad infiniteſimas redigendi, hiſce aliiſque præmitti; cum autem ipſam operationis methodum ſæpius, ſi non ad nauſeam, adhibuerimus, quilibet ſibi compendia ipſe excogitabit. Hoc interim in calculi commodum tyrones obſervent: terminum quemcunque ad infiniteſimas redactum eſſe eundem illum terminum, tot numero terminis infiniteſimalibus junctum quot indeterminatas ipſe terminus continuit, quomodo vero infiniteſimales efformentur ſequens exemplum docebit,

I. In expoſito quocunque cum ſuo ſigno termino $+xxy^3z^4$, prima ordine conſideretur indeterminata x; terminuſque per ipſius x exponentem multiplicatus unicâ x diviſionis ope demptâ, ducatur in e, ſeu propriam ipſi x infiniteſimam, ſuo ſigno debitè affectam; & habebitur terminus infiniteſimalis primus $2y^3z^4xe$.

II. Jam, ſecundâ indeterminatâ y aſſumta, eadem operatio repetatur, emergetque pro infiniteſima y ſecundus terminus infiniteſimalis $3xxz^4yya$.

III. Tertia indeterm: z ſequatur, eodemque ordine ac lege peragantur omnia, orieturq: tertius terminus infiniteſimalis $4xxy^3z^3u$.

IV. Eoque juxta omnium indeterminatarum numerum continuato, obtinebitur hoc in caſu $xxy^3z^4 + 2y^3z^4xe + 3xxz^4yya + 4xxy^3z^3u$ pro termino xxy^3z^4 ad infiniteſimas redacto. determinatæ, ſi termino expoſito miſceantur, quantitates manent. De ſignis non eſt quod

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quod laboremus, modò exponentes ac infiniteſimæ legitimis ſuis ſignis affectæ adhibeantur.

Demonſtrationem omitto, ex præcedentibus non paucis in locis, aut numerorum poteſtates exprimentum vice generaliora ſymbola ſubſtituendo, levi negorio petendam.

NB. In exemplum dedimus terminum pluribus indeterminatis conſtantem; quæ ſi unica tantum ſit, juxta hanc unicam terminum conſiderandum eſſe quilibet videt; ſic $rrx3$ ad infiniteſimas redacta dabit $rrx3 + 3rrxxe$.

Conferet hoc ad pleniorem §. 35, methodique tangentium inibi traditæ, demonſtrationem; & ad legitimam ex terminis infiniteſimalibus ad eos, unde ortum duxerunt, regreſſionem; quam, ut hujus ſcholii converſum, hinc cognitam ſupponimus, in quantum eorum, quæ modo tradidimus, natura patitur.

14. Sit *Fig.* XXXIV, curvilineum *ADFO* idem ac §.11, eademque ſymbola; intelligantur que ſubtangentes *TQ* in *MD*, *VO* in *LE* &c. perpetuo applicari ſervato ad ipſam *AQ* paralleliſmo: erit juxta §.11. oſtenſa trapezium *MDEL* ≕ *ta*; adeoque trilineum *LAF* ≕ omnia *ta* ≕ trilineo *AFO* ſeu omnibus *ye*, ex eadem §. 11.

Coroll. Unde factis *TM* ≕ *DQ*, *LV* ≕ *PE* &c. donec enata ſit curva *LMA*, erit, vocata *AT*: *z*, cum ſit *MT* ≕ *y*, relatio curvæ *LMA* ex relatione curvæ *ADF* cognita; datoque ſpatio *ADQ*, datur ſpatium *TMA* ≕ rectangulo *MDQT* — ſpatii *ADQ* duplo; & contra.

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15. Sit *Fig. XXXV.* curvilineum A F O; curvamque tangens *T D* producatur, donec prolongatæ O F occurrat in *S*; vocenturque *TQ*:t, *AQ*:x, *QD*:y, *DI*:l, *IS*:h; infiniteſimæ *DH*:e, *FH*:a. Statuaturque S O ſeu $h+y$ perpetim in Q B, donec deſcribi poſſit nova curva O B R: erit ſpatium A R O ſemper duplum ipſius A O F.

Eſt enim ſpatium A R O juxta §. 4. = omnia $he+ye$; ſed omnia he ſecundum §. 2. = omnia ye; quare omnia $he+ye$ = totidem $2ye$, unde conſtat propoſitum.

Coroll. Sit A E F arcus circuli; erit, facto radio = r; O S = $\frac{rr}{y}$; unde curvilineum AOR, hanc habens ad circulum relationem æquabitur duplæ circuli portioni A O F.

16. Quod ſi ipſa Q B perpetuo adæquetur ſubtangenti *I S* ſeu h erit ſpatium A R O æquale ſpatio A F O: cum enim ſit perpetuo la = he, ex §. 2. apparet intentum.

Coroll. I. Patet hinc ratio ad eundem axem A Q ſeu x, non duo tantum curvilinea A F O, A R O conſtituendi, quæ ſibi mutuo æquantur, verum & pro lubitu multa; ſiquidem tractato curvilineo A R O denuo, eodem more, quo in curvilineo A F O uſi ſumus, novum ad eundem axem applicabile emerget curvilineum prio-

rioribus æquale, & ſic in infinitum. plura hinc tragoniſmorum genera originem ducunt & ultifaria ſpatiorum inter ſe comparatio.

Coroll. II. Sic *Fig. XIII.* N. 1. ciſſois in ratione l Circulum menſuratur; reſumto enim §. 31. ap. I, erat RL, $t = \frac{3}{3}y + \frac{1}{2}z$, ſeu $3y = 2t - z$; : ductis omnibus in $QP: e$, fiet $3ye = 2te - ze$: um autem ſemper ſit $te = lo$; ſeu (poſita $Q = RL = t$ ad curvam ABC) trapezium $QPC = NLMK$; erunt omnia te, ſeu ſpaum curva $ABCK$ & rectis AO, OS, interctum, æquale integro ſpatio Ciſſöidali ſeu mnibus lo: & ſi omnia ye, ſeu ſemicirculus catur c, ac omnia ze ſeu ſpatium Ciſſoidale ocetur f; cum fuerit omn. $3ye =$ omn. $2te$ omn. ze, erit $3c = 2f - f$, adeoque $3c = f$, ciſſois ſemicirculi tripla.

Coroll. III. Hoc modo Cycloidum menſura dagatur; quæ, prout hic tractatur, exempli co, in cæteris, tyronibus ſervire poteſt.

Sit *Fig. XXXVI.* Cyclöis $ALMD$ ad ſemicircum $ABCG$ relata; hanc tangat recta $ILMS$; centrum Circuli F; vocenturque $IE: t$, $S: s$, $LE: z$, $BE: y$; LV ſeu $EG: x$, arcus $AB: c$; quatio Cyclöidis ſit $p: q :: z - y :: c :: LB: AB$; it, juxta §: 59, Cap. I. IE ſeu $t = \frac{qyz}{pr - qr + qx}$; : quia $IE: EL :: LV: VS$. erit VS, ſeu $s = \frac{xx}{y}$ $\frac{prx - qrx}{qy}$.

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Jam vero appellatis, DE, BP, LN: e, CP: a, MN: i, quia SV: VL:: MN: LN. ſeu s: x:: i: e, erit ſemper $xi = se$; unde, poſitâ SV ſeu s in ET, deſcriptâque curva GHT erunt omnia xi, ſeu rectangula LVR, ſive totum Cycloidis AOG ſpatium, æquale omnibus se, ſeu rectangulis TED, hoc eſt integro ſpatio $GATT$; quare inventâ hujus ſpatii GAT, ſeu omnium se, ratione ad Circulum, inventa erit ſpatii Cycloidalis, ſeu omnium xi, ratio ad eundem.

Quod ut fiat: ſit primo ALO cyclois primaria, erit $p = q$, adeoque $SV = ET = \frac{xx}{y}$: Unde patet curvam GAT eſſe Ciſſoidem: ſed, ex modo dictis, ciſſoidale ſpatium hoc cycloidali expoſito æquatur, ſeu omn. se = omn. xi; adeoque & hoc eſt ſemicirculi triplum juxta coroll. præc. Hoc notabo: quia ſemper eſt: $se = xi$, ſi ad eundem ſemicirculum, tum cyclois AOG, tum ciſſois $AGTT$, referantur; ductâ LV ipſi AG parallelâ, eique normali LT, fore ſemper ſpatium LVO æquale ſpatio ETG.

Quod ſi cyclois alterius ſit generis, & $s = \frac{xx}{y} + \frac{prx - qrx}{qy}$; quoniam $\frac{xx}{y}$ manet ad ciſſoidem; ſi jam innoteſceret, quam rationem curvilineum, cujus applicata $\frac{rx}{y}$, haberet ad ſemicirculum; ex

§. 8. ac applicatarum ſummâ notum foret ſpatium omnium se, unde quæſtio ad ſequentem revolvitur :

Sit centro F deſcriptus ſemicirculus ASG, voceturque radius r, $GD:x$, $DC:y$, $DA:2r-x=l$ $=MH$; *Fig. XXXVII.* ſit item alia curva GHT, cujus applicata $DH=\frac{rx}{y}=f$, tangat vero HK curvam in H, dicaturque $KM:t$, PC, ED, $NH:e$, $NT:o$, $BP:a$; erit *I.* æquatio $KM:$ $MH::NT:NH$, unde $lo=te$. *II* Æquatio ex natura curvæ $\frac{rx}{y}=f$, ſeu $\frac{rx}{f}=y=\frac{lx}{y}$, quare $ry=fl$; & $rx=fy$ redacta ad infiniteſimas $re=fa+yo$.

III. Ex natura circuli $lx=yy=2rx-xx$; unde $re-xe=ay$ jam autem ex I. & II æquationibus, eſt $o=\frac{te}{l}=\frac{re-fa}{y}$, unde $a=\frac{rle-tye}{fl}$ $=\frac{re-xe}{y}$, quâ ordinatâ & facto $fl=ry$ dabit $rl-ty=rr-rx$, & loco l poſito $2r-x$, fit $t=\frac{rr}{y}=KM$: quæ ſi ponatur in ID ad curvam VSI, erit; cum ſit $te=lo$, ſpatium rectis ZA, GO & curva VSI comprehenſum, ſeu omnia te, æquale ſpatio expoſito rectis AG, AL, & curva GHT interjecto, ſeu omn. lo.

Jam autem denuo oporteret rationem curvilinei

linei (cujus applic. ID ſeu $\frac{rr}{y} - t$) ad ſemicirculum indagare. Sed in hoc particulari ad Circulum caſu aliam viam ingrediemur; Ad radium FS ducatur normalis SR, ſecans radium FC productum in R, voceturque $BC: u$, erit (quia DC, y: FC, r:: SF, r; FR, $\frac{rr}{y} = t$) $y : r :: r : t ::$ PC, e: BC: u: quare ſemper $ru = te$; jam autem ſemicirculus eſt æqualis omnibus triangulis BFC ſeu $\frac{ru}{2}$, hinc omnia ru ſemicirculi duplum; adeoque & omnia te = omnia lo = ſpatium KAGT, quod æquivalet omnibus fe. Jam concludendo, fiat $\frac{xx}{y} = h$; &, quia $\frac{rx}{y} = f$, erit $se = he + \frac{p-q}{q} fe$; vocato jam cycloidali ſpat. k, ſemicirculo: d, emerget, quia omnia $he = 3d$. & omnia fe, $= 2d$, hæc æquatio $k = \frac{2pd + qd}{q}$;
& orietur *Torricellii* regula, quia ratio q ad p eſt ratio peripheriæ ſemicirculi ABG ad baſin Cyclöidis cujusvis GOA.

Coroll. IV. Eodem modo & *Fig.* XXXVIII. Complementum Cyclöidis AQO metimur. Vocetur enim BE: y, LE: z, AB: c, AE ſeu KL: x, EF: $l = r - x$ (ſi radius dicatur r) & KV: s; recta LV tangat curvam, erit exiſtente æquatione Cyclöidis, $p : q :: z - y : c$, linea IE $= \frac{qyz}{pr + ql}$ (uti præced: Coroll. inſinuatum) &, quia $l = r - x$,

erit I E $= \frac{q\,y\,z}{pr + qr - qx}$; jam vero I E: EL :: KL: KV $= s = \frac{prx + qrx - xx}{qy}$;quia autem poſita SL $= i$, & SM: $= e$, eſt KV: KL:: SL: SM. Sive $x :: i : e$, erit ſemper $x\,i = s\,e$, & poſito R T $= s$ ad curvam Q R O, ſpat. A Q O ſeu omn. $x\,i$, = ſpat. QRO ſeu omn. $se = \frac{prxe + qrxe - xxe}{qy}$, hoc eſt, facto ſemicirculo $= d$, (quia per Coroll: præc. omnia $\frac{rx}{y}\,e = 2\,d$, & omn. $\frac{xx}{y}\,e = 3\,d$) omnia $s\,e = \frac{2\,p - q}{q}\,d =$ complem. Cycloidis A Q O. Quod ſi $p = q$ ponatur, cyclois erit primaria, ac compl: A QO æquale ſemicirculo d. ſed & cum hoc caſu ſit $s = \frac{2\,rx - xx}{y} = y$; ex proprietate ſemicirculi, patet RT ac BE ſemper æquari & QRO eſſe ſemicirculum; adeoque ſemper eſſe $s\,e = y\,e = x\,i$; ſeu ſpat. A KL continuo æquari ſpat. QRT ſeu A BE.

Corol.V. Sit jam ALO alterius naturæ Cyclöis *Fig. XXXVIII.* cujus æquatio (retentis per omnia ſuperioribus ſymbolis) $p : q ::$ E L: arc. $AB ::$ $z : c$. Unde $q\,z = p\,c$; & juxta tangentium methodum $IE = \frac{qyz}{pr}$, ac KV ſeu $s = \frac{prx}{qy}$; quare omnia $s\,e =$ omn. $x\,i =$ comp: $QOA = \frac{2\,pd}{q}$; quod ſi ſubtrahatur ex rectangulo $QAGO$ ſeu $\frac{2\,pkr}{q}$ (poſito k æquali ſemiperipheriæ $ABCG$)

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ſeu $\frac{4dp}{q}$, remanebit ſpatium $AOG = \frac{2pd}{q}$, æquale ſuo complemento. Si $p = q$, erit alterutrum $= 2d$ ſeu ſemicirculi duplum.

Coroll. VI. Repetitis ſymbolis §. 62. C. 1, aſſumtâque inibi traditâ hypotheſi; *F. XVII.* ſi *ABCD* ſit quadrans circuli, ac *KE* curvam tangat, erit, juxta §. 63. cap. 1, *KM* ſeu $t = \frac{rh}{d}$, quare $d:r::h:t$; & juxta lemma 46, (quia ratio d ad r perpetuo manet eadem) $d:r::ha:ta$; ſed citato §. 62, $ha = uy$, & $ta = ye$, quare $d:r::uy:ye$; verum ex proprietate ſemicirculi $BQ:BN::CB:BR$ ſeu $r:y::u:i$; unde $uy = ri$; adeoque erit $d:r::ri:ye$; hoc eſt radius r ad quadrantem peripheriæ d, ut ſpatium *AFDQ* ſeu omnia ye ad quadratum in *AQ* ſeu omn. $ri = rr$. Secundum 1. II. Euclid.

Coroll. VII. Reſumtâ curvâ §. 30. cap. 1, *Fig. XII.* cæteriſque ut inibi ſuppoſitis, nimirum $r^p - x^p = y^p$ pro æquatione curvæ *AEF*, factis ſcilicet $QO:x$, $QD:y$; tum & $y^s = r^{s-1}z$, pro æquatione curvæ *ANG*, cujus applicata $QN:z$, erit, juxta §. 30. cap. 1,

$$RL \text{ ſeu } t = \frac{sy^{s-p}}{r^{s-p-1}} - sz;$$

unde cognitâ figurâ *ADFO* ſeu omnibus ye cognoſcetur figura *AOG* ſeu omnia ze.

Sit enim primo $s = p + i$; erit æquatio inter utriuſ-

triuſque curvæ applicatas, $y^{p+1} = r^p z$, ac
L ſeu $t = \overline{p+1}\,y - \overline{p-1}\,z$. Cum autem
ixta 30. cap. 1, ſit continuo $te = xo$; hoc eſt
poſito *RL* ſeu *t* in *QS* donec deſcripta ſit cur-
a *ASVF*) figura *ASVFO* ſeu omnia *te* æqua-
is figuræ *ANKGO* ſeu omnibus *xo*, quæ æ-
uantur omnibus *ze*; quia eandem figuram jux-
a §. 1. & 2. hujus conſtituunt: erunt & omnia
te = omn. *ze*. unde factis omnibus *te* ſeu om-
ibus $ze = b$, & omnibus $ye = c$, erit (quia
nodo ſunt $te = \overline{p+1}\,ye - \overline{p-1}\,ze$) quoque
$= \overline{p+1}\,c - \overline{p-1}\,b$, ſeu $\frac{\overline{p+1}\,c}{p+2} = b =$ figura
IGO; unde poſita figura *ADFO* ad quadran-
em circuli erit $p = 2$, adeoque $\frac{3}{4}c = b$: & æ-
uationes $\overline{rr - xx} = yy$ ac $y^3 = rrz$, ſeu
$\overline{rr - xx}\sqrt{\overline{rr - xx}} = rrz$.

Jam, manente quadrantis circularis hypo-
heſi, fiat curvæ *AGO* applicata *h*, & æquatio
$= r^4 h$; fiet $s = 5$, & $p = 2$, unde *RL* ſeu *t*
$= \frac{5y^3}{rr} - 5h$; eritque factis omnibus $he = d$,
cum ex modo inventis omnia $\frac{y^3}{rr}e$ ſeu omnia ze
uerint $= \frac{3}{4}c$) $d = \frac{15}{4}c - 5d$, ſeu $d = \frac{5}{8}c$.

Schol. Ne autem particularibus nimium inhæreamus, methodum generalem addo, quælibet curvilinea in alia infiniti generis transmutandi, tum & plurimorum menſuras inveniendi; ex ſolo præcedenti § 4. deductam.

17. Sit curva, *Fig. XXXIX. A E F*, vocenturque *D Q*: y, *A Q*: x, ſubtangens *T Q*: t, tangens *T D*: s, huic normalis *D R*: k, ſubnormalis *Q R*: l, curva *A D*: c, infiniteſimæ *D H*: e, *EH*: a, *H E*: u; jungaturque *Fig.* XL, $\alpha\varphi o$ ejus naturæ, ut $\alpha\varphi$ per omnia exactè quadret curvæ *A E F*, ſitque ϑ æqualis parti curvæ infiniteſimæ *D E* ſeu u, tandem $o\varphi$, $\pi\vartheta$, $\kappa\vartheta$ ſint applicatæ. Aſſumanturque indeterminatæ h & z, ac fiat quælibet hypotheſis; ſitque

I. $he = za$; erit applicatâ perpetuò h in *QC* ad e, nec non z in *I M* ad a, juxta §. 4, omnia he ſeu ſpatium *ASO* æquale omnibus za ſeu ſpatio *OFK*.

II. Fiat $he = zu$; erit, poſitâ curvâ *ADEF* in directum in $\alpha\varphi$, ac ſemper applicatâ $\kappa\vartheta$ ſeu z ad ϑ ſeu u, area $\alpha o \varphi$ ſeu omnia zu æqualis areæ *A S O* ſeu omnibus he.

III. Sic & facto $za = hu$; erit, exiſtente continuò $\kappa\vartheta = h$, & $IM = z$, trilineum $\alpha o \varphi$ æquale trilineo *O F K*.

Unde patet, datâ curvæ *A D E* proprietate, ac ſumta ipſarum z & h alterutrâ ad arbitrium, reliquam ſemper per infiniteſimarum rationem determinari; hanc autem rationem ex curvæ *A D F* naturâ dari plus quam cognitum eſt, eſt enim $s : t :: u : e$. item $s : y :: u : a$, nec non $t : y :: e : a$.

Coroll. I. Hinc cujuslibet curvilinei transmutationes infinitæ. reſumtâ enim primâ hypotheſi $he = za$ cum ſit ex proprietate curvæ cujuscun-

que

ue $ye = ta$ erit $\frac{th}{y} = z$; unde ſumpta h pro ar-
itrio, Verb: G: ad circulum $= \sqrt{2rx - xx}$, ſi
$= DQ$ ſit ad parabolam $= \sqrt{rx}$, erit $z = 2x$
$\frac{2r - x}{r}$; quæ mediante parabolæ æquatione
$x = yy$ expreſſa per y (cum hic z ad a, ſeu ipſius
infiniteſimam, applicari ſupponatur) dabit
$= \frac{2yy\sqrt{2rr - yy}}{r \; rr}$; adeoque relationem curvæ
M*K*. cum autem, retento ipſius h valore & lo-
o, poſſit *D Q* ad infinitas curvas ſtatui; & mo-
o prædicto ex ſingulis emergat aliûs naturæ cur-
ilineum, quorum ſingula nihilominus circulo
eu omnibus he æquantur; patet dato cuilibet
patio infinita æqualia hac methodo exhiberi poſ-
e: quod in multis caſibus non exigui eſt uſus.

Sic & ex hypotheſi ſecundâ $he = zu$, manen-
ibus h ad circulum & y ad parabolam, cum ſit se
$= tu$, erit $\frac{th}{s} = z$; unde s & t ex naturâ parabolæ,
ex natura circuli cognitis, ipſa z magnitudine
ota eſt: quæ ad $\alpha\varphi$ curvæ *A* F æqualem appli-
ata efficiet trilineum $\alpha o \varphi$ æquale ipſi *A S O*.

Coroll. II. Quod ſi z ad quamcunque curvam,
patium cognitum continentem, ſtatuatur, ad
riangulum, parallelogrammum, paraboliſor-
nium quandam aut aliûs generis quadrabilem;
rit & h ad locum cognitum) cujus ſpatium men-
uram patitur; unde mutatâ rurſus curvâ *A D* F

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in infinitum, infinita dantur ſpatia menſurabilia, & aſſumto cuidam curvilineo æqualia.

Ex: G: in hypotheſi primâ $he = za$, factâ z æquali determinatæ r, erit $h = \frac{ry}{t}$; unde, y & t ex curvæ ADF relatione cognitis, erit ASO æquale omnibus ra ſeu ry ſeu rectangulo $FOGK$. Idem ſi fiat in hypotheſi ſecundâ $he = zu$ erit $h = \frac{sr}{t}$, & ſpatium ASO æquale rectangulo $\alpha\varphi o\mu$, ſeu r in curvam AF; quod ſi aſſumatur $z = c$ erit $he = cu$; & ſpatium AOS æquale ſemiquadrato curvæ AF.

Coroll. III. Nec hypotheſis in duobus tantum terminis conſiſtat neceſſe eſt, cum infinitis fere modis variata aſſumi poſſit. Sit in exemplum $ye + xa = he$; eſt ye ad curvilineum AFO, xa ad ejus complementum AFV, & he (facto $QC = h$) ad trilineum ASO; unde hoc in caſu ſemper $AQC =$ rectangulo $XDQA$. quod ſi proprietatem curvæ ACS invenire quis velit (cum ex data curva AEF etiam x & y data ſint) eſt $ye = ta$; adeoque $h = \frac{xy + ty}{t}$; quæ ope curvæ ADF expreſſa per interceptam x dabit quæſitum.

Sic aſſumto $za + he = fu$ (poſitâ f pro tertia indeterminata ad $\alpha\iota$ applicabili) unicâ Verb. Gr. f pro lubitu determinatâ $= r$ & eliſis infiniteſimis

nis emerget $zy+th=rs$; unde altera quoque ex ipſis z vel h, præter ipſam f, adhuc arbitrio relinquitur.

18. Hoc addam, licet præcedentis tantum ſit conſectarium, in hypotheſi prima $he=za$ fiat $z=y$ erit $h=l$; adeoque $le=ya$; & omnia le ſeu ſpatium *AOS* (cujus applicata *QC* ſemper æqualis eſt ſubnormali reſpectivæ *QR*) æquatur *FOK* ſeu ſemi quadrato ipſius FO. Cum enim perpetuo ſit IO = IΠ erit IΠ ad triangulum iſoſceles tum & ex §. 13, notum eſſe omnia $ya=\frac{yy}{2}$.

Coroll. I. Hinc opulenta rurſus menſurabilium ſpatiorum copia, & ex quâvis curva *ADE* infiniti generis; poſitâ enim quâcunque ſubnormali *QR* in *QC*, naſcetur *AOS* ſpatium menſuræ capax; cujus ſi rurſus ſubnormalis inquiratur, aliud rurſus ſpatium & ſic ſine fine, enaſcentur ex unâ eâdemque curvâ *ADE* curvilinea magnitudinem cognitam habentia, quod ex præcedenti ſatis manifeſtum eſt.

Coroll. II. Quin & hæc, præter alias methodos, curvarum generalem abſolveret quadraturam; ſi ſolvi poſſet problema §. 95 cap. I. propoſitim; hoc eſt ſi, datâ qualibet *QC* per *QA* ſeu x expreſſâ, inveniri poſſet curva *ADE*, cujus ſubnormalis *QR* ipſi *QC* perpetuo ac reſpecti-

pecti *QR* ipſi *QC* perpetuo ac reſpectivè adæquaretur unde, ſi $QC = QR$ datâ foret in terminis æquationis curvæ *ADE* proximis, obtineretur intentum, juxta Cap. I, §. 93, & ſeqq. Modi hoc inquirendi varii Cap. I. 95. &c. videri poſſunt.

Quibus hunc adjungo; cum referat omnium ſubnormalium ad curvas tum geometricas, quæ per ſolas x & y; tum & alias, quæ per curvam c quocunque modo efferuntur, pertinentium terminos proximos.

Fiat $z + f + g + b + d + h + k + m = 0$.

Ac terminorum quorumlibet ſumma, in quibus ſub quibuſcunque ſignis, invenitur poteſtas quælibet ipſius x deſignetur per z, ipſius y per f, ipſius c per g, tum ipſarum x & y per b, ipſarum x & c per d, ipſarum y & c per h, omnium denique x & y & c per k; determinati termini vocentur m. tum littera quamcunque terminorum ſeriem in ſingulis terminis per exponentes reſpectivos ipſius x multiplicans dicatur A; quæ hoc idem in omnibus ratione ipſius y præſtat, ſit B; quæ ratione ipſius c appelletur D: NB. Has terminorum ſeries per A, B, D, &c. hoc eſt in ſingulis terminis per ejuſdem indeterminatæ exponentes reſpectivos multiplicatas, aut diviſas, in poſterum *ſubtangentialiter* multiplicatas aut diviſas appellabo.

Sequitur I. omnes curvarum deſignatas æquationes per modo propoſitam exprimi; uti cuilibet patet; cum omnis generis & multitudinis terminos involvat.

II. Hanc juxta regulam §. 35. Cap. I. tractatam omnium curvarum, tum geometricarum, tum c continentium, ſubnormales l, generali hac æquatione exhibere; juxta denotationem §. 96. Cap. I. adeoque l æquabitur.

$$\frac{\overline{(Az)+(Ab)+(Ad)+(Ak)} \text{ in } \overline{cyy}}{\overline{-(Bf)-(Bb)-(Bh)-(Bk)} \text{ in } cx \overline{-(Dg)-(Dd)-(Dh)-(Dk)} \text{ in } sx}$$

Exem-

Exemplum in tyronum gratiam addam: ſit *Fig.* XXXIX. curva ADE; cujus æquatio $2rx - xx = yy + xy + ry + rr$; Quæritur QR, ſeu l, ejus ſubnormalis.

Poſitâ æquatione, ut ſupra, nihilo æquali; erit $2rx - xx - yy - xy - ry - rr = 0$, videre eſt c hanc æquationem non ingredi; quare ad hanc pertinentes quantitates g, d, h, k, nihilo fiunt æquales; ſolæque, z quæ x, f quæ y, b quæ x & y continet, reſervandæ; fietque per c diviſione inſtitutâ $l = \frac{(Az)yy + (Ab)yy}{-(Bf)x - (Bb)x}$, unde loco z ſubſtituto $2rx - xx$, loco f poſito $-yy - ry$, tandem pro b ipſo $-xy$, ſervatis in ſingulis additis ſignis, fiet, omnibus juxta A, B, &c. ſubtangentialiter multiplicatis, ad diviſis per xy, ſubnormalis $l = \frac{2ry - 2xy - yy}{2y + r + x}$.

Schol. Hoc ſolum ac unicum reſtare videtur: ut ipſa l, per terminos hoſce generales explicata, in curvis geometricis per ſolam x poſſet efferri, juxta canonem omnibus curvis convenientem & univerſalem; unde primo fere intuitu appareret in qualibet curvâ, num juxta præcedentis tenorem poſſet quadrari; & an inter varias, quæ hinc enaſcerentur, relationum expreſſiones aliqua reperiretur, quæ expoſitæ curvæ proprietati congrueret.

Videbantur hæc inter ſubnormalium inventiones Cap. I. tractatas ſibi locum vendicare; verum quoniam curvilineorum hinc orientium quadraturæ primario conſiderantur, malui hanc methodum huic potius loco, tanquam ei, quod intendimus, convenientiſſimo inſerere. Sed ad corrollaria revertor.

Coroll.

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Coroll. III. Hinc eadem, quæ in §. 11. coroll. ex alio principio oſtendimus, originem ducunt; ſerierumque juxta *Clar: Wallis*: tum auctarum tum multatarum aggregata & reſidua, horumque poteſtates quælibet in debita ſua ratione ad totidem maxima conſtituuntur; quæ §. præcedentis non niſi particularem caſum efficiunt.

Ex: Gr: *Fig. XXXIX* poſitâ $AQ = x$, & $QB = y$: ſit z quotlibet terminorum ſimplicium x continentium ſumma, & curvæ ABS æquatio $n^p = z$; quæruntur omnia $n^p e$ ſeu omnia ze. Quapropter fiat $z^1 = n^p = ml$; ſitque $l = QC$, cique ſemper æqualis curvæ ADE ſubnormalis QR; erit ergo $l = \frac{z}{m}$, & juxta Cap. I. §. 97. Coroll. I. æquatio curvæ ADE $\left(\frac{zx}{\text{A}}\right) = \frac{myy}{2}$; ſed omnia $le = \frac{yy}{2}$ ex præced: ergo $\left(\frac{zx}{\text{A}}\right) =$ omnia $mle =$ omnia ze, quæ erant invenienda.

Unde hæc regula; omnia ze, (ſeu facto in exemplum $z = rx \mp xx$) omnia $rxe \mp xxe$ æquantur ipſi zx ſubtangentialiter per A diviſæ; hoc eſt ſerëi datæ z unicâ ipſius x dimenſione auctæ, & poſtea in ſingulis terminis per poteſtatem ipſius x diviſæ $= \frac{1}{2}rxx \mp \frac{1}{3}x^3$. hinc §. 12. rurſus demonſtrari poteſt. Poſſe autem poteſtates ipſius x, in terminis ipſam z componentibus, nunc per numeros fractos nunc per negati-

vos

…os deſignari ; tum & dignitates quaslibet aggre-
…atorum ſerierum auctarum aut multatarum, per
…mbolum z efferri ; notius eſt quam ut hic
…ſtendi mereatur.

Coroll. IV. Oritur hinc, & ex §. præced:, …ultifaria methodus, quâ ex dato curvilineo …Q C, in terminis quibuscunque algebraïcis, …ſius applicatæ Q C ad interceptam A Q rela-
…o exhibetur. Hæc inter ſequentia ex fundamen-
…o magis generali deducemus.

19. His annectere liceat facillimam curvili-
…eorum quorumcunque transmutationem, ſub-
…ractionem, additionem &c. ex §. 6, 7, 8, 9. di-
…ectè manantes.

Sit in exemplum (reſumtis §. modo citatorum …mbolis ac *Fig. XXVII.*) $z + h = y$. ſupponatur $z = \frac{xx}{\sqrt{2rx - xx}}$ ad Ciſſöidem; $h = \sqrt{2rx - xx}$ …d ſemiciculum correſpondentem ; fiet …$= \frac{2rx}{\sqrt{2rx - xx}}$. Jam autem omnia ye æquantur …mnibus he quater ſumtis, ſeu quatuor ſemi-
…irculis, ex §. 16. Cor: III. & juxta hypothe-
…n omnib. $he + ze$ ſeu ſpatio infinito Ciſſöidali …um ſemicirculo : quare Ciſſöidale ſpatium ſe-
…micirculi triplum eſt.

Coroll. Poſſet y ſupponi æqualis quotlibet ap-
…licatis $z + s + f + g$ &c. quibuslibet ſignis affectis;
unde

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unde poſitâ *y* ad curvam datam quamlibet, cæteriſque, exceptâ unicâ, ex arbitrio determinatis, ultima definitione ſui patefaciet curvæ naturam, cujus ſpatium cum reliquarum omnium areis nunc ſimul addits, nunc pro variâ ſignorum naturâ à ſe mutuo ſubductis, æquabit datum ſpatium ſeu omnia *y e.* verum, quæ hic ob tantam applicatarum copiam arbitrio relictam ſe pandunt, ſpeculationes præteribo.

20. Hoc addo: ſi aſſumta ſpatia quadraturam admittant, etiam quod ex iis per additionem aut ſubſtractionem componitur, erit quadrabile: poſſuntque hic curvilinea quælibet, quæ menſuram pati aliunde novimus, adhiberi; horum autem æquationes ſi ad paraboloidum hyperboliformiumque loca conſiſtant, erunt & omnes ipſius *y* poteſtates cognitæ: quod leviter ad præcedentia attendenti ignotum eſſe nequit.

Coroll. Poſſent hæ omnes poteſtatum, ac ſpatiorum menſurabilium hinc oriundæ diverſitates in ordinem redigi, a ſimplicioribus ad magis compoſita progrediendo, ac theoremata quædam efformari, quod provectioribus facile, in minus exercitatorum tamen commodum hoc generalius paulo ſpecimen ſubnecto.

Detur. $y^f = \overline{\frac{z^d}{g^s} \mp \frac{q^m}{c^l}}^{\,t}$ in $\overline{\frac{h^r}{b^p}}^{\,\frac{k}{n}}$

ac

Ac litteræ, $d, m, ſ, l, f, k, n, p,$ quantitatum ræpoſitarum poteſtatem deſignent; item t, erum hæc binomii, lineolâ ſuperinductâ noiti, dignitatem generalem ſignificat; reliuæ, $z, g, q, c, h, b,$ ſunt ſingulæ ad locum quenam ſpatio menſurabilem; qualia ſunt rectangu-um, triangulum, paraboloïdea quæcunque, ut hyperboliformia; quæ hoc in exemplo ntum conſiderabimus.

Dico, ſi fiat $f = 1$, curvilineum, cujus applica- y, quadraturam admittere, conſequentiæ ratio § 11. cum annexis corollariis manifeſta eſt.

Ex: Gr: factis $m = d$, item s & l nihilo æquabus, omnibuſq. ad dignitatem p evectis, ac ndem in analogiſmum reſolutis, fiet $b^n : h^k ::$ $\overline{\frac{d}{z} \mp \frac{d}{Q}}^{\frac{tp}{p}} : y$. Unde poſito rurſum $d = 1$, emeret hæcce, cum y ſuppoſita ſit unius dimenonis, inter poteſtatum exponentes æquatio $\frac{k}{p} - \frac{n}{p} = 1$, ſeu $pt + k - p = n$:; denique â ſimpliciſſimis incipiamus, aſſumtis $b = r$, $= x$, $z = r$, $q = x$, oritur proportio nova, uæ, facto $t = 1$, in ſequentem abibit $r^k : x^k :: \overline{r \mp x}^p$ $: {}^p$, quæ, ſigno negativo retento, eſt curvihei â Clar: *Sluſio* menſurati.

Sic & pro variâ tum quantitatum, tum expontium aſſumtione variæ continuo enaſcuntur

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curvarum proprietates, quæ omnes ſunt ſpatio menſurabiles, ut factis $b = x$ & $h = r$ erit $x^k : r^k :: \overline{r \mp x}^p : y^p$.

Schol. Quanta vero hinc theorematum, ac menſurabilium curvilineorum copia enaſcatur, nulli non manifeſtum eſſe poteſt; qui ex aliis tum poteſtatum, tum reliquarum quantitatum, hypotheſibus alias in infinitum oriri curvas; tum & pro ſingulis, in æquatione generali, fractionum numeratoribus, quotlibet terminorum, ſimpliciter x cum determinatis continentium, aggregata tuto aſſumi poſſe perſpexerit. modo hæc ſequentia obſerventur; I. ne plures uno terminos ipſi fractionum denominatores contineant II. ne poteſtas per t deſignata, niſi per numerum affirmativum aut integrum explicetur; cæteris d, m, s, &c. ſupponentis bene placito relictis.

Quod, ſi ſecus hæc ſeſe habeant, ac deveniatur igitur ad hanc aut ſimilem æquationem $y = \frac{rr}{r \mp x}$ aut $y = \sqrt{rr - xx}$; nulla adhuc hic regula tradita eſt (niſi forte inter ſubnormales repertæ ex §. 18. menſuram invenirent) quâ omnium *ye* valorem abſolutum indagare poſſimus; niſi adhibitâ prius reductione, quam per *diviſiones Mercator*, per *extractiones* inclytus *Newton* ad optatam metam perduxerunt. utraque hæc; tum diviſionis tum extractionis operatio poteſtatum in binomiis deſignatione perficitur; uti mox ſubnectendo theoremate patebit.

21. Eſt autem theorema hujuſmodi: ſit binomium quodcunque $z + f$ evehendum ad poteſtatem quamcunque deſignatam p: erit (poſito termino hujus poteſtatis in ordine primo $= 1\, z^p$ cujus genitura $= 1$) hæc, quæ ſequitur, generalis pro terminorum reliquorum genituris congruis

genituris inveniendis regula. Aſſumto quolibet termino, exponens ipſius z multiplicetur per genituram ejuſdem termini; ac productum dividatur per numerum loci, quem aſſumtus terminus in Ordine poteſtatis obtinet; & habebitur genitura termini ſequentis.

Ex: Gr: $z+f$ ſit attollendum ad poteſtatem quadratam: erit $1\ z^4$ terminus *I*; tum 4 per 1 multiplicetur, ac productum diviſum per 1. (cum terminus ſit ordine primus) erit ſequentis termini genitura $=$ 4, adeoque $4fz^3$ terminus *II*. hic denuo multiplicato 3 per 4 fiet productum 12, quod diviſum per 2 (cum terminus ſit ordine ſecundus) dabit ſequentis ſeu tertii genituram 6, qui idcirco fiet $= 6\,ffzz$: & ſic pergendo obtinebitur levi operâ binomii cujuscunque quælibet pöteſtas. Quare ſi $z+f$ elevanda ſit ad dignitatem p, erit terminôrum hanc componentium ſeries ea, quæ ſequitur. Et

Genitura termini

I ... 1 — — — $= A..$ in z^p

II ... p — — — $= B..$ in fz^{p-1}

III ... p in $\frac{p-1}{2}$ — — $= C..$ in ffz^{p-2}

IV ... p in $\frac{p-1}{2}$ in $\frac{p-2}{3}$ — $= D..$ in f^3z^{p-3}

V ... p in $\frac{p-1}{2}$ in $\frac{p-2}{3}$ in $\frac{p-3}{4}$ $= E..$ in f^4z^{p-4}

 Et

Et ſic pergendo, donec in geniturarum progreſſione numerus (ipſi p ex ſuppoſito æqualis) ex ipſo p ſubducendus veniat; unde iſthæc genitura, cæteræq. eam ſubſequentes nihilo æquales evadent, adeoq: & reliqui termini evaneſcent.

Poteſt p in numeri fracti modum, ut $\frac{k}{n}$, proponi, ad *extractiones radicum* perficiendas; tum & inſtar numeri negativi $-p$, quod *diviſionibus* inſervit & hoc pacto nova in his caſibus geniturarum & terminorum tabula efformari.

22. Hinc primo hyperbolici ſpatii aſymtotis interjecti, Fig. XXX. menſura per infinitam ſeriem explicatur: facto enim $AQ = QD = r$, & $QO : x$, ac $BO : y$, erit. $\frac{rr}{r \ast x} = y$; quæruntur jam (poſitâ e aſymtoti infiniteſimâ) omnia ye, ſeu ſpatium, $VBDQOV$ verſus V interminabile.

Quare Juxta *Cl: Mercatorem* rr dividendum venit per $r + x$; ſeu (quod ex poteſtatum denotatione cognitum eſt,) rr eſt multiplicandum in $\overline{r \ast x}^{\,-1}$; cum mutatio ſigni exponentis ex affirmativo in negativum, quantitatem præpoſitam vertat ex diviſore in multiplicatorem & contra, conſulantur lemmata 50, 51, & ſeqq.

Ut ergo $r + x$ evehatur ad poteſtatem -1, erit $p = -1$, & terminorum genituræ ſucceſſivæ

$= 1 - 1$

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=1−1+1−1+1 &c. adeoque facto $z = r$, & f = x, erit.

$$\overline{r+x}^{\,-1} = r^{-1} - x r^{-2} + xx r^{-3} - x^{3} r^{-4} \ \&c.\ in$$

finitum. Hinc, quantitatibus, quarum exponentes negativi ſunt, ad diviſores reductis, ngulisq: terminis per rr multiplicatis emerget.

$$\frac{rr}{r+x} = y = r - x + \frac{xx}{r} - \frac{x^{3}}{rr} + \&c.$$

Adeoque omnia ye = Seriëi terminorum, quorum ſinguli in e ducti ſunt; ac per § 12 omnia ye

$$= rx - \frac{xx}{2} + \frac{x^{3}}{3r} - \frac{x^{4}}{4rr}\ \&\ \textit{ſic continuando ſeriem}$$

in infinitum.

§. 23. Tum & eodem fonte ſcaturit infinita ries, in Figurâ XXXII, quamlibet quadrantis irculi AOF portionem determinans. Sit enim dius $AO: r$, $OQ: x$, $QD: y$, unde $yy =$ $rr - xx$, & $y = \sqrt{rr - xx} = \overline{rr-xx}^{\frac{1}{2}}$, ſeu eam $rr - xx$ ad dignitatem ½ evecta, hinc deribendo ex eodem §. 21. Poteſtatem binomii $rr - xx$ juxta exponentem ½, hoc eſt facto $p = \frac{1}{2}$ $z = rr$, & $f = -xx$; erunt, manentibus genituris, uilibet exponentes, qui ipſi z & f adhærent, er 2 multiplicandi, (cum z & f per quantitates, uæ numerum binarium pro exponenti habent,

 deſig-

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deſignentur) adeoque progreſſio poteſtatis $Ar^{2p} - Bxxr^{2p-2} + Cx^4 r^{2p-4} - Dx^6 r^{2p-6}$ − &c. unde poſito $p = \frac{1}{2}$ ac ipſis *A*, *B*, *C*, &c. per numeros explicatis fiet, $y = r - \frac{xx}{2r} - \frac{x^4}{8r^3} - \frac{x^6}{16r^5} - \frac{5x^8}{128r^7}$ − &c. hoc eſt ſecundum § 12 omnia *ye* $= rx - \frac{x^3}{6r} - \frac{x^5}{40r^3} - \frac{x^7}{112r^5} - \frac{5x^9}{1152r^7}$ − &c. hinc factâ maximâ $x = r$, dabitur totius quadrantis circuli per infinitam hanc ſeriem dimenſio.

Schol. Quomodo vero, diviſione ac extractione hoc modo inſtitutâ, ſemper ſeries inveniri poſſit, quæ omnium *ye*, ſeu curvilinei dati cujuſcunque valorem abſolutum comprehendat, modo ipſius *y* valor per quantitates determinatas ac interceptam *x* efferri poſſit, licet cujuſvis generis aſymmetriis fractionibuſque implicitus, erudiſſimâ ſuâ diſſertatione proſecutus eſt *Cl. D. Gregorius*: quo lectorem curioſum remittimus. Neque aliud quid ad abſolvendam per infinitas ſeries omnium curvilineorum quadraturam reſtare videtur, niſi ut ipſa applicata *y* modo ſtatim recenſito poſſit determinari. Decreveramque huc ſpectantia nonnulla hiſce annectere, niſi, felici omine, cum hæc prælo ſubmittenda eſſent, me honore præſentiæ ſuæ dignatus inſignis Mathematicus *J. Makreel* anglico idiomate *celeberrimi Newtoni* cogitata circa ſerierum proprietates in publicum emiſſa eſſe aſſeveraſſet: quare cum nihil niſi omnibus numeris abſolutum è tanti viri lucubrationibus prodeat, meis edendis, ut & demonſtrationi §. 21. addendæ ſuperſedi; & nullus dubitans, quin ibi loci profnndè tractata reperiantur, ad alia me confero.

Hacte-

Hactenus quidem ea præcipuè curvilineorum ſymtomata, quæ ex ordinariâ infiniteſimarum triga, *e*, *a*, *u*, ſeu triangulo *DHE* enaſci videbantur, diſcuſſimus, uti toto hujus capitis decurſu tum §. 17, 18, &c. videre eſt; quoniam vero non hæ tantum in ſingulis curvis, ſed & in quibuslibet præterea lineis ſubtangentibus, tangentibus, normalibus ac ſubnormalibus cæteriſque innumeris ſuæ conſiderari poſſunt infiniteſimæ, (quale ſpecimen Cap. I. §. 78, 79, &c. exhibuimus) nova hic ſeſe pandit, curvilinea tranſmutandi menſurandique methodus, ac theorematum non exigua multitudo.

24. Sit *Fig*: *XLII*. curvilineum *ADEP*, vocenturque ut ſemper *AQ*: *x*, *QD*: *y*, tangens *TD*: *s*, ſubtangens *TQ*: *t*, normalis *DR*: *k*, ſubnormalis *QR*: *l*, curva *AD*: *c*, creſcet *x* per reſpectivam infiniteſimam *QP*: *e*; *y* per *EH*: *a*; *c* per *DE*: *u*; *AT* ſeu $t-x=f$ per *BT*: *i*; adeoque *TQ* ſeu *t* per $QP+BT$ ſeu $e+i$, hinc & illinc; ſic & $DT=9C$ ſeu *s* utrinque creſcit per infiniteſimas 9*E*: *u*, & *BC*: λ; ſubnormalis *QR*: *l*, creſcit per *RV*: *o*, decreſcit per *QP*: *e*, unde $PV=l-e+o$, $BE=s+u+\lambda$, $BP=t+e+i$. Sic & *TC* ac *R* 10, ipſis *BE* & *EV* reſpectivè normales, ſunt infiniteſimæ; & *DR* creſcit per infiniteſimam *V* 10, & ſic in cæteris.

Quæ jam hinc ſuccreſcant curvilineorum proprietates uno aut altero exemplo declarabo.

25. Sit *P* 2 curva ſubtangentialis (hoc eſt cujus applicata *H* 2 curvæ *ADE* ſubtangenti reſ-

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pectivæ TQ ſeu t perpetim adæquatur) erit $E♀ = BP = t+e+i$, adeoque $w♀ = e+i$; & factâ ſubtangenti $H^{\Delta}: h$, eſt $h : t :: a : e+i$; unde $ta = he+hi$, ſed $ta = ye$, ergo $ye = he + hi$, quare factìs $Q3 = T8 = H\Delta$, donec enata ſint curvilinea AZP & AFB, erit hoc = omnia hi, illud = omnia he, & utrjuſque ſumma æqualis curvilineo expoſito AEP ſeu omnibus ye, undę hoc theorema:

Si in quâlibet curvâ ADE ad applicatam EP ordinetur curvilineum ſubtangentiale $P2H$, hujuſque inquiratur ſubtangens $H\Delta$: erit expoſitum curvilineum AEP ſemper æquale duorum curvilineorum ſummæ AZP & AFB; quorum hæc eſt proprietas, ut applicatarum $\mathcal{Q}3$ & $T8$ ſingulæ, utrique ſubtangentis reſpectivæ $T\mathcal{Q}$ extremitati inſiſtentes, inventæ curvæ ſubtangentialis ſubtangenti $H\Delta$ jugiter adæquentur.

N. B. quomodo jam, curvæ ADE proprietate cognitâ, cæterarum relationes inter applicatas interceptaſque conſtitutæ inveniri poſſint, cuilibet mediocriter in analyſi verſato plusquam cognitum eſt.

26. Sit, *Fig. XLII*, idem curvilineum APE, item duo alia AGP & $A8FB$, quorum alterum ad interceptam, $A\mathcal{Q}: x$, alterum ad ſubtangentis abſciſſam AT, $= t - x = f$, ordinatum eſt; ſintque in utroque reſpectivæ applicatæ $Q4$

&

T 8 ſemper æquales = *h*; tandem ſubtangens rimi *QI* applicetur in *T*, 12, ad ſecundi intereptam *AT*; item ſubtangens ſecundi *TO* ad rimi interceptam *A* Q in Q 5, donec exorta ſint uo nova curvilinea *A*, 12, *F*, *B*, & *ATP*. Dico æc ultima ſibi mutuo perpetim æquari.

Vocatis enim *TO*: *n*; & Q*I*: *m*; ſit 14, *G*, = 13, *F*, = μ; quæ eſt infiniteſima, quâ *h* in utroq urvilineo creſcit: erit; *n*: *h*:: *i*: μ; tum: *m*: *h*:: : μ; unde emerget *mi* = *ne*, eſt autem ex hyothesi area *A*, 12, F *B* = Omn. *mi* & ſpatium *ATP* = omn. *ne*; unde conſtat propoſitum.

27. Notari poteſt, ſi fiat *h* = *t* = *T*Q, fore erpetuò *m* + *n* = *t*; hoc enim caſu ſiet μ = *e* + *i*; deoq. *n*: *t*:: *i*: *e* + *i*; tum *m*: *t*:: *e*: *e* + *i*; unde, xploſis ex æquationum lege infiniteſimis, efati veritas eluceſcet.

28. Sit idem curvilineum *APE*, item aliud d eundem axem poſitum, *AGP*; cujus hæc ſt proprietas, ut Q 4 jugiter adæquetur prioris ubnormali *QR*: *l*; hujus indagetur ſubtangens *QI*: *m*; erit *G*, 14, = *o* − *e*; adeoque *m*: *l*:: *e*: − *e*; unde *me* + *le* = *mo*, quare facto 4, 5, = *m*, rit Q 5 = *m* + *l*; adeoque quadrilineum *A*, 20, *Y P*, ſeu omnia *me* + *le* æquabuntur quadrilineo, 30, 15, 17, *V*; cujus hæc eſt natura, ut ad ſingulas normalis *DR* ac axis *AR* interſectiones *R* & *V*, &c. elevata perpendicularis *R*, 16,

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æquetur inventæ ſubtangenti : Q *I* ; quamobrem quadrilineum, 30, 15, 17, *V* = omnia *m o*; demonſtratio patet, curvarumque omnium relationes ex curva *A P E* proprietate cognitâ manifeſtæ ſunt.

29. Quod hic circa infiniteſimas *B T*, *i*, & *R V*, *o*, præſtitum eſt, eodem modo circa cætera, verbi gr: *u* & λ ſeu *D E* & *B C* obtinet; factisque 6, *C*, & 7,9, = *h*; erit ſpatium *M N B*, omnia = *h*λ; & *W X E* = omnia *hu*; ſed de his in ſpecie alibi latius; cum punctum *N*, determinatum per interſectionem curvæ *A K N* & tangentis B *E*, ipſam tangentem dirimat in duas partes; quarum prior N E æquatur curvæ *A D E* = *c*, unde *N* B = *s* — *c*; ſuffecerit hic modum indicaſſe, quo innumeri generis curvilinea transmutantur.

30. Quomodo vero hinc multa curvilineorum ſpatia ad notam redigantur menſuram, ex methodo §. 17. adhibitâ innoteſcet. factâ enim quâlibet hypotheſi *h e* = *zi*, erit factâ Q 3 = *h*, & *T*, 12, = *z*, ſpatium *AZP* = omnia *h e*, & *B F*, 12, *A* = omnia *z i* : ſumtâque *h* ad curvilineum quadrabile, erit & ipſa *z* ad curvilineum notum.

Quopacto autem ex datâ *h* inveniatur *z*, ſequentia manifeſtant. Cum hic infiniteſimæ *e* & *i* ſint adhibitæ, notum eſt eas conſtituere ſubtangentis *T* Q incrementum, quare deſcribatur alia curva *A* 4 *G*, in qua ſemper Q 4 = *T* Q = *t*, erit

rit G, $14 = e + i$, vocatâque hujus ſubtangen- i $QI = n$, fiet $n : t :: e : e + i$, unde $\frac{te - ne}{n}$ $: i = \frac{he}{z}$ adeoque $z = \frac{hn}{t - n}$, hinc, datâ curvâ ADE, datur z ſeu curvilinei BFA relatio.

31. Sic & aſſumtis quibuslibet hypotheſibus, per quaslibet infiniteſimas deſignatis, ultima indeterminatarum poſt rejectas infiniteſimas (cæteris ex arbitrio ad quaslibet curvas determinatis) innoteſcet.

Verb: Gr: fiat $he + do = zi + g\lambda$, quomodo hinc infiniteſimæ ejiciantur, ſequenti paradigmate patebit. In genere, ſi e in ſuppoſita æquatione reperiatur, illa ad curvam ADE pertinens linea, quæ per infiniteſimam, quam ejicere conamur, creſcit, ad eundem axem AP applicetur; curvæque hinc enatæ ſubtangens reperiatur, habebiturque æquatio, qua expoſitam infiniteſimam tollere poterimus.

Sic I. ad ipſam o tollendam, cum ſubnormalem ſpectet, juxta §. 24. fiat curvilineum AGP tale, ut Q 4 æquetur ipſi QR, erit G 14, $= o - e$; adeoque vocatâ ſubtangenti Q$I = b$, eſt $b : l :: e : o - e$, unde $o = \frac{le + be}{b}$ II. ad i tollendam, cum ſpectet ſubtangentem t, concipiatur eadem Q4 eſſe $=$ TQ $= t$, ejusque ſubtangens Q$I = n$, erit $n : t :: e : e + i$,

unde

unde $i = \frac{te - ne}{n}$.

III. Ad tollendam λ, cum hæc tangentem ſpectet, ſit Q 4 $= s$, Q $I = m$, erit $m : s :: e : u + \lambda$, ſeu $mu + m\lambda = se$; ſed ex ordinariâ curvarum proprietate eſt $tu = se$, cujus ope ſublata u ex modo inventa æquatione, erit $\lambda = \frac{tse - mse}{tm}$. qui tandem valores in ipſorum o, i, λ, locum repoſiti, ac diviſione per ultimam e inſtitutâ, hanc æquationem reliquam faciunt:

$$h + \frac{dl + db}{b} = \frac{zt - zn}{n} + \frac{gst - gsm}{tm}$$

Unde, tribus ex h, d, z, g, pro arbitrio aſſumtis, ultima manifeſtatur.

Schol. Notetur, cum ad ſingulas lineas, quæ pro vario ad curvas reſpectu ac poſitione magnitudinem variant, ſingulæ pertineant infiniteſimæ, quibus vel ab unâ vel ab utrâque ſui extremitate augentur aut minuuntur; ac nulla ſit curva, ad quam infinitæ, ſaltem pro lubitu multæ, tales indeterminatæ multis modis referri non valeant; quarum ſingulæ in quamlibet infiniteſimam ductæ, ſi infinities ſumantur, curvilineum aliquod efformabunt; hoc ipſo methodus patet ac ratio, tranſmutationes ac ſpatiorum quadraturas plurimis modis tentandi; qui hactenus, ni fallimur, ferme intacti Mathematicis remanſerunt. Exempla quilibet ac theoremata proprio marte, ad præcedentium normam, ſibi cudet; unicum hoc addam in minus exercitatorum gratiam; ut pateat infiniteſimarum curvilineorumq: hinc ortum ducentium numerum ac varietatem, ſi velimus, in ſingulis curvis nullo limite cöerçeri.

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32. Repetitis *Fig. XXII.* ac Cap. I. §. 78 ſymbolis; in axe AQ producto aſſumatur punctum aliquod fixum W, unde ad prolongatas tangentes $EB3$, $ET5$, ducantur perpendiculares $W3$, $W5$, appellatiſque $AW: r$, $W5: k$, erit $WT = r + x - t$; tum & 4, 5, erit infiniteſima quæ vocetur: μ, unde $W3 = k - \mu = W4$, juxta lemma 36; & ex hypoth. angulus $E, 3, W$, eſt rectus, quare triangula $W3B$, & EPB ſunt ſimilia, ideoq. $r + x - t - i : k - \mu :: s + u + \lambda : y + a$, unde $ra - xa - ta - yi = ku - s\mu + k\lambda$, quibus rectangulis, tanquam curvilineorum indicibus, infinities ſumtis, juxta infiniteſimarum μ & i multitudinem; hinc & inde naſcetur æqualis curvilineorum ſumma; nec ſchemata nec demonſtrationem addo, cum mox prægreſſa intelligenti hæc obſcura eſſe nequeant. Hoc adjungo, puncta 3 & 5, eſſe ad novam hinc generatam curvam 6, 3, 5, 7; cujus relatio ad axem WA, ſi opus ſit, exhiberi poteſt, mediante normali in eundem demiſſâ; rectula 3, 4, eſt infiniteſima, & triangula $W, 3, 4$, & $E, 4, 5$, ſunt ſimilia, unde alia denuo propullulant.

Sic reſumtis Cap. I. §. 101, ob magnam illic indeterminatarum, infiniteſimarumque copiam, nulla fere ibi loci reperitur æquatio, quæ variorum curvilineorum multifariam æqualitatem aliaque ſymtomata non inferat: verum hæc ſufficiant.

33. Quo-

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33. Quoniam vero in trapezia non ſolum, ſed & plurimum in triangula, idque variis modis, quæ menſuranda præ manibus ſunt, polygona diſpeſcere ſolent Geometræ; ſit *Fig. XLI.* Curva *ADEF*, ejuſq. tangens *TDE*, cui normalis eſt *AR*; dividaturq. Curvilineum *ADEFO* (quod juxta Lem. 26, & 27 cum polygono coincidit) ex puncto *A* in triangula *ADE*, *AEF* & ſimilia; ac cæteris vocatis; ut ſæpius, dicatur *AR*: n, erit *TA*: *AR*:: *DE*: *EH*; ſeu $t - x$: n:: u: a; quare $ta - xa = nu$; hoc eſt poſito *TA* in *IM*, figura *FKO* ſen omnia $ta - xa$ æqualis duplo bilinei *ADEF*, quod conſtat totidem triangulis *ADE* ſeu $\frac{nu}{2}$ hoc eſt $\frac{AR \, in \, DE}{2}$.

Coroll. ſit, *ADE*, paraboloides, cujus æquatio $r^{p-q} x^{q} = y^{p}$, erit $t = \frac{p}{q} x$ adeoque $ta - xa = \frac{pxa - qxa}{q}$; ſunt autem omnia xa æqualia complemento parabolico *SAF*, quod vocetur f, ergo $\frac{pf \div qf}{q}$ æquale duplo bilinei *ADE*; quare $\frac{xy}{2} - \frac{pf + qf}{2q} = f$, ſeu triangulum *SAF* minus bilineo *ADEF* æquale trilineo *SADF*: unde reductâ æquatione $\frac{qxy}{p+q} = f$. uti ſupra jam aliquoties inventum.

34. Sit

34. Sit eadem *Fig. XLI.* multangula ſeu
urvilinea A*DEFO*, ducaturque *NE* cur-
am in F tangens; ut ſpatii AF*O* magni-
udo indagetur, dividatur trilineum *N*F*Z*A in
riangula *D*B*T*, ac ſimilia; cujus alterum la-
us *DT* infiniteſimæ *ED* in directum jaceat,
lterum B*D* ſit in infiniteſimâ *ZD* productâ & ſic
n cæteris; vocatiſque BT infiniteſima i, ac
P: $y+a$, poſitâ *D*Q $= y$, erit triangulum
B*DT* $= \frac{yi}{2}$, adeoque trilineum *N*F*Z*A æ-
quale omn. $\frac{yi}{2}$: fiat jam $yi = ze$, factoque
Q2 $= z$, erit ſpatium A4*O* ſeu omnia ze æ-
qnale duplo trilinei *N*AF ſeu omn. $\frac{2yi}{2}$; cum
autem hic ſit y: z:: e: i & poſito *YI* $= h$, ac
IM $= t$ juxta §. 79. Cap. I. h: $y-h$:: e: i,
erit $z = \frac{yy-hy}{h}$.

Coroll. Unde poſita *AE*F ad idem cum
præcedenti paraboloïdes, erit juxta tangentium
methodum *YI* ſeu $h = \frac{qy}{p}$, quare $z = \frac{py-qy}{q}$;
& omnia ze ſeu *A*4*O* ſeu $\frac{pye-qye}{q}$ ad para-
bolöides *A*F*O* ſeu omnia ye ut $p-q$ ad q,
ſed ſpatium *A*4*O* æquale ex præcedenti duplo
trilineo *NA*F, ergo duplum trilinei *AN*F
ad

ad parabolam *AFO* triangulum *NFO* complentem ut $p-q$ ad q, unde rurſus parabolôideôn quadratura innoteſcit.

35. Si vero quis aliam curvilineorum in triangula diviſionem expetat, aſſumto *Fig. XLIII* & *XLIV.* in axe *AP* puncto quolibet B, ducatur ex eo tangenti *DE* productæ normalis B*R*, quæ vocetur z, ac BT: h, & porro ex puncto B curvilineo B*ED*A diviſo in triangula B*DE* &c. æquabunt hæc triangula ipſum curvilineum; & propter triangulorum T*D*Q & TB*R* ſimilitudinem T*D*:*D*Q::BT:B*R*::*DE*:*EH*, ſeu $s:y::h:z::u:a$. unde $zu = ha$, factâque *M*B ipſi *D*Q parallelâ, ac ſumtâ in eâ B*C* = *D*Q = y, & *GC* = *EH* = a, cui in *C* normaliter inſiſtat *CL*:h, idque toties, donec per extremitates ipſorum h ſeu puncta *K*, *L*, &c. deſcribi queat curva *NKL*A, erit ſpatium *NM*BA æquale omn. ha, quod æquatur totidem zu, ſeu rectangulis ex B*R* in *DE*; jam B*R* in *DE*, eſt duplum trianguli B*ED*, per elementa: quæ triangula complent trilineum mixtum A*DE*B, quod proinde = omn. $\frac{zu}{2}$, quare omnia ha ſeu ſpatium *NM*BA æquatur duplo trilinei *ADE*B ſeu omn. $\frac{2zu}{2}$ unde variæ denuo curvilineorum transmutationes.

Coroll.

Coroll. Poſitis enim, *Fig. XLIII*, $BQ:x$, $CD:y$, ſit æquatio curvæ ADE ad hyperbolam $rr+yy=\frac{l}{q}xx$; erit juxta tangentium methodum BT ſeu $CL=h=\frac{qrr}{lx}$, quæ æquatio expreſſa per y, cum CL ad y ſeu CB ordinetur, erit $h=\frac{rr\sqrt{\frac{q}{l}}}{\sqrt{rr+yy}}$ curvæ NLA relationem deſignans æquatio; eſtq. hoc in caſu curvilineum $AGKA$ æquale trilineo hyperbolico BEDA.

36. Sponte hinc naſcentem obſervationis loco conchöidum dimenſionem adjicere liceat.

Sit *Fig. XLV.* Polo M, vertice A, normâ DQ, deſcripta Conchois ABC; vocenturque $AD=Q$B$:r$, $DM:s$, $GD:f$, GB$:x$, ſit radio AD ſeu r deſcripto quadrante circuli AHJ, appellatâque $HG:y$, conchoidis proprietas, $x=\frac{ſy}{f}+y$; &, poſitâ $\frac{ſy}{f}=z=GN$ ad curvam ANR, $z+y=x$. Unde trilineum AGN ſeu omnia ze, una cum trilineo AHG ſeu omnib. ye æquatur omnib. xe ſeu ſpatio conchoidali AGB.

Jam autem, cum omnia ye ſint ad quadrantem circuli, inveniendum reſtat trilineum AGN ſeu omnia ze, quem in finem ſumtâ NP ſeu f pro applicatâ, & DP ſeu z pro interceptâ,

 erit

erit (cum ſit $\frac{sy}{f} = z$, & $yy = rr - ff$) $f = \frac{rs}{\sqrt{ss + zz}}$ æquatio curvæ ANR relationem ad axem DP manifeſtans, huic autem curvilineo $ADPN$ per præcedens corollarium, æquale fuit inventum trilineum hyperbolicum; quæ ut hinc caſui approprientur, erat $h = \frac{rr\sqrt{\frac{q}{l}}}{\sqrt{rr+yy}}$ in Coroll. præmiſſo, hic vero $f = \frac{rs}{\sqrt{ss+zz}}$. fiat ergo terminorum comparatio, & collatis denominatoribus erit $s = r$ & $z = y$, ex numeratoribus autem emergit: $\frac{q}{l} = \frac{rr}{ss}$; quare loco æquationis hyperbolicæ $rr + yy = \frac{l}{q}xx$, ſi loco x ſumatur k, orietur hoc in caſu hæc, quæ ſequitur, $ss + zz = \frac{ss}{rr}kk$, hyperbolam ſpectans, cujus latus transverſum $= 2r$, rectum $= \frac{2ss}{r}$. ad cujus conſtructionem, quadrilineo $ADPN$ circa DP revoluto in $SDPT$, latere recto $\frac{2ss}{r}$ & transverſo AS ſeu $2r$, deſcribatur hyperbola modo inventa SF, ſecans productam NP in F, erit ductâ rectâ DF, mixtilineum

ADNP

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DNP ſeu *DSTP*, duplum trilinei hyperbolici *SDF*.

Ergo ſpatium *AGN* æquale duplo trilineo *SDF*, demto rectangulo *GDPN*; cui ſpatio *AGN* jam invento ſi addatur ſpatium *ANB*, ſeu illi æquale ſegmentum circulare *AHG*, ex modo datis emergit ſpatium conchôidale *AGB* æquale duplo trilineo hyperbolico *SD* F, una cum ſegmento circulari *AGH*, demto rectangulo *GDPN*.

§. 37. Repetantur ſymbola §. 24. aſſumta, ac ſchema XLII. oportet curvilineum *AZP* deſcribere, quod æquale eſt rectilineo cuicunque inter quaſlibet rectas, curvam *ADE* ſpectanti, efformato.

Dicatur curvilinei quæſiti applicata Q 3 : *h*, et *AZP* = omnia *he*; jam quæritur *h*, ſi

I. Sint omnia $he = \frac{yy}{2}$; ſeu area *A* Q 3 = ſemiquadrato in Q*D*. Hæc & ſequentia ex §. 13. ſolutionem admittunt, fiat enim *he* ſemper æqualis terminorum infiniteſimalium ſummæ ex rectilineo dato, ſeu hoc in caſu ex $\frac{yy}{2}$ ortum ducentium, erit $he = ya$; adeoque juxta §. 13. tracta, omnia $he =$ omn: $ya = \frac{yy}{2}$, jam per æquationes ex curva *ADE* conquiſitas eliminentur

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infiniteſimæ e, & a; habebiturque, cum ſit $he = ya$, & $ye = ta$, amotis infiniteſimis, $h = \frac{yy}{t} = l = QR = Q3$.

II. Sint omnia $he = ry =$ rectangulo ex r determinatâ in DQ, erit ra terminus infiniteſimalis ex ry oriundus, & facto $he = ra$, erit $ry =$ omnia $he =$ omnia ra, ex §. 13. adeoque cum ſit $ye = ta$, fiet $h = \frac{ry}{t}$.

III. Sint omnia $he = xy =$ rectangulo ex AQ in QD, erit juxta *Schol.* §. 13. $xa + ye$ terminorum inſiniteſimalium ex xy naſcentium ſumma; hæc fiat $= he$, erit $xy =$ omn. $he =$ omn. $xa + ye$ ex §. 13. & eliſis infiniteſimis per æquationem: $ye = ta$, fiet $h = \frac{xy + ty}{t}$

IV. Sint omnia $he = ty =$ rectangulo ex QT in QD. erunt, juxta §. 24. ipſius t reſpectivæ infiniteſimæ $= e + i$; adeoque termini infiniteſimales per *Schol.* §. 13. hinc prodeuntes $ye + yi + ta$, quibus factis $= he$, erit $ty =$ omn. $he =$ omn. $ye + yi + ta$ ex §. 13; unde ex hac æquatione $he = ye + yi + ta$ hoc modo ejiciantur infiniteſimæ, primo, eſt $ta = ye$, quare $he = 2ye + yi$; ſecundo, ut tollatur i, cum hæc pertineat ad ſubtangentem TQ ſeu t, fiat AGP curvilineum ſubtangentiale, in quo ſemper,

pr, $Q4 = QT$, erit $G,14, = e+i$, & vocatâ hujus ſubtangenti $QI = n$, eſt $ne+ni = te$, ex quâ cum priori combinatâ emergit $h = \frac{ty+ny}{n}$.

V. Rurſum ſint omnia $he = rt$ = rectangulo ex r determinata in QT, erit juxta modo memorata $he = re+ri$, tum $ne+ni = te$, adeoque $h = \frac{rt}{n}$.

VI. Sint Omnia $he = ſt$ = rectangulo ex D in QT. erunt, juxta §. 24, $u+\lambda$ ipſius $ſ$, & $e+i$ ipſius t infiniteſimæ; adeoque infiniteſimales termini ex $ſt$ oriundi, juxta §. 13. Schol. $tu+t\lambda+ſe+ſi$; qui ſi fiant = he, erunt omnia he = omnia $tu+t\lambda+ſe+ſi = ts$ ex §. 13 ...; quomodo jam hinc infiniteſimæ tollendæ ſint, ut ipſius h valor inveniatur, jam §. 31. in genere oſtenſum eſt; & exemplis præcedentibus ritè intellectis ſatis liquido apparet.

§. 38. Hoc notetur, neceſſario non requiri, ut h ſeu $Q3$ perpetuo ad AQ applicetur; unde curvilineum AZP per omnia he explicatur, ſeu per h & infiniteſimam e, qua AQ ſeu x creſcit, verum & ad alias quaſcunque rectas ipſam h ordinari poſſe.

Sit enim $T, 8, = h$, erit curvilineum EFA = omnia hi; ſit hoc æquale ipſi xy ſeu omnibus $xa+ye$, adeoque cum ſit $xa+ye =$

 hi,

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hi, tum $ta = ye$, & juxta §. præc. Ex. IV. $ne + ni = te$, ejectis infiniteſimis fiet $h = \frac{nxy + nty}{tt - tn}$.

Nec ulteriori diſcurſu adſtruere opus videtur, poſſe h ad quaſlibet rectas quibuslibet infiniteſimis creſcentes applicari; modo infiniteſimarum eliminatio, juxta ea quæ in §. præc. tum & §. 31. tradidimus, legitime adhibeatur: ſic enim ty poterit ſupponi = omnia he in curvilineo AZP; tum = omnia ha in curvilineo $AP\Omega$, item = omnia hi in curvilineo AFB; nec non = omn. ho in curvilineo 30, 15, 17, V. & ſic infinitis modis.

§. 39. Neque ſemper neceſſarium eſt, ut rectilineum, cui alia figura curvilinea æqualis quæritur, ſit inter rectas ad curvam ADE ſpectantes conſtitutum; quinimo generaliter ſolvi poteſt problema, ac inveniri relatio, quam habet applicata Q3 ad interceptam AQ, ſi curvilinei AQ3 area ſemper ſit æqualis rectangulo quarumcunque applicatarum Q4 & Q5. factis enim Q4 $= f$, & Q5 $= z$, ſubtangentibuſque, Q$I = b$; Q, 18, $= d$, ac infiniteſimis, G, 14, $=$, $^{\theta}$, & Υ, 19, $=$ $^{\mu}$:

Cum ex hypotheſi ſint omnia $he = fz$; erunt $f^{\mu} + z_{\theta}$ termini infiniteſimales ex rectangulo fz oriundi, qui ſi fiant $= he$, erunt omnia $he =$

om-

[omnia] $f\mu$ + omnia $z\theta = fz$ ex §. 13. jam ex
rvilineorum AGP, AYP naturâ eſt $\theta = \frac{fe}{b}$
em $\mu = \frac{ze}{d}$, unde ſuffectis in æquatiöne $he =$
$\theta + f\mu$ inventis valoribus erit $h = \frac{fz}{b} + \frac{fz}{d}$;
are fz rectangulo per x interceptam expreſſo
iam h per eandem x exprimetur.

Ex. Gr. ſit $fz = \sqrt{rrxx + rx^3}$, fiatque $f = x$,
$z = \sqrt{rr + rx}$, erit $b = x$, & $d = 2r + 2x$;
[i]nde ablatâ aſymmetriâ emerget curvilinei
ZP relationem deſignans æquatio, $4r^3 +$
$rrx + 9rxx = 4rhh + 4xhh$.

§. 40. Quod ſi f & z, quolibet modo,
oc eſt in relatione ad quaſlibet lineas tum rec-
s, tum curvas, exprimantur, tamen h de-
rminabitur; modo ſubtangentes b & d cog-
tæ ſint, uti patet.

§. 41. Animadverſu dignum videtur, ſi cur-
ilineum quæſitum per omnia he deſignetur,
oc eſt ſi Q 3 ſeu h ad AQ ſeu x tanquam
terceptam ordinata fuerit, ipſius h valorem
giter adæquari ſubnormali QV curvæ A,
, P; cujus hæc eſt proprietas, ut exiſtente
ſius applicatâ Q, 22, $= g$, ſit perpetuo
$= \frac{gg}{2}$, vocatâ enïm hujus ſubtangenti Q,
1, $= m$, & ſubnormali QV: q, cæteris ut

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ſupra manentibus, erit juxta Cap. 1, §. 35. æquatio ad tangentem $fzbm + fzdm = ggbd$: quare $\frac{fz}{b} + \frac{fz}{d} = h = \frac{gg}{m} = q$ ex proprietate trianguli rectanguli, 21, 22, V.

Poſſent hinc omnia exempla §. 37. exhibita deduci; ſi enim fiat, $f = t$, & $z = y$, erit $b = n$, & $d = t$, adeoque $h = \frac{ty}{n} + y = q$, ut in exemplo tertio.

Schol. quod, ſi quis hinc converſum meditetur; & ex data h ſeu $Q3$ velit inquirere curvilinei $AQ3$ magnitudinem; patet ex præcedenti totum hoc negotium ad probl. §. 95. Cap. I. propoſitum referri, nempe ad inventionem curvæ A, 23, P, cujus ſubnormalis QV ſit data $= h$; de quo alibi latius actum eſt. Interim licet §. 37. & 39. exhibita exempla varios videantur innuere modos, quibus idem, à poſteriori veſtigia relegendo, obtineri poſſit; ex §. 41. tamen ſatis eſt manifeſtum, totam hanc, qnæ primò ſeſe offerre videtur, principiorum varietatem in unicum hoc, quod modo propoſuimus, deſinere.

Quænam etiam ſit eorum, quæ §. 38. tradidimus, cum modo recenſitis differentia, adhibito calculo generalioriad methodum §. præcedentis, eluceſcet.

Poſſent & hic relationes curvilinei menſurabilis A Q3, naturali quodam ordine diſponi; a ſimplicioribus ad magis compoſita progrediendo, plnrimorumq. adeo curvilineorum menſuræ tradi, ex variis ipſius g ſeu Q, 22, ad AQ, relationum hyphotheſibus oriundæ; at quorſum? cum non niſi generaliter ſoluto ſæpius inſinuato ſubnormalium problemate, deſideratum obtineri queat.

§. 42. Sit, *Fig. XXXIV.* $AQ = ND : x$,

$QD = TM: y$, $QT = DM: t$; $QP: e$, $PT: i$; erit area $AEP =$ omn: ye, hujus quæritur menſura. I, cum e ſit infiniteſima ad AQ ſeu x referenda, hoc in ipſius locum repoſito habebitur: yx, cujus termini infiniteſimales $ye + xa$; & juxta §. 13, omnia $ye +$ omn. $xa = yx$; unde patet cognitis omnib. xa, cognoſci omn. ye: eſt autem xa ad dati ſpatii complementum KEA, ſati notum.

II. Cum infiniteſima e ſpectet ſimul ſubtangentem, TQ ſeu t, emerget, t ipſi e ſurrogato, yt; cujus infiniteſimales $ye + yi + ta$, ſeu $2ye + yi$; & per §. 13, omn. $2ye +$ omn. $yi = yt$; unde inventis omnibus yi, nota erunt omnia ye; jam yi eſſe ad curvilineum KVA nemo ignorat. Hæc autem methodi gratiâ.

Schol. ut vero pateat, non ſemper tangentium inventionem aut ſubnormalium ad menſuras curvilineorum neceſſariam eſſe; verum & ſolam æquationis curvæ ad infiniteſimas redactionem ſæpenumero levi operâ pandere, quæ, non niſi intricatiori calculo, per tangentium methodum inquirere moris eſt, ſequentia ſubjungo.

§. 43. Sit, in *Fig. XXXIX*, $AQ = x$, $QD: y$, infiniteſimæ $DH: e$, $EH: a$, & curvæ æquatio ad quamlibet paraboliformem $r^{p-q} x^q = y^p$; erit hæc æquatio ad infiniteſimas redacta, ac in yx ducta $q r^{p-q} x^q ye = p y^p xa$, quæ per æquationem datam diviſa dabit $qye = pxa$; quocirca juxta lemm 43, $p: q::$ omnia

nia xa ad omn. ye, hoc eſt ita complementum AFV ad paroboloidem AFO. Unde eadem, quæ §. 11. oſtenſa ſunt, deduci poſſent abſq. tangentium auxilio.

§. 44. Sit *Fig. XIII. N.* 1. eademque, quæ §. 31. Cap. I. ſymbola; adeoque ciſſöidis AKG æquatio $xx = yz$, ſeu $x^3 = rzz - xzz$; erit ea ad infiniteſimas redacta, juxta *Schol.* §. 13, $3xxe = 2rzo - 2xzo - zze$; quæ diviſa per z, ac loco $\frac{xx}{z}$ poſito y juxta hypotheſin, tum & l ipſi $r - x$ ſubſtituto abibit in hanc. $3ye = 2lo - ze$.

Jam vocato ciſſoidali integro ſpatio $= k$, ſemicirculo integro $= c$, cum omnia lo ſint $=$ omnia ze, (ſi utrumque aream ciſſöidis infinitam deſignet) erit $3c = 2k - k = k$ ſeu Ciſſöis ſemicirculi. tripla.

Hoc notetur; ſi tum omnia lo, tum omnia ze, non ad infinitum ciſſöidis ſpatium, ſed ad partes ejus reſpectivas referantur, fore AKP $=$ omnia ze (hoc dicatur: f) tum $AKMO$ $=$ omnia lo (hoc appelletur: $p + f$, vocato ſcilicet rectangulo $PKMO$: p) ac omnia $ye = APE$. (hoc fiat $= d$) quare cum ſit ex ſuperioribus omn. $3ye =$ omn. $2lo -$ omn. ze, erit $3d = 2p + 2f - f$, ſeu $3d - f = 2p$, hoc eſt triplum portionis circularis APE, detractâ

reſ-

reſpectivâ Ciſſoidali portione AKP, erit æ-
quale bis ſumto rectangulo $PKMO$ ſeu AP
in PE: eſt enim ex naturâ Ciſſoidis $rz - xz$
$= xy$.

Tum & alia ratione ex æquatione inventa
$3d - f = 2p$ totius Ciſſoidis ad circulum ratio
cognoſcitur; eſt enim rectangulum $PKMO$
: $p = rz - xz$; unde facto $x = r$, ſeu AQ
$= AO$, pertinebit ipſa f ad integram Ciſſoi-
dem, ac d ad totum ſemicriculum, & p eva-
net æqualis nihilo; quare $3d - f = 0$, & $f =$
$3d$, ſeu ciſſois ad ſemicirculum ut 3 ad 1.
Similia hic poſſent circa ciſſoidem ex hyperbola
genitam aliaſque curvas adjungi.

§. 45. Sit Conchöïs ABC *Fig. XLV*, ver-
tice A, normâ DQ ac polo M deſcripta;
vocenturq. $AG: x$, $GB: z$, $GD: f$, $GM: l$,
$AD: r$, $DM: p$, tum radio AD ducto circu-
li quadrante ADI, appelletur $HG: y$, tan-
dem dicantur infiniteſimæ, $G2: e$, $4, 5: a$;
$V3: o$.

Notum eſt æquationem conchöidis proprie-
tatem exprimentem eſſe $ly = fz$, hæc redacta
ad terminos infiniteſimales dabit $fo - ze =$
$la - ye$, quæ, ſi ducatur in l, ac loco ly
ponatur fz, exhibebit $flo - lla = lze - fze$;
unde (cum ſit $l - f = p$, ac ex naturâ conchoi-
dis $l : z :: f : y$.) omnibus per f diviſis fiet $fo - la$

$= p$

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$= \frac{pye}{f} = \frac{prye}{fr}$: jam vero ſi $\frac{ry}{f}$ ſeu tangens *A 7* vocetur *n*, ipſiq. jugiter æqualis aſſumatur *G N*, donec exorta fuerit curva nova *A N R*, erit $fo = la + \frac{pne}{r}$; & ſumtis omnibus infinities, omnia *fo* ſeu ſpat: *AWVD* æquantur omnib. *la* ſeu ſpatio 5 *Z M A*, una cum ſpatio *A*, 2, 8, ſeu omnib *ne* ducto in $\frac{p}{r}$.

Unde rurſum conchoïdis ad alîus generis curvilinea ratio innoteſcit.

§. 46. Sit, *Fig. XXXIX.* radius *A O*:*r*, & *A D* arcus circuli, *A Q*:*x*, *Q D*:*y*, *A C* portio Ciſſoidis cujus applicata *Q C*: $\frac{xx}{y} = z$; tum *A B* pars curvæ, cujus applicata *Q B*: $\frac{rx}{y} = k$; quæritur Areæ *A B Q* ratio ad ſemicirculum; ſeu omnium *ke* ad omnia *ye*. æquatio ſemicirculi eſt: $2rx - xx = yy$, quæ redacta ad infiniteſimas dabit, $re - xe = ya$, & ducta in *x* ac diviſa per *y* exhibebit. $\frac{rxe}{y} - \frac{xxe}{y} = xa$, ſeu $ke - ze = xa$; & vocatis omnib. *ke* ſeu *A B Q*: *d*, tum omnib. *ze* ſeu *A C Q*: *f*, ac omnib. *xa* ſeu complemento portionis circularis *A D X*: *b*, eſt $f = d - b = 3c - 2p$ ex §. 44. (facto nimirum ſegmento ſemicirculi *ADQ*: $= c$,

& rec-

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rectangulo $xy = 2rz - xz = p$) unde $d = 3c$ — $b - 2p$, & cum ſit $b + c = p$, tandem $d = 2c - p$.

Notetur, ſi ſit $x = 2r$ fore $p = 0$, adeoque $= 2c$, hoc eſt totum curvilineum d, cujus applicata: $\frac{rx}{y}$, æquatur duplo totius ſemicirculi, ſeu $2c$.

Hujus curvilinei aream §. 16. *Coroll. III.* alio modo in ratione ad circulum tangentium ope menſi ſumus

§. 47. Quinimo & brevius ſic efficitur, diviſâ circuli æquatione $2rx - xx = yy$ per y, ductiſq. omnibus in e, fiet: $\frac{2rxe}{y} - \frac{xxe}{y} = ye$; ſeu ſumendo ſymbola præcedentis $2d - f = c$; & cum æquationem ad integra curvilinea determinando ſit juxta §. 44. $f = 3c$, erit $d = 2c$, ut ſupra.

§. 48. Stantibus iisdem, quæritur curvilinei ABQ, cujus applicata $QB = \frac{rr}{y}$, ratio ad circulum.

Æquatio ſemicirculi infiniteſimalis $re - xe = ya$, ducta in r, ac diviſa per y, dabit $\frac{rre}{y} - \frac{rxe}{y} = ra$; fiant omnia $\frac{rre}{y}$ ſeu $ABQ = q$, omnia $\frac{rxe}{y}$ ſeu $AQC = d$, omnia $ra = AX$

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$IO = ry = n$, erit $q - n = d = 2c - p$, juxta §. 46, adeoque, $q = 2c - p + n$.

Et æquationem ad integra curvilinea extendendo, erit $x = 2r$, & hinc tum p tum n nihilo æqualia; unde tandem: $q = 2c$, ſeu integrum hoc curvilineum eſt ſemicirculi duplum; quod §. 16. *Coroll. III.* tum & §. 15, alio pacto per tangentes evictum eſt.

§. 49. Quomodo vero hæc omnia in §. 10. modum curvis ad punctum Q relatis applicari queant, levi opera explicabo; cum enim ſit Fig. XLVI. continuo $TD : s, DQ, y :: DE, u : EH : a$, erit ſemper $sa = yu$; unde radiis, DQ & EQ ductis arcubus DI, EG, erit $IG = a$; poſitâque TD ſeu s, in IM, erit Figura QFK ſeu omnia sa æqualis Figuræ $q\kappa\kappa\varphi\varphi$ ſeu omnibus yu, Fig. XLVII. in qua Fig. $q\varphi\varphi$ recta $q\varphi$ ſuppoſita eſt æqualis curvæ expoſitæ $QDEF$, ita ut DE ſit æqualis $\delta\epsilon$ ſeu u, ac applicatæ $\delta\kappa$, $\epsilon\kappa$, ſunt æquales rectis DQ, EQ ſeu y.

Quoniam vero tot continentur in rectâ QF infiniteſimæ ut $IG = HE$ ſeu a, quot in Curvâ $QDEF$ infiniteſimæ DE; vel, quod eodem redit, quot in recta $q\varphi$ (quæ cnm curvâ QDF eadem eſſe ſupponitur) infiniteſimæ $\delta\epsilon$ ſeu u, vocatis $\delta\kappa : z$, & $IM : h$, factoque ſemper $ha = zu$, Figura QFR ſeu omnia ha, æqualis erit ſpatio $q\varphi\varphi$ ſeu totidem zu.

Quare

Quare poſitâ z vel h ad locum quemlibet, ..tera h vel z innoteſcet; nam quia $ha = zu$, .. $a : u :: z : h :: y : s$, unde $hy = zs$, & $h = \frac{zs}{y}$.

Cum vero quidem æquemulta ſint DH ſeu e, ..iam HE ſeu a, ac DE ſeu u; ſed infini..ſimæ DH tam commodè in rectam projici ..queant, ac in Figuris ad axem relatis; deſcrip.. .. Curvâ ſecundâ $QSNO$ ad idem punctum .. relatâ, productiſque QD in S, & QE .. N, fiat QS ſeu $QR = z$, & arculus in..nitesimus RS dicatur i, erit juxta §. 10, ſpa..tium $QSNO =$ omnia $\frac{zi}{2}$; facto jam $zi =$ ..u, ipſa IM dicatur h; eritque ſpatium ..KF ſeu omnia ha æquale duplo ſpatii ..SNO ſeu omnium $\frac{zi}{2}$, nam $ha = \frac{2zi}{2}$. ..uare poſitâ alterutrâ h ad locum quemlibet ..innoteſcit z, cum enim ſit QD, $y : DH$, $e ::$ ..S, $z : SR$, i. ob arculos DH & SR pa..allelos, erit. $e = \frac{iy}{z} = \frac{at}{y}$, hinc $\frac{atz}{yy} = i = \frac{h\ldots}{\ldots}$, quare $tzz = hyy$. unde data h noſcitur z .. contra.

Simili modo, poſito $zi = hu$, factâque ..ctâ $q\varphi$ æquali curvæ $QDEF$, ac $\delta =$ DE,

DE, $= u$, item $\delta\varkappa = h$, erit ſpatium $q\phi\varphi$ ſeu omnia hu æquale duplo ſpatio QSo, ſeu $\frac{2zi}{2}$; quare z vel h pro arbitrio poſitâ, quæ reſtat hoc modo determinatur, eſt enim, $TD, s : DE, u :: QT, t : DH, e$. unde $e = \frac{tu}{s} = \frac{yi}{z}$, quare $\frac{tzu}{sy} = i = \frac{hu}{z}$, & tandem $tzz = shy$; unde patet propoſitum.

Coroll. I. Hoc in exemplum: ſit F. XLVIII. ſpiralis *DVSH P* ad Circulum *APX* pertinens, cujus radius, r, peripheria, d, *SD*: z, *DF*: t recta FS tangat ſpiralem in S, infiniteſimæ, S*I*: e, *HI*: a, ſitque juxta quemvis tum in radio tum in circumferentia motum, vocato arcu *P A* ad lineam. S*D* relativo: c, æquatio ſpiralis, $r^k : d^s :: r^k : c^s$, erit juxta §. 67 Cap. I. de tangent. *D* F ſeu $t = \frac{kcz}{sr}$, & per curvæ æquationem ipſo c amoto $\frac{s^s r^{k+s}}{d^s k^s} t^s = z^{k+s}$, quæ æquatio eſt ad parabolarum unam; ducto enim ad ſpiralis principium *P* radio *D P*, factâque *D M* $=$ *D* S $= z$, & *D L* $=$ *D H*; erit *L M* $=$ *H* S $= a$, poſitâq. *D* F ſeu t in *M N*, donec per omnia *N* duci poſſit Curva *D N O R*; erit t applicata, & *D M* ſeu z intercepta, ac $RP = \frac{kd}{s}$, & rectangulum

gnum $PDQR = \frac{kdr}{s}$: ſed juxta §. 11. Cor. I. et paraboloides DRP $\frac{kdr}{k+2s}$ = omnia ta, $\frac{2ze}{2}$ = duplum areæ ſpiralis ex §. 10; quare ſpatium ſpirale æquatur $\frac{\frac{1}{2}kdr}{k+2s}$, quod eſt ad Circulum ſeu $\frac{1}{2}dr$, ut k ad $k+2s$; hæc quoque, ſe alio modo, demonſtrata dedit doctiſſimus *Sluſius* in miſcellaneis.

Corol. II. Quod ſi ſpiralis ejus ſit naturæ, ut circuli radios BD ad eundem ſemper angulum ſecet, erunt triangula SHI perpetuo ſimilia, angulus enim I rectus eſt, & H ex ſuppoſitione in omnibus conſtanter æqualis. Sit ergo laterum SI: e ad HI: a ratio permanens ut p ad q, erit $pa = qe$, adeoque $qze = pza$; & per §. 13. converſum ſpatium ſpirale infinitum VD ſeu omn. $\frac{ze}{2} = \frac{pzz}{4q}$; ſeu cum ſit p: q:: GD, n: PD, r, (poſitâ GP ſpiralem in P tangenti, ac GD ipſi DP normali) erunt omnia $\frac{ze}{2} = \frac{nzz}{4r}$; unde calculum ad totum ſpirale ſpatium $PSVD$ aptando, erit $z = r$, adeoq. hoc ſpatium infinitum $= \frac{nr}{4}$ = ſemitriangulo PDG. Notari poteſt, ſi AB vocetur: o, fore $e =$

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$\frac{zo}{r} = \frac{pa}{q}$, unde hæc proportio $\frac{pr}{q} : o :: z : a$; jam poſita *o* conſtanter æquali, hoc eſt *PB*, *PA*, &c, in progreſſione arithmeticâ, habebit *z* ad *a* rationem ſemper eandem) adeoque omnia *z* erunt in progreſſione geometricâ; cum quantitatibus, quæ differentiis ſuis proportionales ſunt, hoc eſſe proprium ex elementis notum ſit.

50. Forſitan & hinc ortum trahentes *involutarum* (quas vocant) & *evolutarum figurarum* proprietates addidiſſe operæ fuerit pretium.

Sit *Fig. L.* ſpatium *ABHG* in trapezia infiniteſima *CEFD* præcedentium more diviſum; concipianturq. curvæ partes *CD* earumq. ſinus *CL* filorum in modum flexiles, poterunt igitnr manentibus applicatis, *A* B, *EC*, *FD*, *GH*, rigidis, ſingula trapezia *CEFD* mutari in trilinea *κγδ* in *Fig. LI.* ſi puncta ninurum *E* & *F* cöincidant; quod ſi fiat in ſingulis *Fig. L.* trapeziis, & omnia diviſionum axis puncta ſupponantur in *G* cöire, orietur *Fig. LI.* *κβη*, & punctum *κ* referet omnium punctorum *AEFG* coalitum; diciturque Figura *κβη* Figuræ *ABHG* *involuta*, & hæc illius *evoluta*, quarum ſequentes obviæ ſunt proprietates.

I°. Cum rectangulum *CLFE* cogatur in cir-

rculi ſectorem infiniteſimum $\gamma\lambda\kappa$, hoc eſt di-
idium ſui, ob angulos in γ & λ rectos, ac
nearum $\gamma\lambda$ & *C L*, item $\gamma\kappa$ & *E C* æqua-
atem: quod ſi idem fiat in omnibus, erunt
mnia rectangula *C E F L*, ſeu Figura *A B H G*,
pla omnium triangulorum $\kappa\gamma\lambda$ ſeu Figuræ
volutæ $\kappa\beta\eta$.

2°. Quia ex hypotheſi $CL = \gamma\lambda$, & $CD = \gamma\delta$,
unt, ob angulos in *L* & λ rectos, triangu-
C L D & $\gamma\lambda\delta$ ſimilia & æqualia: quare pö-
o angulo $\tau\kappa\gamma$ recto, quia $\gamma\kappa = EC$, erunt
hiſce ſimilia triangula $\tau\kappa\gamma$ & *T E C* ſimilia
æqualia inter ſe.

3°. Sic & arcum $\beta\rho$ radio $\kappa\beta$ deſcriptum
A G minorem, arcum vero $\mu\eta$ radio $\kappa\mu$
ſcriptum eodem axe majorem eſſe ex præ-
iſſis ſponte fluit.

Duo vero ſequentia problemata fundamenti
co cæteris huc ſpectantibus ſubſterni poſ-
nt.

§. 51. Quorum primum ſit; ex data in-
luta $\kappa\beta\eta$ ejus evolutam *A B H G* inve-
re.

Ad cujus problematis ſolutionem, (cum den-
r $\kappa\eta = GH$, $\kappa\delta = FD$, $\kappa\gamma = EC$ ex invo-
tæ natura) requiritur *E G* axis longitudo in
ſpectu ad datam applicatam *E C* ſeu $\kappa\gamma$; quæ
c modo indagatur, ſit $\tau\kappa = TE : t$, $\kappa\gamma = EC : y$,

$\gamma\lambda = CL : e$, $\lambda\delta = LD : a$, $\gamma\delta = CD : u$, $\tau\gamma = TC : s$. quia $\tau\kappa : \kappa\gamma :: \gamma\lambda : \lambda\delta$, ſeu $t : y :: e : a$, erit $e = \frac{ta}{y}$, & omnia $\frac{ta}{y}$ = omnia e = omnia EF = axis AG.

Quia vero y eſt indeterminata, ſiat. $\frac{ta}{y} = \frac{fa}{r} = e$, erit $f = \frac{rt}{y}$, quæ ſemper poſita in ιo erit trapezium $\iota o \sigma \xi = fa$, & ſpatium $\iota o \varphi \eta$ = omnia fa, quæ Figura diviſa per r dabit omnia $\frac{fa}{r}$ ſeu omnia e inter η & γ, hoc eſt axem EG, ad quem in puncto E applicanda $\kappa\gamma$ ſeu $\kappa\iota = EC$.

Habeturq. adeo punctum C in evolutâ, ad quem modum cætera inveniuntur; diviſâ enim verb. Gr. Figurâ integrâ $\rho\psi\varphi\eta$ per r oritur AG ſeu integer evolutæ axis, cui ſi $AB = \kappa\beta$ applicetur in A habetur denuo punctum B in evolutâ, & ſic in cæteris.

Coroll. I. Reſumtâ *Fig. XLVIII.* ac eâdem cum ſymbolis ſpirali, quæ §. 49. Cor. I. diſcuſſa fuit; cum ſit $ze = ta$, adeoque $e = \frac{ta}{z}$, ſit hoc $= \frac{fa}{r}$, erit hoc in caſu, facto $MN = f$ æquatio Curvæ DNO: $rt = fz$; ejectâque, t per valorem §. 67. de tang. inventum ſeu $\frac{kcz}{srs}$ re-

reductâque æquatione ad indeterminatas f & z oritur $k^s d^s z^k = s^s r^k f^s$ ſeu paraboliformis ; unde ſpatium DMN ſeu omnia fa, juxta §. 11. Cor. I, æquatur $\frac{sfz}{k+s}$, & omnia $\frac{fa}{r} = \frac{sfz}{kr+sr} =$ omnia e ſeu longitudo axis AE in Fig. XLIX. ubi $EC = DS = z$, & $EF = e$, vocetur autem hæc $AE : h$, ſeu omnia e, oreitq. curvæ ADH æquatio $\frac{sfz}{kr+sr} = h$, quæ reducta per ſuperiora ad indeterminatas h & z, ſi AE & CE, dabit æquationem rurſus paraboliformem: $\overline{k+s}^{s}\, r^{s+k}\, h^{s} = k^{s}\, d^{s}\, z^{k+s}$ curvam ACH reſpicientem ; adeoque cum maxima z ſit GH ſeu r, & maxima h ſeu AG ſit $\frac{kd}{k+s}$, erit rectangulum $AQGH = \frac{kdr}{k+s}$; & juxta §. 11, Coroll. I. curvilineum $ADHG = \frac{kdr}{k+2s}$; eſt autem hoc ſpatium ſpiralis ſpatii evolutum juxta præcedentem, ergo juxta §. 50, hujus duplum, unde ſpatium ſpirale æquatur $\frac{2kdr}{k+2s}$, uti in §. 49. Coroll. I. malui autem iiſdem exemplis inſiſtere, ut principiorum conſenſus dilucidius ob oculos ponatur.

§. 52. Idem & alio modo problema ſolvitur; ſiquidem, Fig. LI. $\tau\gamma : \gamma\delta :: \tau\kappa : \gamma\lambda$. ſeu

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$s : u :: t : e$; unde $\frac{tu}{s} = e$, ſit hoc $= \frac{hu}{r}$ erit $rt = sh$: quare in *Figurâ LII.* poſitâ curvâ involutâ $\beta\gamma\delta\eta$ in directum, ut $\gamma\delta$ in utrâque ſit æqualis $= u$, applicetur & in γ linea $\gamma A = h$, erit $A\gamma\delta B = hu$, & $\beta D B \delta =$ omnia hu; quod ſpatium diviſum per r dabit omnia $\frac{hu}{r} =$ omnia e, ſeu longitudinem axis AF in *Fig. L.* factâ nimirum EC in *Fig. L.* æquali $\varkappa\gamma$. eſtque, uti patet, ſpat. $ABHG$ in *Fig. L.* ſpatii $\varkappa\beta\eta$ in *Fig. LI.* evolutum.

§. 53. Quin & hinc naſcitur aliis jam exculta ex involutâ evolutam deſcribendi methodus.

Radio quolibet $\varkappa P$ ſeu r deſcribatur *Fig.* LI. arcus PO; cujus infiniteſima, inter $\varkappa\gamma$ & $\varkappa\delta$ productas intercepta QR ſit: o, erit, ob radios arcubus proportionales, $\varkappa\gamma : \gamma\lambda :: \varkappa R : RQ$. ſeu $y : e :: r : o$, unde $\frac{yo}{r} = e$. poſito ergo arcui $ORQP$ in directum, *Fig. LIII.* applicetur in puncto R linea $RI = \varkappa\gamma = y$, erit $IRQK = yo$, & $OQKH =$ omnia yo; quod ſpatium diviſum per r dabit omnia $\frac{yo}{r} =$ omnia $e = AE$ in evoluta *Fig. L.* ubi $EC = \varkappa\gamma = RI = y$, & curva $BD =$ curvæ $B\delta$.

Coroll. I. Sit *Fig.* XLVIII. eadem ſpiralis §. 49. & 51. in Coroll. tractata $DVSP$, factâ circuli

[c]uli infiniteſimâ $AB = o$. (quia DS eſt $= z$) [e]t $zo = re$. & poſitâ circumferentiâ circuli [i]n directum in $AEFG$, *Fig.* XLIX. ut EF ſit $=$ [A]$B = o$, & EC huic applicata $= DS = z$, erit [E]$A = c$; & curvæ $ACDH$ æquatio eadem [cu]m ſpirali $d^s z^k = r^k c^s$, quæ ad Figuram XLIX [re]lata eſt paraboliformis.

Unde, §. 11. Cor. I. AEC ſpatium eſt $= \frac{kz}{k+s} =$ omnia zo ſeu omnia $ECDF$; ergo [o]mnia $\frac{zo}{r} = \frac{kcz}{kr+sr} =$ omnia e; ſint autem [o]mnia $e = h$, & in *Fig.* L. ei æqualis ſtatuatur [A]E, cui applicetur EC ſeu z, erit hujus [cu]rvæ æquatio $\frac{kcz}{kr+sr} = h$; & per æquationem [ſp]iralis curvæ ablato c, ac æquatione inter z & h [c]onſtitutâ, erit $d^s k^s z^{k+s} = \overline{k+s}^s r^{k+s} h^s$; [E]t autem in *Figurâ*. L. ſpatium $ABGH$ ſpi[r]alis evolutum, adeoque hujus duplum, & jux[ta] §. 11. Cor. I. æquale $\frac{kdr}{k+2s}$, unde ſpirale $\frac{kdr}{k+2s}$, ut ſupra jam ſæpius inventum.

§. 53. Alterum, quod conſiderationem hic [m]eretur, problema eſt, ex datâ evolutâ inve[n]ire involutam.

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Sit *Fig. L.* data evoluta $ABHG$; cujus ut inveniatur involuta $\kappa\beta\eta$, *Fig. LI.* determineturq. ideo in hæc punctum γ reſpondens evolutæ puncto C, patet quidem lineam $\kappa\gamma$ ſeu $\kappa\iota$ æquari lineæ EC, ſed & præterea ad definiendum punctum γ requiri anguli $\gamma\kappa\iota$ determinationem, ſeu arcus $\gamma\iota$ aut PR: quod ut fiat, reſumto §. præced. erat $yo = re$; adeoque $\frac{re}{y} = o$, ſit hoc $= \frac{he}{r}$, erit $\frac{rr}{y} = h$; quæ applicetur in EN erit $NEFS$ ſeu he duplum ſectoris circuli $\kappa R\mathcal{Q}$, ſeu $= \frac{2ro}{2}$; adeoq. omnia he ſeu ſpat. $NEGK$ duplum ſectoris κRP; hoc autem ſpatium diviſum per r dabit omnia o, ſeu arcum RP, quo dato angulum $R\kappa P$ dari notum eſt; unde ſi in radio κR ſumatur linea $\kappa\gamma$ ipſi EC æqualis, habetur in involutâ punctum γ reſpondens evolutæ puncto C, quo invento & cætera eodem modo inveſtigari poſſunt.

CA-

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CAPUT III.

De Corporum ex Curvilineis ortorum dimenſionibus.

§. 1. EX ſuperioribus *Fig.* L*IV.* ſpatium *ADE*F*O* polygonum eſt, quia curva *ADE*F rectulis *DE* & ſimilibus conſtat; in quo ſpatio, tanquam baſi, ſi erigatur corpus columnare in altitudine $A\alpha$, ſeu Oo conſtabit hoc infinitis columnulis, quarum baſes ſunt trapezïola *DQPE*, altitudines $Q\kappa$, $P\pi$, quod ex geometriæ vulgaris præceptis notum eſt.

§. 2. Cujus columnulæ valor algebraicus ut inveniatur, fiat ut ſupra $DQ:y$, $QP = DH:e$, $EH:a$, $DE:u$, altitudo $A\alpha:r$. erit quæſita columnula rye, hoc eſt æqualis baſi in altitudinem ductæ; adeoque integra columna = omn. rye; ſeu, juxta præcedentem omnibus parallelopipedis in baſi rectangulâ *DQPH*, cum priſma triangulo *DHE* inſiſtens ob ipſius trianguli nullitatem nihilo ſit æquale; eſt enim triangulum $DHE = \frac{ae}{2}$, & priſma ei inſiſtens $= \frac{rae}{2}$, quod ex lemm. 10 nullum eſt.

 Quod

Quod hic, licet jam ad nauſeam fere inculcatum, monemus, ne poſtea de ſectione columnari acturi illud repetere neceſſe habeamus. Cum omnia corpora, tum pyramidalia, tum cylindrica &c. baſin nihilo æqualem habentia etiam nihilo æquivaleant, adeoq. in calculo prætermitti debeant.

§. 3. Hinc patet, ſi fuerint duæ columnæ, quarum baſes ſunt in eadem latitudine *AO*, & applicata primæ baſeos: *y*, ſecundæ: *z*, tum altitudo primæ columnæ: *r*, ſecundæ: *s*, ſi infiniteſima *e* ſibi perpetuo conſtans & æqualis aſſumatur, fore primam columnam ſeu omnia *rye* ad ſecundam ſeu omnia *sye*, ut omnia *ry* ad omnia *sy*, hoc eſt, ut ſumma rectangulorum *Q*×*δD* ex applicatis ſingulis in altitudinem ductis in primâ columnâ: ad totidem reſpondentia rectangula in ſecundâ.

Plura hinc quilibet ad §. 6, 7, 8, 9, &c. Cap. præc. normam facile deducet.

§. 4. Quod ſi *Fig. LV. N°.* I. columna hæc plano per lineam *CB* ipſi *AO* æquidiſtantem tranſeunte, quodq. plano per utriuſque baſeos axem F*O* incedenti normale eſt, obliqué ſecetur, ortum hinc ducent duo columnæ ſegmenta, ſuperius & inferius, iisdem corpuſculorum generibus inverſé conſtantia; quæ *truncorum* nomine vocant Geometræ.

Hos conſideranti innoteſcit truncum inferio-

rem

rem conſtare infinitis Corpuſculis, quorum baſes ſunt traperia *DQPE*, altitudines ab unâ parte *PG*, *Q*I, ab alterâ *EN*, *DK*; quæ ut calculo explicentur, cæteris, ut ſupra, appellatis, dictiſq. *QI*: l, & *KD*: s, erunt primo omnia baſi triangulari *D*H*E* directe inſiſtentia ob ipſius trianguli nullitatem ex Lemm. 10 rejicienda; unde tantum pro calculo reſtat rectangulo *PQD*H inſiſtens priſma *PQIG*L*KD*H $= \frac{sye+lye}{2}$, æquale parti infiniteſimæ trunci inferioris a plano reſecti; quare factis *CT*: b, *TS*: h, *QC*: d, erit Q*I* ſeu $l = \frac{hd}{b}$, & *KD* ſeu $s = \frac{hd}{b} + \frac{hy}{b}$; unde hoc in caſu conſtabit truncus omnibus, $\frac{h}{b}$ in $d+\frac{1}{2}y$ in ye; & ſuppoſito $h=b$, hoc eſt angulo *RBV* ſemirecto, componetur truncus omnibus, $d+\frac{1}{2}y$ in ye, ſeu $\frac{2dye+yye}{2}$; iterumq. ſi fiat *CD* ſeu $d+y=z$, conſtabit idem truncus omnibus $\frac{zze-dde}{2}$.

§. 5. Unde expoſito alio cylindrico, (cujus baſis ejuſdem latitudinis cum *AO*) oblique ſecto, factâq. diviſione per e, erit truncus inferior primi cylindrici ad truncum inferiorem ſecundi, ut omnia trapezia OQ*DK* in primo ad

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ad omnia trapezia correſpondentia in ſecundo.

Hoc notetur, cum hæc trapezia I Q *D K* ſint continuo $= \frac{2dy+yy}{2} = \frac{zz-dd}{2}$, ſeu æqualia differentiis quadratorum in *CD* & *C* Q conſtitutorum, truncorum rationem per has differentias exprimi poſſe.

§. 6. Quod ſi planum ſecans non tranſeat per *BC*, ſed per baſis axem *AO*, erit Q *C* ſeu $d = 0$: quique hinc reſultabit truncus, *ungulæ* nomine alias dictus, conſtabit infinitis $\frac{hyye}{2b}$ ſeu priſmatibus *P* H L *KD* Q.

Eritque in ratione ad aliam ungulam, ubi *e* ſibi ſemper æqualis diviſione excidit, ut omnia $\frac{hyy}{2b}$ ſeu triangula *K* Q *D*, ad totidem triangula in ſecundâ ungulâ.

§. 7. Si vero fiat $h = b$, & angulus ideo quo planum ſecans lineam *AO* tranſit ſemirectus, erit $\frac{yye}{2}$ hujus ungulæ, quæ hâc de cauſâ ſemiquadrantalis vocari ſolet, pars infiniteſima; atque triangulum *K* Q *D*, hoc in caſu $= \frac{yy}{2}$.

§. 8. Si autem non $\frac{hyy}{2b}e$, ſed $\frac{hyye}{b}$, pro parte Corporis infiniteſimâ aſſumatur, aliud ſolidi genus efformabitur, duplum ungulæ: factâ ni-

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imirum *Fig.* L*VI.* linea *D* Q ut ſupra $= y$, at continuo, ut *b* ad *h* ita *y* ad $\frac{hy}{b}$, quæ ſi ſta-uatur in *M* Q, normalis Figuræ *AO* F, erit ompletum parallogrammum Q*M*I*D* $= \frac{hyy}{b}$; uod ſi ducatur in infiniteſimam Q*P* ſeu *e*, ori-ur parallelopipedum ſolidi hujus infiniteſimum; um cætera extra hoc parallelopipedum extantia orpuſcula juxta lem: 10 ex calculo evaneſcant.

Hoc autem perpetuis his parallelopipedorum ggregatis conflatum ſolidum *AO*F*CB* ejus eſt roprietatis, ut quælibet ſectio, rectangulo *O*FC parallela, faciat parallelogrammum *M*Q*D*I ſeu $\frac{hyy}{b}$. quod monuiſſe ſufficiat, cum & hoc principii loco pro Curvilineorum menſu-ris ſtatui poſſit; quoniam vero ungulæ ſimpli-ciores forſan viſæ fuerunt, iis potius inſiſtere voluiſſe geometras vero eſt ſimile.

§. 9. Cum autem *Fig.* L*VII.* trunci ungulæ ve baſis *AO*F, juxta ſuperiora, non tantum reſpectu *AO* in trapezia *D*Q*P E*, verum & reſpectu *O*F in trapezia *DG*I*E* dividi poteſt, conſtabit, hoc ſuppoſito, ungula truncuſve *AOFV* ſolidis trapezio *DG*I*E* inſiſtentibus, quale eſt *DG*I*EMSK*L.

Qnod ut Calculo exprimatur, ſiat IL: *s*, *EI*: *x*,

$EI:x$, infiniteſimæ GI ſeu $KT:e$, $LT:i$, erit KG ſeu $IT:s-i$; jam vero cum triangula KTL, MRN, DHE, SNR, juxta lemm. 10, nihilo æquivaleant, reſtabit ſolum parallelopipedum $GITKNREH: = sxe - ixe = sxe$, (ex eodem lemm. 10) pro ſolidi hujus infiniteſimâ; adeoque ungula ſive truncus quovis modo per AO reſectus in ratione ad alium eliſâ utriuq. e, ut omnia sx ſeu omnia rectangnla $MLIE$.

Unde ſi ſolidum ſit ungula ſemiquadrantalis per AO reſecta, erit LI ſeu $s = OI$ ſeu y, adeoque $sx = yx =$ rectangulum $MLIE =$ rectangulum $OIEP$; quare hoc in caſu ungula ſemiquadrantalis $AOFV$ algebraice per omnia yx efferetur.

§. 10. Sit $OC:d$, & BC ipſi AO parallela, erit $CI: d+y$ ſeu $CO+OI$, ſeceturq cylindricus ipſi AOF inſiſtens plano ſeminormaliter per BC tranſeunte, erit trunci inferioris infiniteſima $= dxe + xye$; & ratio trunci ad alium ejuſdem latitudinis calculo deſignabitur per omnia $dx + xy$, ſeu rectangula ex CI in IE.

§. 11. Quod ſi perpetuo fuerit CI ad IL ut b ad h, erit trunci inferioris cylindrici, in angulo, quem data ratio ipſius b ad h determinat, per BC reſecti infiniteſima abſoluta =

$dx+yx$

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$x+yx$ in $\frac{he}{b}$, reſpectiva vero ad alium ejuſ-
em latitudinis truncum = rectangulum $dx+yx$
in $\frac{h}{b}$.

§. 12. Hoc notetur, *Fig.* LV. *N°*. 1. trun-
um cujuslibet cylindrici inferiorem æquari
ummæ cylindrici in eâdem baſi conſtituti in al-
itudine PG, & ungulæ ejuſdem cylindrici baſi
uperiori inſiſtentis, eodem plano, per IG ad
undem angulum tranſeunte, reſectæ.

Demonſtratio ex appoſitæ Figuræ inſpectione
atet. Tum & calculus eandem ſtatim prodit,
: enim $\frac{dhye}{b}+\frac{yyhe}{2b}$ infiniteſima trunci jux-
a §. 5. eſt $\frac{dhye}{b}$ ex §. 2. ad cylindricum, &
$\frac{yhe}{2b}$ per §. 6. ad ungulam. ſi vero trunci infini-
eſima, juxta §. 11, ponatur. $\frac{h}{b}dxe+\frac{h}{b}xye$,
dem eveniet; conſequentiæ ratio ex §. 9. elu-
eſcit.

Corollaria caſus particulares concernentia hic non ad-
ungo, cum in ſequentibus abunde circa ſolida ex rotatio-
ibus genita ſeſë ingeſtura ſint.

§. 13. Neque tantum ad curvilinea, quæ
xem ac baſin rectilineam habent, modo de-
ducta

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ducta extenduntur; verum &. *Fig.* L*V*. *N*°. 2. ad Figuras quibuslibet peripheriis terminatas, applicari poſſunt; cum enim triangula *DHE* ac Q*P*O, ex lemmate 10, nihilo æquivaleant, erunt & ſolida iis ſuperſtructa nihilo æqualia: adeoque & hoc in caſu, trunci ungulæque per eadem, quæ modo ex calculo inventa ſunt, priſmata deſignabuntur.

De Corporibus ex Curvilineorum rotatione genitis.

§. 14. RECtangulum *ABEP*, *Fig.* L*VIII*. conſtat infinitis rectangulis Q*PEF*, *R*QF*T*, &c. ſeu (vocatis *PE*: *b*, Q*P* infiniteſimâ: *e*) omnib. *be*. quæ rectangula Q*PE*F, &c. ſi circa reſpectivos axes *P*Q, Q*R*. &c. rotari concipiantur, efficient infiniti hi cylindruli integrum cylindrum ex *APE*B circa *AP*; exiguorumq. horum cylindrorum valor algebraicus, juxta cylindri menſuram in elementis traditam, erit $\frac{bbce}{2r}$. (poſitâ nimirum ratione radii ad circumferentiam ut *r* ad *c*, quod & in ſequentibus ſemper obtinebit) adeoq. totus hic cylindrus explicabitur per omnia $\frac{bbce}{2r}$: quæ

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uæ (appellatâ $AP:d$, æquivalebunt termi-o abſoluto $\frac{bbcd}{2r}$.

§. 15. Huic rectangulo $APEB$ inſcriptum t triangulum APE, vocatiſque $AR:x$, $X:y$, SR infiniteſima: e, erit trianguli pro-rietatem exprimens æquatio $bx = dy$, con-abitque hoc triangulum juxta Cap. præc. §. 4. mnib. $ye =$ omnib. $\frac{bxe}{d}$ hoc eſt infinitis rectan-ulis $SRXZ$; hæc autem omnia rectangula u triangulum APE, ſi circa axes ſuos SR onvertantur, generabunt conum AEM innu-eris cylindrulis hiſce conſtantem; quorum ylindrulorum valor analyticus ex §. præc. eſt $=$ $\frac{yye}{2r} = \frac{cbbxxe}{2rdd}$; unde conus AEM æqui-alet omnibus $\frac{cbbxxe}{2rdd}$, ſeu (juxta Cap. II. 13.) abſolutæ quantitati $\frac{cbbx^3}{6rdd}$. quæ, po-tâ x maximâ $= d = AP$, tranſit in hanc $\frac{bbd}{6r}$.

Unde patet conum AEM eſſe ad cylindrum ircumſcriptum $BENM$, ut $\frac{cbbd}{6r}$ ad $\frac{cbbd}{2r}$ deoque ut 1 ad 3.

Malui hoc in caſu, methodi gratiâ, Figuras circum-

ſcriptas adhibere; rationemq. coni ad cylindrum, utpote ex Cap. II. §. 13. immediate profluentem, eodem cum cæteris principio demonſtrare; cum conus, ut ut ex figurâ rectilineâ ſeu triangulo ortum ducens, rotationis tamen nomine curvilineis annumerandus videatur.

§. 16. Hoc notetur, cum in ſequentibus ſemper ſubintelligendum veniat, eadem hæc omnia in converſione partiali locum habere. Si enim rectangulum *APEB* circa axem *AP* per arcum *IE* tantum convertatur, erit (arcu ſimili in radio *r* poſito $= f$) pars cylindri converſione iſthâc imperfectâ geniti $= \frac{bbfe}{2r}$.

Poſſent ex eodem, quo §. 15. uſi ſumus, fundamento, ſeu curvilineorum in rectangula diviſione, generalia ſolidorum ſymptomata deduci, cum vero eorum in trapezia diſtinctio tum hanc, quam intendimus, ſolidorum diſcuſſionem, tum & ſuperficierum conöidalium naturam magis directe ſpectet, hanc viam eligemus.

§. 17. Ex præc: *Fig. LIX*, ſpatium *AOF* conſtat infinitis trapeziis *DQPE*, *EPOF*, &c; quæ ſi ſingulæ rotentur circa ſuos axes, *QP*, *PO*, efficient totidem conos truncatos, vertices habentes in *T* & *V*, ubi tangentes ſeu Curvæ coincidentes *DE*, *FE*, productæ axem *OA* ſecant; omnesque hi coni truncati ex trapeziis *DQPE* circa axes *QP* volutis conficient ſolidum ex integrâ Figurâ mixtillineâ

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lineâ *ADEFO* circa axem ſuum *AO* rotatâ.

§. 18. Ut ergo ejus modi coni truncati valor indagetur: retentis, quæ ſupra ſymbolis, & ratione radii ad circumferentiam poſita, ut r ad c, erit circulus radio *QD* ſeu y deſcriptus = $\frac{cyy}{2r}$; qui ductus in $\frac{1}{3}$ *QT* ſeu $\frac{1}{3}t$, dabit $\frac{\frac{1}{3}tcyy}{2r}$ pro ſoliditate coni minoris ex triangulo *QTD* circa *QT* voluti. Jam vero circulus radio *PE* deſcriptus eſt $\frac{cyy + 2cay + caa}{2r}$, qui multiplicatus per $\frac{1}{3}$ *PT*, ſeu $\frac{1}{3}t + \frac{1}{3}e$, dabit, rejectis per lemma 10. rejiciendis, $\frac{\frac{1}{3}tcyy + \frac{2}{3}tacy + \frac{1}{3}ceyy}{2r}$ pro contento coni ex triangulo *TPE* circa *TP* circumacto; ex quo ſi detrahas conum minorem modo inventum $\frac{\frac{1}{3}tcyy}{2r}$, reſtabit coni truncati ex trapezio *DQPE* circa *QP* rotati quæſitus valor: $\frac{\frac{2}{3}tacy + \frac{1}{3}ceyy}{2r}$, ſeu $\frac{ceyy}{2r}$, aut $\frac{atcy}{2r}$ (eſt enim $at = ye$ ex §. 11. Cap. præc) jam vero $\frac{ceyy}{2r}$ eſt æquale cylindro ex rectangulo *QDPH* circa *QP*, & pars conöidis quæſiti infiniteſima.

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§. 19. Hinc ſi fuerint *Fig. LX.* duo curvilinea *A*F*O* & *AGO* ad eundem axem *A O* conſtituta, ſintq. applicatæ in primo $y = DQ$, in ſecundo $z = RQ$, erit, ut Conoides ex primo *A*F*O* circa *A O* rotato ad conoides ex ſecundo *AGO* circa eandem *A O*, ita omnia $\frac{c}{2r}\, eyy$ ad omnia $\frac{c}{2r}\, ezz$; & diviſis per e, ut omnia $\frac{c}{2r}\, yy$ ad omnia $\frac{c}{2r}\, zz$, hoc eſt ut omnes circuli ex applicatis, tanquam radiis, deſcripti in primâ ad omnes circulos correſpondentes in ſecundâ Figurâ; tandemque, rurſus diviſione per $\frac{c}{2r}$ factâ, ut omnia yy ad omnia zz, ſeu ut omnia applicatarum quadrata in primâ Figurâ ad omnia applicatarum quadrata in ſecundâ; poteſt & hoc addi, omnia $\frac{c}{2r}\, eyy$ eſſe ad omnia $\frac{c}{2r}\, ezz$, ut omnia $\frac{yye}{2}$ ad omnia $\frac{zze}{2}$; hoc eſt conoides ex *A*F*O* circa *A*O ad conoides ex *AG*O circa eandem *A*O, ut ungula ſemiquadrantalis cylindrici in baſi *A*FO per *A*O reſecti, ad ungulam ſemiquadrantalem in baſi *AG*O; juxta 7 hujus.

§. 20. Rurſus, cum *Fig. LX.* conöides ex

*A*FO

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AFO circa *A*O ſit æquale omnibus $\frac{cyye}{2r}$
& conôides ex *AG*O circa eandem *A*O æqua-
le omnibus $\frac{cezz}{2r}$, erit ſolidum ſecundum dem-
to ſolido primo, ſeu ſolidum cavum ex *AGF*
circa *A*O æquale omnibus $\frac{cezz}{2r} - \frac{ceyy}{2r}$:
quâde cauſa, ſi aliæ duæ curvæ *AIO* & *AKM*
ad eundem aut æqualem axem applicentur, ſit-
que *QI*:*h*, & *QK*:*k*, erit ſolidum cavum ex
AOM circa AO æquale omnibus $\frac{cekk - cehh}{2r}$.

Unde ſolidum ex *AGF* circa *AO* ad
ſolidum ex *AM*O circa *A*O, ut omnia
$\frac{cezz - ceyy}{2r}$ ad omnia $\frac{cekk - cehh}{2r}$ & divi-
ſis per *e* ut omnia $\frac{czz - cyy}{2r}$ ad omn. $\frac{ckk - chh}{2r}$
ſeu ut omnes differentiæ circulorum in radüs
RQ & *DQ* ad totidem differentias circulo-
rum in radüs *QK* & *QI*; aut potius ut om-
nes armillæ ex omn. *RD* circa *A*O ad totidem
armillas ex omnibus *IK* circa AO; rurſumq.
diviſione per $\frac{c}{2r}$ inſtitutâ, ut omnia $zz - yy$
ad omnia $kk - hh$, hoc eſt ut differentiæ qua-
dratorum *R*Q & *D*Q ad differentias quadra-
torum in Q*K* & Q*I*; unde factis $z - y = f =$

RD, ac $k-h=g=IK$, erunt ſolida inter ſe, ut $2fy+ff$ ad $2gh+gg$.

Coroll. I. Hoc notetur, Corpora cava ſeu annularia, ex armillis compoſita per quadratorum in applicatis conſtitutorum differentiam exprimi.

Coroll. II. nec opus eſt addi; cum omnia $\frac{czze-cyye}{2r}$ ſint ad omnia $\frac{ckke-chhe}{2r}$ ut omnia $\frac{zze-yye}{2}$ ad omnia $\frac{kke-hhe}{2}$; eſſe ſolidum annulare ex *AG*F circa *A*O, ad ſolidum ex *AM*O circa *A*O, ut truncus inferior cylindrici baſi *AG*F inſiſtentis ac per *A*O ſeminormaliter reſecti, ad truncum inferiorem in baſi *A*O*M* per eandem *A*O reſecti: juxta § 4. hujus.

§. 21. Sit jam *r* ad *s* ratio immutabilis: ac ſemper $r:s::zz:yy$; erit conoides ex *AG*O circa *A*O, ad conoides ex *A*FO circa *A*O, ut *r* ad *s*, juxta lemma 43.

Ac ſi fuerit perpetuo $r:s::zz-yy:hh$; erit ſolidum annulare ex *AG*F circa *A*O ad conoides ex *A*FO circa *A*O, ut *r* ad *s*; & poſito $r=s$, erit ſolidum annulare illud æquale hinc corpori conoidali.

Coroll. I. patet hinc, ſi loca duorum, hoc eſt ipſorum *z* & *y* ad arbitrium ſumantur; locum reliquum, ad quem *h* futurum eſt, hinc pate-

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atefactum iri. poſitâ enim $AQ : x$, ſit z ad parabolam AGO, ut ſit $2rx = zz$; & y ad iangulum iſoſceles AFO, unde $y = x$; erit ubſectis in locum z & y valoribus, $2rx - xx$ hh, unde QI ſeu h ad ſemicirculum AHO, ujus centrum W. quare ſi parabola latere reco $2r$ deſcripta ARQ circa axem AQ roteur; & triangulum ADQ circa eandem AQ olvatur; erit differentia, quâ conoides paraolicum ſeu omnia $2rx$ ſuperat conum ſeu mnia xx, æqualis portioni ſphæricæ ex ſegmento ſemicirculi AQI circa eandem AQ evoluto genitæ.

Coroll. II. Si vero portionis hujus ſphæricæ on ratio ad alia ſolida, verum abſolutus valor uæratur; eſt ſegmentum ſphæricum ex AQI irca AQ rotatum $=$ omnia $\frac{hhce}{2r}$ $=$ omnia $\frac{rcxe - cxxe}{2r}$, ex modo demonſtratis: ſed x Cap. II. §. 12. Sunt omnia $2rcxe = rcxx$, um omnia $cxxe = \frac{1}{3}cx^3$, adeoque omnia $\frac{hce}{2r}$ ſeu portio ſphærica $= rxx - \frac{1}{3}x^3$ in $\frac{c}{2r}$ cono, cujus baſis circulus radio QI deſcriptus, altitudo $\frac{3rx - xx}{2r - x}$ ſeu $\frac{QO + WO}{QO}$ in QA. i vero quæratur integra ſphæra, fiat $x = 2r$, erit-

que $rxx - \frac{1}{3}x^3$ in $\frac{c}{2r} = \frac{4}{6}rrc$; adeoque hæc ſphæra ad cylindrum circumſcriptum rrc, ut 2 ad 3.

§. 22. Potuiſſet & in F*ig.* *LX.* aſſumtâ Fig. quartâ A*M*O, cujus applicata Q*L* = *k*, fieri perpetuo $kk - hh = rr - yy$ hoc eſt ſolidum annulare cavum ex A*M*O circa AO æquale ſupponi ſolido annulari modo dicto ex AFG circa eandem AO, unde locis trium ex hiſce cognitis cognoſcetur ſemper locus quarti.

§. 23. Neq. poſt hæc intellecta dictu opus autumo, ſi axis rotationis *BC* ſit extra Figuræ cujuslibet, ut *M*A*G*, ambitum, omnia ſimiliter obtinere. Facto enim *SC*: *z*, & *CL*: *y*, ſolidum annulare ex Figura *M*A*G* circa *CB* rotata in ratione ad aliud ſolidum æque altum, explicabitur per omnia $zz - yy$, aut poſitâ *LS* ſeu $z - y = f$: per omnia $2yf + ff$.

§. 24. Cum autem non tantum trapeziis *DQPE* planum mixtilineum AOF F*ig.* *LXI.* conſtet, verum & trapeziis *DIGE*; quæ ſi rotentur circa axem *AO*, facient ſingula trapezia ſingulos tubos ſeu ſolida tubulata; quæ ſimul ſumta integrum conſtituent conôides ex Figurâ *AFO* circa *A*O rotatâ.

§. 25. Pro ſingulorum ergo valore inveniendo, retentis ſuperioribus ſymbolis, ſit præterea

ea QO ſeu $DI: x$, HE ſeu $GI: a$ infini-teſima; unde, quia $OI: y$, erit $OG: y+a$, $GE: x-e$; jam autem juxta §. 18. erat conus truncatus ex $DQPE$ circa $QP = \frac{ceyy}{2r}$; cui adde cylindrum ex rectangulo $POGE$ circa $QO = \frac{cyyx}{2r}$ $\frac{+2acyx - ceyy}{2r}$, orietur ſolidum ex pentagono $QDEGO$ circa $QO = \frac{cyyx+2acyx}{2r}$; ex quo ſi dematur cylindrus ex rectangulo $QDIO$ circa QO ſeu $\frac{cyyx}{2r}$ orie-tur tubus quæſitus ex trapezio $DIGE$ circa $QO = \frac{acyx}{r}$; ſeu, ut ex calculo patet, tubus ex rectangulo $DIGS$ circa QO, quæ co-noidis hoc modo conſiderati infiniteſima eſt.

Coroll. I. Quod ſi $OF = AM$ ſit $= p$, & $AO = MF = b$; erit cylindri ex rectangulo $AMFO$ circa AO infiniteſima $= \frac{byac}{r}$.

§. 26. Sint jam *Fig. LX.* duæ Figuræ AGO & AFO ad eundem axem AO poſitæ; vo-catiſq. $RQ: z$, $DQ: y$, infiniteſima $QP: e$, intercepta $AQ: x$, erit ex præcedenti ſolidum ex AGO circa $^{\beta}A^{\pi}$, quæ per verticem appli-catis parallela eſt, æquale omnibus $\frac{zxec}{r}$; & ſoli-

ſolidum ex *AFO* circa eandem $\beta A \pi$ æquale omnibus $\frac{yxec}{r}$.

Unde illud ſolidum ad hoc, ut omnia $\frac{zxec}{r}$ ad omnia $\frac{yxec}{r}$; & diviſione facta per e, ut omnia $\frac{zxc}{r}$ ad omnia $\frac{yxc}{r}$, ſeu ut omnes ſuperficies cylindricæ aut tubulatæ ex applicatis *R*Q, in diſtantia ſua AQ & poſitione ſuâ ad AQ normali, circa $\beta A \pi$ rotatis, ad totidem ſuperficies cylindricas ex omnibus *D*Q in eadem diſtantia AQ circa eandem $\beta A \pi$ volutis; & factâ rurſum diviſione per $\frac{c}{r}$, erit ſolidum primum ad ſecundum, ut omnia zx ad omnia yx; hoc eſt ut ſumma rectangulorum ex lineis RQ axi rotationis $\beta A \pi$ parallelis in reſpectivas ab eodem axe diſtantias ductis factorum, in primâ Figurâ, ad ſummam rectangulorum reſpondentium in ſecundâ Figurâ, manente eâdem ab axe diſtantiâ AQ; hoc addatur, cum $\frac{zxec}{r}$ ſemper ſit ad $\frac{yxec}{r}$ ut zxe ad yxe; fore ſolidum primum ad ſecundum ut truncus cylindrici ungulave in baſi A*G*O ſeminormaliter per $\beta A \pi$ reſecti, ad ſimilem ungulam in baſi AFO; conſequentiæ ratio ex 9 hujus manifeſta eſt. *Scholl.*

Schol. Hoc notetur *Fig. LXI.* ſi *QD* ſeu *OI* ſit y; & *QO* ſeu *DI* ſit: x; *PQ*: e; *IG*: a, fore rectangulum *QDI* ſeu xy exponens rationis ſolidi ex ſpatio *AOF* tum circa *AO*, tum circa *OF* rotato, juxta præcedentia; ne autem hic error ſubrepat, ſi ſolidum deſignetur per rectangula xy, notetur ſemper illud ad infiniteſimam e vel a reducendum eſſe; cum omnia $\frac{xyec}{r}$ conſtituant ſolidum ex *AOF* circa *OF*, & omnia $\frac{xyac}{r}$ componant ſolidum ex eadem *AOF* circa *AO* rotatâ; ut inquirenti patebit.

§. 27. rurſus, ſint *Fig. LXII.* duæ figuræ *RAF* & *GOQ*, ad æquales lineas *RF* & *GO* poſitæ; ita ut interceptæ *RQ* & *MG*, ac infiniteſimæ *QN* & *ME*, continuo æquales ſint, adeoque Figuræ ad eundem axem ordinari poſſe intelligantur; quæ cautela in hiſce uſque neceſſaria. Vocatiſque *RQ* = *GE*: x, *QN* = *ME*: e, *PQ*: y, *DE*: z, ac *BR*: b, & *BG*: d, utraque determinata; ſitq. *HB* axis rotationis, erit, juxta præcedentes duas, ſolidum ex *RAF* circa *HB* æquale omnibus $\frac{byec + xyec}{r}$, illudque ex *QGO* circa eandem *HB* æquale omnib. $\frac{dzec + xzec}{r}$ unde ſolidum primum ad ſecundum, ut omnia $\frac{byec + xyec}{r}$ ad

ad omnia $\frac{dzec + xzec}{r}$; & diviſis per conſtantem e, ut omnia $\frac{byc + xyc}{r}$ ad omnia $\frac{dzc + xzc}{r}$; hoc eſt ut omnes tubi cylindrici ex *P* Q circa *HB* ad totidem tubos ſeu ſuperficies cylindricas ex *DE* circa eandem *HB*; ac, rurſum inſtitutâ diviſione per $\frac{c}{r}$, ut omnia $by + xy$ ad omnia $dz + xz$, ut omnia rectangula ex applicatis *P* Q & *DE* in reſpectivas ab axe rotationis diſtantias *B* Q & *BE* ductis.

Coroll. I. Unde hoc notetur ſolida cava; ſeu annularia, ex tubis compoſita exprimi per modo recenſita rectangula.

Coroll. II. Cum autem ſemper ſit $\frac{byec + xyec}{r}$ ad $\frac{dzec + xzec}{r}$, ut $bye + xye$ ad $dze + xze$, erit, juxta §. 10. ſolidum primum ad ſecundum ut truncus cylindricus in Figurâ primâ per axem rotationis ſeminormaliter reſectus ad truncum ſimilem in ſecundâ Figurâ.

Coroll. III. Nec opus eſt addi ipſum ſolidum ex *RA*F rotatione genitum, ſeu omnia $\frac{byec + xyec}{r}$, æquari trunco cylindrici inferiori, baſi *RA*F inſiſtentis, ac per *HB* reſecti ad angulum, quem ratio radii circuli ad circumferen-

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ᵻrentiam determinat; cum pateat hoc ex præ-
ȝeſſâ §. 11. Idemq. circa §. 20. animadverti poſ-
ᵻt in ſolidis armillaribus.

Coroll. IV. Notetur hinc, ſi ſolidum quod
bet annulare ſeu tubulatum in æqualem trun-
um poſſit commutari, rationem radii ad cir-
umferentiam circuli cognitam eſſe.

Coroll. V. Cætera hinc facile eliciuntur, ut
: ea, quæ ſolida ex Figuris circa axi aut baſi
arallelas rotatis genita ſpectant, cum *HB* ut
lis conſiderari poſſit. Item *BC* circa ſolida
rmillaria: §. 23.

§. 28. Pro ſpecialibus ſolidorum menſuris
ræcedentis Capitis §. 11 Coroll. V, VI, VII,
VIII. ut & §. 12. & 13. dilucide ſatis funda-
nenta tradunt. ne tamen nihil dixiſſe videar, ſo-
idorum ex parabolis genitorum contemplatio-
em primo ex præcedentium more aggrediemur.

I. Sit parabola *AFO*, F*ig. LXI.* in quâ
$AQ : x$, $QD : y$, æquatio $r^p - q\, x^q = y^p$.
uæritur ratio ſolidi ex eâdem circa *AO* rotatâ
geniti, ſeu omnium $yy =$ omnium $r^{\frac{2p-2q}{p}}$
$x^{\frac{2q}{p}}$ ad omnia bb, ſeu cylindrum ex rectangulo
AOFM circa A*O*; eritq. ex §. 11. Coroll. V.
Cap. præced. ut p ad $p + 2q$. cum vero omnia
$\frac{yy}{2}$ ſint juxta §. 7. ad ungulam ſemiquadranta-

lem

lem in baſi parabolicâ per Ao reſectam, erit & hæc ad omnia $\frac{bb}{2}$ ſeu ſimilem ungulam in baſi rectangulâ AMFo per eandem AO reſectam in eadem ratione.

II. Si parabola circa baſin OF rotetur, factâ AO: c, erit QO: $c-x$, & ratio ſolidi hac ratione geniti ad cylindrum ea, quæ eſt omnium $yc-yx$ (§. 27.) ad omnia $bc-bx$; quæ & ungularum ſemiquadrantalium per OF reſectarum in reſpectivis baſibus AFO & AMFO (§. 10.) exponentes ſunt; ergo omnia $yc-yx$ ad omnia $bc-bx$, ut omnia $cr^{\frac{p-q}{p}}x^{\frac{q}{p}}-r^{\frac{p-q}{p}}x^{\frac{p+q}{p}}$ ad omn. $bc-bx$; hoc eſt per §. 11. Capitis II. ut $2pp$ ad $2pp+3pq+qq$, ita ungula ad ungulam, aut ſolidum ad cylindrum.

III. Si circa AM baſi in vertice parallelam revolvatur, erit ſolidum ad cylindrum circumſcriptum, aut ungula ſemiquadrantalis reſecta per eandem AM in baſi parabolicâ ad ſimilem ungulam in baſi rectangulâ AMFO (juxta §. 26. & 9.) ut omnia yx ad omnia bx, ſeu ut omnia $r^{\frac{p-q}{p}}x^{\frac{p+q}{p}}$ ad totidem bx, hoc eſt ut $2p$ ad $2p+q$.

IV. Si circa MF axi parallelam fiat rotatio,

o, erit ſolidum hoc modo productum in ratio-
r ad cylindrum, ut omnes differentiæ quadrato-
rm in QK, & DK, ſeu omnia $2by-yy$
a omnia bb, hoc eſt ut omnia $2br^{\frac{p-q}{p}}x^{\frac{q}{p}}$
$r^{\frac{2p-2q}{p}}x^{\frac{2q}{p}}$ ad omnia bb, adeoque
$\frac{2p}{p+q}-\frac{p}{p+2p}$ ad 1. conſequentia ex loco
tato nullo negotio colligitur.

§. 29. Et hæc de parabolis; quæ ſi q ſit ma-
jr quam p ad earum complementa A F *M*
ferri poſſunt; ſin autem q ſigno negativo
ficiatur, erit curvæ æquatio ad aſymtotos hy-
rbolæ; quarum ſolida ſimili modo genita &
nc dimenſionem, determinationem ſaltem,
cipiunt.

§. 30. Sit *Fig. XLV.* polo *M*, normâ *DQ*,
rtice A, deſcripta conchöis A*BC*; item
dio A*D* ſeu r deſcriptus circuli quadrans
DI vocenturque *D*M: s, *GD*: z, *GB*: x,
G: y, erit juxta Cap. præc. §. 36. conchoidis
oprietas $sy+zy=xz$.

Adeoq. juxta §. 27. ſolidum ex quadrante
rculi A*DI* circa *Z*M (quæ normæ æqui-
iſtat) rotato, ſeu omnia $sz+zy$ æquale
lido infinito ex ſpatio conchöidali circa nor-
am *DQ* in infinitum productam rotato ſeu
mnibus xz.

Tum

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Tum &, juxta §. 10. erit truncus inferior cylindrici quadranti *ADI* inſiſtentis, ac per *MZ* ſeminormaliter reſecti, ſeu omnia $sy+zy$, æqualis ungulæ in baſi infinitâ conchoidali per infinitam *DQ* ſeminormaliter reſectæ ſeu omnibus xz.

§. 31. Sit (*Fig: XIII. N°.* 1.) ſemicirculus *ADO*, cujus diameter *AO*: r, Q*A*: x, Q*O*: l ſeu $r-x$, *D*Q: y; item hunc ſpectans ciſſois *ANKG*, cujus applicata Q*N*: z; erit, juxta calculum §. 31. Cap. de tangentibus, æquatio curvæ ciſſöidis $lz = xy$; unde ex §. 26. ſolidum ex ciſſoidali ſpatio circa infinitam aſymtoton O*R* voluto, ſeu omnia lz, æquantur ſolido ex ſemicirculo circa *AF* aſymtato O*R* parallelam rotato ſeu omnibus yx; aut juxta §. 9. ungula ſemiquadrantalis in baſi ciſſoidali per O*R* infinitam reſectæ, ſeu omnia lz, æqualis ſimili ungulæ in ſemicirculo per *AF* reſectæ, ſeu omnia yx.

§. 32. Quomodo autem hinc ſolidorum tranſmutationes plurimæ levi operâ obtineantur, facile quivis animadvertit. Reſumtâ enim *Fig. XXXIX.* ſymbolisque §. 17. Cap. præced. ſuppoſitis, cum ſit perpetuo $ye = ta$, hoc eſt, poſito *T*Q in *IM*, ſpatium *A*O*F* æquale ſpatio F*K*O, erit & ſi utrumque in y ducatur,

yye

$yye = tya$; unde omnia $\frac{yye}{2}$ ſeu ungula cylin-
drici in baſi *AFO* per *AO* ſeminormaliter
reſecti (§. 7.) ad tya, ſeu truncum inferio-
rum cylindrici in baſi *FKO* per eandem *AO*
ſeminormaliter reſecti (§. 9.) ut omn. $\frac{yye}{2}$
ad omn. tya ſeu omn. yye hoc eſt ut 1 ad 2.

Tum etiam omn. $\frac{yyec}{2r}$ ſeu per §. 17. ſoli-
dum ex *AOF* circa *AO* rotato ad $\frac{tayc}{r}$
juxta §. 25. ſolidum ex *FKO* circa eandem
AO revoluto, ut omn. $\frac{yyec}{2r}$ ad omn. $\frac{tayc}{r}$ ſeu
omn. $\frac{yyec}{r}$ hoc eſt ut 1. ad 2.

Si vero *IM* vocetur h, fiatque ſemper
$ty = hh$, erunt omnia $\frac{tayc}{r}$ = omnia $\frac{hhac}{r}$
= $\frac{yyec}{r}$; & omnia $\frac{hhac}{2r}$ = omnia $\frac{yyec}{2r}$, hoc
eſt ſolidum ex *FKO* circa *FO* æquale ſolido
ex *AFO* circa *AO*, ſecundum §. 17; quod
ungulis ſuperiori modo applicari poteſt. Quin &
aſſumtâ *Fig. XL.* ſi recta αδεφ reſpondeat curvæ
Fig. XXXIX. ADEF, fiatque κδ = QD ſeu
y, ac *IM* = *TD* ſeu s, erat $yu = sa$, quare
$yyu = sya$ hoc eſt $\frac{yyuc}{2r} = \frac{syac}{2r}$, ſeu ſolidum

O ex

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ex $\alpha o \varphi$ circa $\kappa \varphi$ (§. 17.) æquale ſemiſolido ex *FOK* circa *OA* rotato; quod & ungulis, ſi placuerit, applicetur.

§. 33. Sit *Fig. LXIII.* $BK = BF$, & *BA* ſemper æqualis *BC* ſeu *x* interceptæ, angulus *KBF* rectus, applicata *AE*: *y*, & *CD*: *z*, fiatque ſolidum ex *FDB* circa *FB*, ſeu omnia $\frac{zzc}{2r}$ §. 19. æquale ſolido ex *BKL* circa *BN*, ſeu omnibus $\frac{yxc}{r}$ §. 26. erit æquatione conſtitutâ $\frac{zz}{2} = yx$, hoc eſt juxta §. 7. & 9. ungula ſemiquadrantalis cylindrici in baſi F*DB* per F*B* reſecti æqualis ungulæ cylindrici in baſi K*BL* per *BN* ſeminormaliter reſecti; unde poſito *y* ad locum quemlibet, cujus applicata *x*, innoteſcet *z*, & contra.

§. 34. Sint (F*ig. LXIV.*) FK, *RM*, *PN*, normales ipſi F*N*; vocenturq. *FC* & *NP*: *r*, *CN*: *b*, intercepta *NS* & huic ſemper æqualis *BC*: *x*, *BA*: *y*, *ES*: *z*, unde *EG*: $2z$, conſtitutâque æquatione qualibet, juxta §.^os. 7. & 9. verbi gratia. $yx = 2bz$, erit juxta §. 9. *yx* ad ungulam ſemiquadrantalem in baſi *F*K*C*, per *CM* reſecta; & $2bz$, ad cylindricum in baſi *HNL*, cujus altitudo *CN* ſeu *b*, eſt enim $2bz = b$ in $2z$, unde illa ungula huic æquatur cylindrico.

§. 35.

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§. 35. Quod ſi yx ponatur ad ſolidum rotandum, & fiat $\frac{yxc}{r} = \frac{2bzc}{r}$; erit ſolidum e FKC circa RM rotatum æquale cylindrico, cujus baſis HNL ſeu omnia $2z$, & altitudo $\frac{b}{r}$, ſeu circumferentia circuli radio CN deſcripti.

Coroll. I. Poſito hinc y ad locum quemlibet, naſcitur locus ad z, & viciſſim; verbi gratia, ſit y ad ſemicirculum, ut ſit, $yy = rx - xx$, erit, facto $2b = r$, hoc in caſu $2z = \frac{2rx^3 - x^4}{rr}$; adeoq. juxta §. 34. ungula in ſemicirculo per CM ſeminormaliter reſecta, ſeu omnia yx, æqualis cylindrico in baſi PNL, cujus altitudo r. Notetur ungulam in baſi PNL per NP ſeminormaliter reſectam, ſeu omnia $z = \frac{2rx^3 - x^4}{2rr}$ cubari poſſe, juxta Coroll. III. §. 11. Cap. præc.

Coroll. II. Cum vero ſit $yx = 2bz$, erit $\frac{yx}{b} = 2z$, unde poſito y ad quemlibet locum, ejus intercepta x, ductâque curva FTC, cujus applicata BT ſit $\frac{yx}{b}$, erit ſpatium FTC ſeu

ſeu omnia $\frac{yx}{b}$ æquale plano curvilineo *HNL*, ſeu omnibus $2z$.

§. 36. Similia plura tranſmutationum ſpecimnia ſibi quilibet pro arbitrio cudet; retentis enim linearum nominibus, fiat $4bz = yy$ ſeu $\frac{yy}{2} = 2bz$; adeoque cylindricus in baſi *HNL*, cujus altitudo *b*, æqualis ungulæ ſemiquadrantali in baſi F*KC* per *FC* reſectæ; quod factis $\frac{yyc}{2r} = \frac{2bzc}{r}$ ad ſolida rotunda referri in modum præcedentium poteſt.

Coroll. I. Ponatur *y* ad ſemicirculum, erit $yy = 2rx - xx = 4bz$; & locus ipſius *z* ad curvam, cujus ſpatium juxta Cor. IV. §. 11 Cap. præc. eſt quadrabile, nam, facto $4b = r$, erit $2x - \frac{xx}{r} = z$; adeoq. cylindricus in baſi *HNL* cubationem admittit, qui æqualis eſt nngulæ cylindrici in baſi ſemicirculari per diametrum ſeminormaliter reſectæ, unde & hæc ungula cubari poteſt.

§. 37. Affinis hiſce eſt methodus, qua ſolvitur hoc problema, datis *Fig. LXV.* duobus ſpatiis, *A*F*O* & *AOM*, invenire duo ſolida in ratione harum Figurarum.

Factis *DQ*: *y*, & *QS*: *z*, erit ſpatium *AFO* ad ſpatium *AOM*, ut omnia *y* ad omnia

a z; & aſſumtâ b determinatâ, ut omnia
b ad omnia bz; ſi jam expetantur ſolida co-
nidica, reducantur by & bz ad circulos,
fiat $by = \frac{kkc}{2r}$, & $bz = \frac{hhc}{2r}$, erit, ſepoſitâ
tione $\frac{c}{2r}$, applicata TQ ſeu $k = \sqrt{by}$, &
N ſeu $h = \sqrt{bz}$; & ſolidum ex AGO cir-
AO ſeu omnia $\frac{kkc}{2r}$ ad ſolidum ex AOL
rca eandem AO ſeu omnia $\frac{hhc}{2r}$, ut ſpatium
FO ſeu omnia y ad ſpatium AMO ſeu om-
a z; unde poſito $AQ : x$, & $yy = 2rx - xx$,
it $k^4 = 2bbrx - bbxx$; &, dato loco ipſius
, innoteſcet curvæ ANL proprietas, cum
: $hh = bz$.

Si quis autem malit ſolida tubulata ſeu annu-
ria: detur circa axem AO Figura AGL,
ujus applicatæ $QN = TQ = z$; item alia
FM, cujus applicatæ $QS = DQ = y$, erit
igura prima ad ſecundam ut omnia z ad omnia
, ſeu aſſumtâ b determinatâ, & r pro nu-
nero, ut omnia rbz ad omnia rby; quorum
trumque ſi reducatur ad armillas ſeu quadrato-
um differentias pro acquirendis ſolidis annula-
ibus juxta §. 20. fiatq. $rbz = kk - hh$, &
$by = ff - uu$, erit $hh = kk - rbz$, &
$uu = ff - rby$. Cum autem, tum hh, tum uu

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ſint quadrata rationalia, oportet quoque ut ipſorum valores $kk-rbz$ & $ff-rby$ rationali radice gaudeant; quare aſſumtâ quadrati $kk-rbz$ radice $k-td$ erit $k=\frac{rbz+ttdd}{2td}$; ac ſuppoſito quadratum $ff-rby$ provenire a radice $f-ſc$, emerget $f=\frac{rby+ssc c}{2sc}$; quo vero tum k tum f á fractionibus liberentur, factis $r=4$, item $s=t=2$, ac $b=c=d$, evadet $k=z+b$, & $f=y+b$; unde emergit $h=z-b$, aut $b-z$, nec non $u=y-b$ aut $b-y$. Ducta igitur AH ipſi FM parallela vocetur b, completoque $AOKH$ rectangulo, erit ſemper ducta quavis ipſi FK æquidiſtanti recta $DI=b+y$ & $IS=b-y$, unde ſemper armilla ex DS circa HK, ſeu quadratorum in DI & SI differentia æqualitur $4by$, & ſolidum ex ſpatio AFM circa HK rotato $=$ omnia $4by$: eodem modo invenietur ſolidum ex AGL circa eandem $HK=$ omnia $4bz$. Adeoque patet hic ab aliis, alio tamen modo demonſtratus canon.

Si circa axem AO duo quæcunque conſtituantur ſpatia AFM & AGL, fiatque ductâ AH ipſi MO parallelâ quodlibet rectangulum $AOKH$; erit, ut prima Figura ad ſecundam, ſeu ut omnia y ad omnia z, ita ſolidum

ex

e Figura prima circa *H* K rotata, ad ſolidum e Figura ſecunda circa eandem *H* K.

Coroll. I. Quod ſi *A G L* ſit ſpatium menſurabile, & *A* F *M* rectangulum ipſi *A G L* circumſcriptum; erit tubus ex hoc rectangulo circa *H K* rotato ad ſolidum ex *A G L* circa eandem, in ratione rectanguli circumſcripti ad ſpatium *A G L*; cujus ideo ſolidi datur ratio ad cylindrum, cum ſpatii *A G L* ad rectangulum circumſcriptum ratio ſit nota ex hypotheſi.

Coroll. II. Cum autem, juxta Cor. I. §. 18. cap. præc. ex quâlibet curvâ infinita curvilinea cognitæ magnitudinis ortum ducant, per hanc infinita ergo ſolida quorum ratio ad cylindrum nota erit, ex ſingulis curvis, juxta præc. Coroll. exhiberi poterunt.

§. 38. Neſcio anoperæ ſit pretium hic aperire fontes, qui Mathematicorum libros immenſa theorematum ſerie jam olim inundaverunt; cum cuilibet per præcedentia ſatis innoteſcant. In tironum tamen gratiam unum aut alterum ſpecimen addo.

Datum (F*ig. LXVI.*) ſpatium *B D C* (cujus applicata F *E*: *y*) rotetur circa rectam *A S*, ipſi F *E* normalem; voceturq. *A E*: *d*, ſolidum ea rotatione genitum juxta §. 20. explicabitur per omnium quadratorum in F *A* & *A* conſtitutorum differentias, ſeu omnia

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$\frac{2dyc + yyc}{2r}$, ſi vero in baſi *BCD* elevetur cylindricus, cujus altitudo F*K* vocetur *h*, & qui ſecetur plano *HA* per *TAS* tranſeunti, ita ut *AG* ſit: *b*, truncus inferior algebraicè exponetur juxta §. 5. per omnia trapezia *P*F*EQ* ſeu omnia $\frac{2dyh + yyh}{2b}$; qui truncus ergo erit ad ſolidum rotatione genitum, ut omnia $\frac{2dyh + yyh}{2b}$ ad omnia $\frac{2dyc + yyc}{2r}$, hoc eſt, ut *h* ſeu altitudo cylindrici ad $\frac{bc}{r}$ ſeu circumferentiam in radio *GA* ſeu *b*: vel poſito $h = b$, aut angulo *HAG* ſemirecto, ut *r* ad *c*, ſeu ut radius circuli ad peripheriam.

§. 39. Hinc aſſumto, in Figura LXVII. ſpatio altero *DBC*; cæteriſque, quæ tum rotationem tum trunci ſectionem ſpectant, factis, ut in præcedenti; vocentur *AE*: *z*, F*E*: *x*, altitudo FK: *s*, *GA*: *g*; erit juxta præcedentem truncus inferior ad ſolidum tubulatum ut omnia $\frac{2zxs + xxs}{2g}$ ad omnia $\frac{2zxc + xxc}{2r}$, ſeu ut *s* ad $\frac{gc}{r}$; unde, vocato trunco in F*ig*. *LXVI*: *t*, in F*ig*. L*XVII*: *p*, ſolido rotundo in F*ig*. L*XVI*: *f*, in *Fig*. L*XVII*: *l*, factoque $\frac{bc}{r} = q$

gc

$= n$, erit juxta §. 38. $t : f :: h : q$. & quoque
$l :: s : n$; unde ſolidorum quorumvis annula-
rum ratio $f : l :: \frac{qt}{h} : \frac{pn}{s} :: \frac{bt}{h} : \frac{gp}{s}$; trunco-
rum autem ratio $t : p :: \frac{hf}{q} : \frac{ls}{n} :: \frac{hf}{b} : \frac{ls}{g}$; quæ
fuſius demonſtrata apud alios videantur.

§. 40. Reaſſumto ſchemate XLII. ac ſym-
bolis, tum §. 24. tum §. 37. Cap. II. adhibitis,
oportet ſolidum rotundum verb. gr. ex curvili-
neo *AZP* circa *AP* rotato genitum effor-
mare, quod æquale eſt, ſolido cuicunque ro-
tundo per quamlibet expreſſionem analyticam
deſignato.

Caſus, ubi ſolidum datum mediantibus rec-
tis, ad curvilineum cognitum *APE* pertinen-
tibus, explicatur. Sit ergo deſcribendum,
poſito *Q*3 $= h$, ſolidum ex *APZ* circa
AP rotato (ſeu, juxta §. 18. omnia $\frac{chhe}{2r}$)
æquale cono, ex triangulo *TDQ* circa *TP* ro-
tato, ſeu $\frac{cyyt}{6r}$; erit ex §. 13. Cap. II. omnia
$\frac{chhe}{2r} =$ omnib. $\frac{cyye + cyyi + 2ctya}{6r}$ ſeu ter-
minis ipſius $\frac{cyyt}{2r}$ infiniteſimalibus, adeoque,
diviſione facta per $\frac{c}{2r}$ erit $3hhe = yye + yyi + 2tya$;

$+ 2tya$; ex qua æquatione ejectis, methodo §. 31. Cap. II. traditâ, infiniteſimis, ipſa h per lineas cognitas deſignabitur, calculum hic omitto, cum poſt Cap. II. §. 31. & 37. ritè intellectas ſponte ſequatur.

§. 41. Neque ſemper requiritur, ut curvilineum quæſitum ad AP applicetur; verum & ad quaſcunque rectas curvam ADE ſpectantes ordinari poteſt.

Quæratur, exempli gratiâ, curvilineum $A8FB$ ad tangentis abſciſſam AB applicatum, quod rotatione ſui circa eandem AB efficiat ſolidum æquale cono modo expoſito $\frac{cyyt}{2r}$; poſitâ $T8 = h$, ac $BT = i$, erit hoc ſolidum $=$ omn. $\frac{chhi}{2r}$, juxta §. 18, adeoque ſumendo ipſius $\frac{cyyt}{2r}$ partes infiniteſimas, ac diviſione per $\frac{c}{2r}$ inſtitutâ, erit $3hhi = yye + yyi + 2tya$; unde ad modum Cap. II. §. 31. & 38. eliminatis infiniteſimis, applicata h innoteſcet; quæ ordinariâ methodo ad interceptam AT reducta, curvilinei $A8FB$ æquationem exhibebit.

Simile in Figuris planis exemplum Cap. II. §. 38. ſuppeditat.

§. 42. Quin & eadem, quam in Figuris planis Cap. II. §. 39. demonſtravimus, etiam in ſolidis generalis methodus obtinet, retentis enim

nim inibi aſſumtis ſymbolis quæratur ſolidum, x *APZ* circa *AP* rotato æquale cylindro, ujus baſis eſt circulus radio *Q*4 ſeu *f* deſriptus, ac altitudo *Q*5 ſeu *z*, erunt omnia $\frac{hce}{2r}$ æqualia $\frac{cffz}{2r}$; unde juxta Cap. II. §. 13. diiſis omnibus per $\frac{c}{2r}$ erit $hhe = ff\mu + 2fz\theta$; am cum juxta ibi tradita ſit, $\theta = \frac{fe}{b}$ & $\mu = \frac{ze}{d}$ rit $hhe = \frac{ffze}{d} + \frac{2ffze}{b}$ & $hh = \frac{ffz}{d} + \frac{2ffz}{b}$;

Unde, *f* & *z* per quamlibet expreſſionem nalyticam deſignatis, etiam *h* per eandem reſectivè exprimetur.

Coroll. Hinc eâdem operâ conſtat ratio, lato cuilibet ſolido *ffz* ungulam truncumque equalem conſtituendi; tum &, è converſo, nnumera ſolida rotunda ad cylindros, & truncos ngulaſque ad cubos reducendi methodus.

§. 43. Quomodo jam, quæ §. 17. & 30. Cap. II. circa plana demonſtravimus, etiam ſolidis accommodari queant, poſt levem attenionem in aperto eſt; aſſumta ſiquidem quâlibet hypotheſi, $\frac{chhe}{2r} = \frac{czzi}{2r} = \frac{cffu}{2r} = \frac{cgga}{2r}$ &c. habebuntur, poſt ejectas ſecundum §. 31. Cap. II. infiniteſimas, duo ſolida ſibi mutuo æqualia.

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Nec brevitati conſulentes integrum in ſingulis hiſce calculum adjungere operæ duximus pretium, principia tradidiſſe contenti; nec quomodo hæc ad ſolida annularia ſeu tubulata, & truncos ungulaſque referri queant, ulteriori hic diſcurſu proſequor; cum modo prægreſſa §. 42. generaliter hæc omnia comprehendat. Hinc enim datâ cujuslibet ſolidi deſignatione in terminis quibuscunque algebraicis, ſolidum tum rotundum, tum ad ungulas truncoſque pertinens exhiberi poteſt, cujus magnitudo per expoſitam expreſſionem analyticam eſt determinabilis; quin & non uno modo idem effici poſſe ex §. 41. manifeſtum eſt, cum pro variis, quibus ipſa *b* applicatur, infiniteſimis conſtructio varia ſit. Quod, licet ſpiſſo ſatis volumini opulentam materiam exhibeat, num etiam ab aliis, qui ſatis prolixè alioquin ſolidorum ex Curvilineis ortorum proprietates deſcripſerunt, animadverſum ſit, neſcio.

CA-

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CAPUT IV.

De ſuperficierum, lineis Curvis inſiſtentium, dimenſionibus.

§. 1. IN expoſito Curvilineo *ADEOA*, *Fig. LXVIII.* ſi erigatur Cylindricus, cujus altitudo $A\alpha : r$, vocenturque, ut in ſuperioribus, $QP : e$, $DE : u$, conſtabit ſuperficies curva cylindrici curvæ *ADEO* normaliter inſiſtens, propter baſin infinitè polygonam, innumeris rectangulis $DE\delta E$ ſeu omnibur ru. ea vero, quæ inſiſtit rectæ *AO*, omnibus ſeu totidem re; quare integra, demtis baſibus, cylindrici ſuperficies æquatur omnibus $ru + re$.

§. 2. Quod ſi cylindricus hic, *Fig. LXIX.* plano per *CB* tranſeunte in duos ſecetur truncos; ita ut *CT* ſit: b, ac $TS : h$; conſtabit inferioris trunci ſuperficies, quâ parte rectæ *AO* ſuperſtat, innumeris rectangulis *QIGP*; hoc eſt (cum appellato $QC : d$, $QP : e$, ſit $QI : \frac{dh}{b}$) omnibus $\frac{dhe}{b}$; quâ vero parte ſuperficies curvæ *ADEO* inſiſtit, conſtabit omnibus, trapeziis *DKNE*, hoc eſt (cum vocatis $QD : y$, $HE : a$, $DE : u$, $PE : y + a$,

ſit

ſit $DK = \frac{dh + yh}{b}$ & $EN = \frac{dh + yh + ah}{b}$) omnibus $\frac{2dhu + 2yhu + ahu}{2b}$, hoc eſt, juxta lemnia 10. omnibus $\frac{dhu + yhu}{b}$, adeoq propter trianguli *NKM* nullitatem omnibus rectangulis *MEDK*.

§. 3. Quod ſi fiat $h = b$, & angulus *TCS* ideo ſemirectus, æquabitur trunci ſuperficies curvæ *ADEO* inſiſtens omnibus $du + yu$.

§. 4. Sin autem non per *BC*, ſed per *AO* planum ſecans tranſeat, erit ungulæ plano *ADEOA* inſiſtentis ſuperficies æqualis omnibus yu; hoc enim caſu *QC* ſeu d ex hypotheſi evaneſcit.

Coroll. I. Sit *Fig.* LXX. ſemicirculus *AEO* centro F deſcriptus, ac radio F*D*: r; erit ſuperficies ungulæ cylindrici per diametrum ſeminormaliter reſecti, retentis iiſdem ſymbolis, juxta præcedentem æqualis omnibus yu; cum autem ſit *D*F: *DQ*:: *DE*: *DH*, ſeu $r : y :: u : e$. erunt omnia $yu =$ omnia re, hoc eſt, facto *A*L $=$ *D*F $= r$, erit ungulæ ſuperficies æqualis rectangulo *A*L*M*O; adeoq. quadrabilis, tum integra, tum ſecundum partes proportionales, quod & in parabolâ obtinet, conſulatur Cor. III. §. 11. ſeq.

§. 5. No-

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§. 5. Notetur, non requiri ut linea *A*O ecceſſario ſit recta, factis enim, Fig. LXXI. QC: z, & CD: y, ST: h, TC: b, infiniteſi-nes DE: u, QP: i; planum ſecans per SC C B tranſeat; erit ſuperficies trunci curvæ *D E* O inſiſtens æqualis omnibus $\frac{yhu}{b}$, illa ero, quæ curvam AQPO pro baſi habet, quatur omnibus $\frac{hzi}{b}$, juxta §. 2. hujus, adeo-ue tota trunci ſuperficies toti curvæ A*DE*O QA ſuperſtans æquabitur omnibus $\frac{hyu + hzi}{b}$.

§. 6. Patet hinc, poſitâ *y* curvæ cujuslibet pplicatâ, ipſiuſque curvæ infiniteſimâ = *u*, b, h, r, d, lineis quibuſcunque determi-atis; curvæ inſiſtentem ſuperficiem cylin-ricam in altitudine *r* deſignari per omnia *ru*; iperficiem ungulæ ſemiquadrantalis per omnia u; ſuperficiem ungulæ cylindrici, ad angu-im, quem ratio *b* ad *h* determinat, reſecti er omnia $\frac{hyu}{b}$; trunci ſuperficiem per omnia $\frac{du + hyu}{b}$ aut $du + yu$, prout angulus plani ecantis vel ſemirectus vel alterius magnitudi-is eſt, & ſic in cæteris, quod modo traditis tè intellectis ſatis manifeſtum eſt.

De

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De ſuperficiebus rotatione Genitis.

§. 7. RECtangulum A*BEP Fig. LVIII.* diagonali A*E* diviſum, ſi rotetur circa axem A*P*, efficiet linea *BE* hac circumvolutione ſuperficiem cylindricam B*EINM*; & linea A*E* ſuperſiciem conicam *EIM*A. Cujus utriuſque valor Analyticus ut indagetur, circuli *EIM* utriuſqne baſin conſtituentis peripheria vocetur *c*; quæ ſi dividatur in partes infiniteſimas, ut *GI*, quæ appellentur *u*, conſtabit ſuperficies cylindrica infinitis rectangnlis LO*IG*, ſeu (vocata A*P* = O*I*: *d*) innumeris *du*; ideoque totius cylindri ſuperſicies, juxta Cap. II. § 13. æquabitur *dc*.

§. 8. Conica autem ſuperficies componetur infinitis triangulis iſoſcelibus *G*A*I*, quorum ſingula (vocatâ nimirum A*E* = A*I* = A*G*: *l*) erunt = $\frac{lu}{2}$, cum juxta lemni 33. ſint rectangula ob baſin *GI* infinite parvam, unde ſuperficies modo dicta æquabitur omnibus $\frac{lu}{2}$; erit que juxta Cap. II. §. 13. abſolutus ipſius valor = $\frac{lc}{2}$, ſeu triangulo rectangnlo, cujus unum latus eſt latus trianguli per axem coni, alterum circumferentia circuli baſin conſtituentis.

§. 9. Ve-

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§. 9. Verum & alius cylindricæ ſuperficiei va-or hoc pacto inveſtigatur, non contemnendi uſus n hoc calculi genere. Cylindrus, *Fig. LXXII.* ex ectangulo *AO*F*B* circa *AO* rotato conſtat nfinitis cylindrulis ex Q*PED* circa Q*P*, nde ſuperficies cylindri majoris ex infinitis ſu-erficieculis cylindricis ex *DE* circa Q*P* ro-ata genitis: talis autem ſuperficiecula, factâ *E*:*p*, & *DE*:*e* infiniteſimâ rectæ *BF* ſeu *A*O, quabitur $\frac{pec}{r}$, poſitâ radii ad circumferen-am ratione ut *r* ad *c*; unde tota cylindri ſuper-icies erit æqualis omnibus $\frac{pec}{r}$, tam ſecundum onverſionem integram, quam partialem.

§. 10. Conoides *Fig. LIX.* ex Figura *AF*O rca *A*O rotata, juxta 17. Cap. præc:, infi-itis conſtat conis truncatis, ex trapeziis Q*DPE* rca Q*P* volutis; quorum ſingulorum ſuper-cieculæ efficiuntur reſpectivis rectulis *DE* rca eandem Q*P* rotatis; unde patet omnes as conorum truncatorum ſuperficieculas ſimul umtas æquare ſuperficiem conoidicam ex om-ibus rectulis *AD*, *DE*, *EF* &c. hoc eſt ex ntegrâ Curvâ *ADE*F circa *A*O rotatâ.

§. 11. Ut vero inveniatur ejus modi ſuper-cieculæ valor, factis *D*Q: *y*, *TD*: *s*, cui ngenti ſit normalis *DR*: *k*, tum *EH*: *a*,

P *DE*: *u*,

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$DE:u$, & ratione radii ad circumferentiam ut r ad c, erit circumferentia radio QD deſcripta $= \frac{cy}{r}$, quæ ducta in $\frac{1}{2}TD$ dat $\frac{cys}{2r}$ pro ſuperficie coni ex TDQ circa TQ rotati; uti ex prægreſſo §. 8. notum eſt. Eandem ob rationem ſuperficies coni ex triangulo TPE circa TP converſo æquabitur $\frac{cy+ca}{r}$ in $\frac{s+u}{2}$, ſeu (eliminatis per lemma, 10. rejectaneis) $\frac{cys+cyu+cas}{2r}$; unde ſi detrahas $\frac{cys}{2r}$, remanet coni truncati ſuperficies ex DE circa PQ $= \frac{csa+cyu}{2r}$.

Cum vero ob tangentem TD ſit perpetuo $yu = sa = ke$, erit ſingularum ſuperficiecularum valor $= \frac{cyu}{r} = \frac{csa}{r} = \frac{cke}{r}$; quare tota ſuperficies ex AFO circa AO rotata $=$ omnia $\frac{cyu}{r} =$ omnia $\frac{csa}{r} =$ omnia $\frac{cke}{r}$.

Coroll. I. Sit AEO *Fig. LXX.* ſemicirculus centro F deſcriptus, qui volvatur circa diametrum AO, quæritur hinc orta ſphæræ ſuperficies; vocatoq. radio $DF:r$, & $DQ:y$, æquabitur juxta modo inſinuata omnibus $\frac{cyu}{r}$ $=$ omnibus $\frac{cre}{r} =$ omnibus ce; eſt enim

yu

$y = rc$, §. 4. Cor. I. ſed omnia ce efficient rectangulum ALMO, in quo, AO exiſtente circuli diametro, AL æquatur ipſi c ſeu peripheriæ; eſt autem hoc rectangulum $= 2rc$ $= \frac{4rc}{2} =$ quadruplum majoris circuli in ſphærâ repoſita.

Coroll: *II.* Cum autem hoc æque in partibus analogis obtineat; ſuperficies ex ſegmento ADQ circa AQ rotato, facto AQ: x, æquabitur, ipſi cx, ſeu rectangulo ex AQ in circumferentiam integram maximi circuli in ſphæra.

Quod ſi deſiderentur circulus huic ſuperficiei æqualis, ſit radius: z, erit circulus $\frac{zzc}{2r} =$ cx, adeoque $\sqrt{2rx} = AD = z =$ radius circuli quæſiti.

Coroll. III. Rotetur, *Fig. L*XXIII. linea parabolica ADE circa axem AP, vocentq. AQ: x, QD: y, DF tangenti normali: k, erit genita hinc ſuperficies ex præc. $=$ omnia $\frac{cuy}{r} = \frac{cke}{r}$; quod ſi k ſeu DF applicetur ad QP ſeu e, adeoque ad AQ ſeu x erit $\frac{1}{4}rr + rx = kk$; adeoque curva $SLCM$ erit eadem parabola & $SA = \frac{1}{4}r$; unde omnia k ſeu ſpatium $ALMP$ cognitum eſt, & æquale

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æquale ſuperficiei ungulæ cylindrici in baſi APE per AP ſeminormaliter reſecti; vocetur hoc n, erit ideo ſuperficies conoidis parabolici $= \frac{cn}{r}$; cui æquatur circulus radio $\sqrt{2n}$ deſcriptus.

Coroll. IV. Eodem modo res in cæteris curvis peragitur; ſi ADE ſit hyperbola (cujus æquatio $2rx + xx = yy$) erit k ad hyperbolæ alterius ſegmentum $ALMP$, cujus æquatio $rr + 4rx + 2xx = kk$; ſir hoc ſegmentum $= n$; erit $\sqrt{2n}$, radius circuli æqualis ſuperficiei conoidis hyperbolici.

§. 12. Quod ſi rotationis axis LM, *Fig. LXX.* ſit extra Figuræ $ADEO$ ambitum; factâ $QN: d$, determinatâ; $QD: y$, & $DE: u$, conſtabit ſuperficies ex curva $ADEO$ circa LM converſa, omnibus, $\frac{duc + yuc}{r}$; quod coincidentem DE ultra T producendo, donec cum LM concurrat, ac eodem cum §. 11. calculo patefiet.

§. 13. Si vero inquiratur, *Fig. LXXI.*, ſuperficies ex integrâ Figuræ $ADEOPQA$ peripheriâ circa BC rotatâ, ſint infiniteſimæ $DE: u$, $QP: i$, rectæ $CD: y$, $QC: z$, erit eadem

eidem æqualis omnibus $\frac{yuc+zic}{r}$, conſequenter ratio jam ſæpius inculcata eſt.

§. 14. Plurima hinc in modum §. 38. & 39. Cap. præc. deduci poſſent; cum, juxta §. 2. hujus, ſuperficies trunci inferioris cylindrici in altitudine *ST* ſeu *h*, per planum *BC* (ita ut *ZC* ſit = *b*) reſecti conſtet omnibus $\frac{yuh+zih}{b}$ & ſuperficies ex perimetro *ADE*O*P*Q*A* circa *BC* rotatâ ſit æqualis omnibus $\frac{yuc+zic}{r}$ per præced; erit illa ſuperficies trunci ad hanc ut omnia $\frac{yuh+zih}{b}$ ad omnia $\frac{yuc+zic}{r}$ ſeu ut *h* ad $\frac{bc}{r}$.

Unde in truncorum ac ſolidorum rotatione genitorum ſuperficiebus idem obtinet ac in eorum ſolidis. Conferantur hæc cum §. 38, 39. Cap. præcedent.

§. 15. Cæteræ vero, quæ ordinario circumferuntur propoſitiones, facili hinc negotio eruuntur. Reſumtis enim ſchematibus *XXXIX* & *XL* & ſymbolis §. 17. Cap. II. cum ſuperficies conoïdalis ex curva *ADE*F circa *A*O converſa ſit, juxta §. 11. hujus, æqualis omnibus $\frac{cuy}{r}$ = omnib. $\frac{csa}{r}$ = omnib. $\frac{cke}{r}$; erit *Primo*, ut *r* ad *c*

ſeu radius ad circumferentiam ita omnia uy ſeu ſuperficies curva ungulæ cylindrici plano AFO inſiſtentis ac per AO ſeminormaliter reſecti, juxta 4 hujus, ad omnia $\frac{cuy}{r}$ ſeu ſuperficiem conoidalem ex $ADEF$ circa AO.

Secundo, poſitâ $ADEF$ in directum, ut ſit $DQ = \delta x = y$, $DE = \delta t = u$, planum $\alpha\varphi o$ æquale omnibus uy; & poſitâ DT ſeu s continuo in IM, erit FKO æqualis omnibus sa; nec non, DR conſtitutâ ſemper in QC, Figura ASO æquabitur omnibus ke. Unde generaliter: ut r ad c, ſeu radius ad peripheriam, ita quodlibet ex planis $\alpha\varphi o$, FKO ASO ſeu omnia uy, aut $ſa$, aut ke, ad ſuperficiem conoidalem ex $ADEF$ circa AO, hoc eſt omnia $\frac{cuy}{r}$ ſeu $\frac{csa}{r}$ ſeu $\frac{cke}{r}$.

§. 16. Hoc addam; ſi Conöidalis ſuperficies per omnia $\frac{kec}{r}$ explicetur; & figuræ AFO circum ſcribatur rectangulum $AVFO$, ac fiat $AV = p$; fore omnia $\frac{pec}{r}$ ſeu ſuperficiem cylindricam rectanguli $AVFO$ circa AO rotatione genitam ad omnia $\frac{kec}{r}$ ſeu ſuperficiem conoidalem ex AFO circa AO volutâ productam, ut omnia pe ad omnia ke, ſeu ut rec-

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ectangulum, $AVFO$ ad planum ASO.

§. 17. Quo vero pacto immenſa propoſitio-
um meſſis hinc colligatur, hoc exemplo ty-
onibus aperire liceat. Retentis ſymbolis, vocen-
ur AQ, cui æqualis DX: x, & AO: q;
atque juxta ſæpius jam inculcata $t : s :: e : u ::$
z, unde $ze = pu$. Facto ergo conſtanter
$QC = z$, erit planum $ASO =$ omnia ze;
autem p & e in analogiſmo ſint quantitates per-
anentes ac immutabiles, quia fuit ut $p : z :: e : u$.
rit & $pe : ze :: e : u$; &, ductis conſequenti-
us in $\frac{cx}{r}$, ut $pe : \frac{zexc}{r} :: e, \frac{xuc}{r}$, hoc eſt ut
ectangulum $AVFO$ ſeu omnia pe, ad ſoli-
um ex ASO circa VG rotato ſeu omnia
$\frac{zxc}{r}$, (§. 26. Cap. III.) ita omnia e ſeu recta AO
. omn. $\frac{xuc}{r}$ ſeu ſuperficiem conoidalem ex AEF
rca VG rotatâ, ex §. 11. hujus. Quod ſi antece-
entes in idem $\frac{cx}{r}$ ducantur erit $\frac{pexc}{r} : \frac{zexc}{r} ::$
$\frac{ec}{} : \frac{xuc}{r}$, hoc eſt, ut cylindrus ex $AVFO$ cir-
VG, ſeu omnia $\frac{pexc}{r}$, ad ſolidum ex ASO
rca eandem GV, ſeu omnia $\frac{zexc}{r}$, ita om-
a $\frac{xec}{r}$ ſeu $\frac{qqc}{2r}$, (quia omnia $xe = \frac{1}{2}qq$.)

 ſeu

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ſeu circulus in radio *AO* ſeu q ad ſuperficiem conoidalem ex *AEF* circa *VG* ſeu omnia $\frac{xuc}{r}$, quod ablatâ per diviſionem ratione $\frac{c}{r}$ ad ungularum truncorumque ſuperficies transferri poteſt: ſed hæc omitto.

§. 18. Cum igitur, juxta §. 37. Coroll. II. Cap. III. infinitorum numero ſolidorum ratio ad cylindrum nota ſit, per hanc præcedentem quoque infinitarum numero ſuperficierum conöidicarum ad circulum ratio innoteſcet, ſi ex data curva A*CS* inveniri poſit æquatio curvæ curvæ A*EF*.

§. 19. Reſumtâ *Fig. XI.* ſit *QD*: y, A*Q*: x, ſubnormalis *QS*: l; erit circulus radio *DQ* deſcriptus $= \frac{yyc}{2r}$, cui oportet ſuperficiem conöidicam æqualem ex curvæ A*NK* circa eundem axem A*P* rotatione genitam, aſſignare. Sit eum in finem *QN*: f, tangens *NL*: k, ſubtangens *LQ*: h, curvæ normalis *NS*: n, infiniteſima *NK*: o, erit juxta §. 11. dicta ſuperficies $=$ omnia $\frac{foc}{r}$; quæ æquanda eſt circulo $\frac{yyc}{2r}$ $=$ omnia $\frac{yac}{r}$ ex §. 13. Cap. II. adeoque, diviſis omnibus per $\frac{c}{r}$, fiet. $fo = ya$; ſed, quia

$h:k::$

$b : k :: e : o$, eſt $o = \frac{ke}{b}$; tum etiam eſt $ya = le$, ideoque $\frac{fke}{b} = le$ & $l = \frac{fk}{b} = n$.

Unde hic conſequitur canon: Si duarum curvarum *ADE* & *ANK*, ad eundem axem *AP* ordinatarum, ea ſit relatio, ut illius ſubnormalis *QS* hujus normali *NS* perpetuo adæquetur, erit ſuperficies ex hujus curvæ *ANK* rotatione circa axem *AP* genita, ſemper æqualis circulo, qui applicatâ *DQ* alterius curvæ, tanquam radio, deſcribitur.

Coroll. I. Sit *QN* ad circulum, cujus æquatio $2rx - xx = ff$; & *QD* ad parabolam, cujus æquatio $2rx = yy$; erit pars ſuperficiei ſphæricæ ſemper æqualis circulo, qui applicatâ parabolæ in eodem axis puncto *Q*, tanquam radio, deſcriptus eſt ſeu $= \frac{yyc}{2r} = cx$; unde poſito $x = 2r$ erit integra ſuperficies $= 2rc =$ quadruplum circuli maximi in ſphæra, cujus radius r, peripheria: c; cum inter circulum hunc & parabolam requiſita relatio obtineat.

Coroll. II. Patet hinc, datâ methodo ſubnormalium inverſâ, dari omnium ſuperficierum conoidicarum ad circulos reductionem.

Cor. III. Poſſet & hoc principio ſubnormalem *NS* per x expreſſam in ſeriem reſolvendo non

paucarum ſuperficierum ratio ad circulum (in praxi ſatis accuratè) determinari.

Schol. Licet autem de planis & ſolidis tractantes methodum tradiderimus, dato cuilibet plano ſolidoque per analyticam quamcunque expreſſionem deſignato, curvilineum, truncumq. æqualem exhibendi, problema hoc circa ſuperficies non æque feliciter ſolvitur, hàc quidem de cauſâ, quod methodum normalium ſeu, quod eodem recidit, tangentium inverſam requirat; potuiſſetque ſuperficierum, quæ corpora conica determinant, tractatio ulterius longè extendi; cum modis variis, præter jam recenſitos, formari, adeoque menſurationis ſuæ diverſa à modo traditis principia admittere poſſint: cum tamen ex eodem cum præcedentibus fundamento originem ducant, juxta quod curvilinea tanquam infinitorum laterum polygona conſiderantur, ſpecialem magis horum inquiſitionem curioſorum ſtudio relinquimus. Elegans exemplum ſuppeditat Erudit. *Barovii* Lect. XII. appendicula II.

CA-

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CAPUT V.

De Curvarum dimenſionibus.

§. I. EX lemmate 26. curva *ADEF Fig. XXXIX.* conſtat rectulis infinitis, *AE*, *ED*, &c. ſimul ſumtis, ſeu omnibus *u*; quorum valor ut reperiatur, manentibus §. 17. Cap. II. ſymbolis, noti ſunt, propter tangentem *TD*, eique normalem *DR*, ſequentes analogiſmi, qui exhibent $u = \frac{ke}{y} = \frac{se}{t} = \frac{ke+le}{s} = \frac{sa}{y} = \frac{ka}{l} = \frac{ta+la}{k}$.

§. 2 Hinc ergo curva *ADEF*, ſeu omnia *u*, æquantur omnibus $\frac{ke}{y}$, cum ſit *y*:*k*::*e*:*u*; unde, cum *e* ſibi ſemper æqualis ſupponi poſſit, ſi & *y* talis foret, eſſet ut omnia *y* ad omnia *k* ita omnia *e* ad curvam ſeu omnia *u*. Hæc autem immutabilitatis conditio cum in denominatore *y* deſideretur, factâ *r* determinatâ, fiat $\frac{z}{r} = \frac{k}{y}$ unde $\frac{ze}{r} = u$, jam poſitâ *z* in *QC* ad curvam *ACBS*, erit, factâ *AG* = *r*, ut omnia *r* ſeu rectangulum *AGSO* ad omnia *z* ſeu ſpatium *ASO*, ita omnia *e* ſeu axis *AO* ad omnia *u* ſeu curvam *ADEF*. Noſcitur autem

tem locus ipſius z ex Curvæ $ADEF$ relatione cognita & r determinatâ. Quare dato ſpa. $ABSO$ ſeu omnibus z datur curvæ $ADEF$ ratio ad axem ſuum AO.

Notari poteſt, quia $ze = ru$, fore $\frac{ze}{r} = u$; hoc eſt, curvam ſeu omnia u, æquari ſpatio ASO per r determinatam diviſo, ſeu $\frac{\text{omnibus } ze}{r}$.

Coroll. I. Quo vero hac methodo indagetur, num paraboliformium quædam ad menſuram poſſit revocari, ſit curvæ $ADEF$ æquatio $r^p - q\,x^q = y^p$, erit ex tangentium methodo TQ ſeu $t = \frac{px}{q}$, unde QR ſeu $l = \frac{qyy}{px}$, & kk ſeu quadratum $DR = yy + \frac{qqy^4}{ppxx}$: Jam vero, cum ſit $kk : yy :: zz : rr$, erit, poſito pro kk in vento valore, ac eliſo yy per æquationem curvæ, hæcce curvæ ASO proprietas

$$zz - rr \text{ in } \frac{pp}{qq} r^{\frac{2q-4p}{p}} = x^{\frac{2q-2p}{p}};$$

unde ſi dimenſiones ipſius x ſint $\frac{1}{f}$, erit curvilineum ASO ex Cap. II. §. 11. Coroll. ſemper menſurabile; ſiat ergo $\frac{2q-2p}{p} = \frac{1}{f}$, erit

rit $q = \frac{p + 2pf}{2f}$, & $q:p:: \frac{2pf + p}{2f} :p:: 2f + 1:$ f, ſeu ut numerus impar ad numerum proximè minorem. Obſervetur, cum q hic major ſit ipsâ p, ſemper r adhæſuram ipſi y; ſed hæc nihil habent difficultatis.

Coroll. II. Sit *Fig. LXXIV.* cyclöis (cujus dimenſionem hiſce cognatam hic adjungo) primaria A*DEG*, cujus axis AF: $2r$, A*C*F ſemicirculus genitor, vocenturq. AQ: x, Q*C*: y, *F*: h, *DB*: s, quæ cycloidi normalis eſt, DQ: f, *DE*: u, *DH*: e; erit ex §. 60. Cap. I. $f:: h: y:: u: e:: z: r$, unde omnia $ru =$ omnia e; ponatur z in QS, donec deſcripta ſit curva *SI*; cujus ut noſcatur relatio, fuit $h: y:: z: r$, AC: AQ, ſeu $2rx: xx:: zz: rr$, unde $r^3 = zzx$. Quare curva *ISL* ex hyperboliformibus una eſt & juxta Cap. II. §. 11. Coroll. 2. curvilineum *K*A*FLI* ſeu omnia e æquale $4rr$; adeoque omnia u, ſeu $\frac{\text{omnia } ze}{r}$, ſeu curva cycloidis $= 4r$, hoc eſt duplo diametri circuli genitoris. Obſervetur hic, cum *K*AQ*I* ſeu quælibet pars ſpatii hyperbolöidis cognita ſit, etiam A*D* ſeu quamlibet curvæ partem hoc ipſo notam eſſe.

§. 3. Addam & hoc a pluribus animadverſum, cum abſque lemmatum auxilio immediatè

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tè ex hoc calculo profluat. Sit *Fig. LXXV.* curva *ADE*F, eademque, quæ in §. 1, linearum ſymbola, erat $y:k::r:z::e:u$, unde $ze = ru$, quæritur jam (ſi z ponatur in *QC* ad ſpatium curvæ *BCNG* ex hypotheſi quadrabile) curvæ *ADE*F, quæ idcirco, ex §. 1. in rectam eſt mutabilis, proprietas; aut applicatæ *DQ* ſeu y ad *AQ* ſeu x relatio; cum itaque ſit $y:k::r:z$, erit & $yy:kk-yy::rr:zz-rr$, &, facto $zz-rr=ff$, erit quoque $y:l::r:f::e:a$, unde $fe=ra$; poſitis ergo f in Q*X*, & r in *IM*, eſt curvilineum *AX*X*O* = omnia fe, & rectangulum F*KLO* = omnia ra, quæ æquantur rectangulo dicto ry;

$$\text{quare } DQ \text{ ſeu } y = \frac{\text{omnia } fe}{r} = \frac{\text{ſpat. } AXXO}{r}.$$

Unde ſtatim apparet ſequens theorema: data Figura quadtabili *ABGO*, cujus applicata Q*C* ſeu z, ad eundem axem applicetur ſecunda Figura *A*XX*O*, cujus applicata QX ſeu f; ita tamen, ut ſit perpetuo $zz-rr=ff$; ſi deinde aſſumatur tertia Figura *A*F*O*, cujus applicata *D*Q ſeu y, ejus naturæ, ut ducta determinata r in applicatam quamlibet *D*Q ſeu y, ſit continuo rectangulum ry æquale ſpatio curvilineo adjacenti. *A*XXQ ſeu omnibus fe, erit curva *ADE*F in rectam convertibilis, & æqua-

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qualis ſpatio primo aſſumto *A B G O* diviſo per
determinatam.

Coroll. I. Sit *A B G O* trapezium paraboli-
m , & $zz = rr + rx$, erit $ff = rx$, &
X X O parabola, quæ, juxta Cap. II. §. 11.
or. I, eſt menſurabilis & æqualis $\frac{2}{3}\sqrt{rx^3} =$ om-
a *fe*. Cum autem ſint omnia $fe = ry$, erit
$\frac{2}{3}\sqrt{rx^3} = ry$, & $x^3 = \frac{9}{4}ryy$, quæ eſt æqua-
o curvæ A *D E* F.

Coroll. II. Hoc notetur, cum ſit $ru = ze$
quavis curvâ, datâ omnium curvilineorum
adratura, verb. gr. omnium ze, dari omnium
rvarum longitudines ſeu omnia *u*.

§. 4. Hunc in modum, aſſumto quolibet
nnium *u* valore, proceditur; ex. gr. ſit $u =$
kl ; fiatque, ſi *l* ſeu Q *R* F*ig*. XXXIX. ſit
antitas mutabilis, $\frac{k}{l} = \frac{z}{r}$; erunt omnia
$=$ omnia $\frac{za}{r}$, hoc eſt, poſito *z* continuo
I M, ſpatium ſpatium F O *K* ſeu omnia
a diviſum per determinatam *r*, æquale om-
bus *u* ſeu curvæ A *D* F.

uin & cum ſit $za = ru$ erit ſpatium F O *K*
quale rectangulo *αμοφ*, *Fig. XL.* cujus al-
rum latus *αφ* curvæ, alterum *φο* determina-
r æquatur.

Quod

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Quod ad quoſlibet omnium *u* valores §. 1. recenſios quivis juxta ſuperiorum normam accommodabit.

§. 5. Quo vero modo hæc methodus curvis, tum verſus axem concavis applicari queat; tum & quo pacto, ſi curva non ad axem, ſed ad punctum referatur, hæc tractanda ſint; præcedentia intelligenti ſatis manifeſtum eſt.

In exemplum, repetatur *Fig. XLVIII.* ſitque *DVSP* eadem ſpiralis, quam Cap. II. prop. XLIX. Cor. 2. diſcuſſimus, talis nimirum, quæ circuli radios *BD* ad eundem ſemper angulum ſecat; fiatque $SH = u$, tum vocatis $SF: s$, $SD: z$, $HI: a$, curvâ $DVS: c$, factâque conſtanti ratione ipſius *u* ad *a* eâ quæ eſt *b* ad *q*, erit perpetuo $qu = ba$; quæ ad terminos abſolutos per Cap. II. §. 13. reducta dabunt $qc = bz$; ſed $b:q::s:z$, ergo $qs = br = qc$, adeoque $s = c$, quod ſi *c* extendatur ad infinitam ſpiralem, erit $GP = s$; unde infinita hæc ſpiralis ipſi *GP* ſpiralem in puncto *P* tangenti adæquatur.

§. 6. Sit *Fig. LXXVI.* curva *AZXDE* conſtans infinitis rectulis *ED*, *DX*, *XZ*, &c. ſeu totidem *u*, quam contingat recta *BD* in *E*; ipſæ autem curvam conſtituentes infiniteſimæ productæ axem *BA* interſecent in punctes *T*, *M*, *N*, *R*, &c. aſſumaturque $EO = DE$; item $VO =$

DI =

D = DX; ut & SV = 3, 1, = 2X = XZ; co[illegible]tinuando hoc modo, donec tandem ultima [illegible] facta ſit = 7, 8, = 9, 6, = 10, 5, = 11, 4, = [illegible] 4. Porro in triangulo BET, ductâ CT, [illegible] CE = TE; item in triangulo TDM duc-[illegible] M, 12, fiat D, 12, = DM; ſit & in [illegible]ngulo MXN ducta N, 14, fiat X, 14, [illegible] N; & ſic in cæteris. Tandem factâ LC [illegible], 12, nec non LF = 12, 13, = M, 14, [illegible]eliquis idem continuetur, donec fiat ulti-[illegible], GI = 15, 8, = 16, 9, = 17, 10 = R 11. [illegible]atiſq. BT, TM, MN, ſemper: *i*; & [illegible], ac T 12, ut & M, 14, perpetim λ; [illegible]entes ſe pandent proprietates.

[illegible] Recta EI æquabitur curvæ ADE: cum [illegible] & A4, KS & Z4, SV & ZX &c. co[illegible]tinuo ex hypotheſi æquentur, & ex partium æ[illegible]alitate totorum æqualitas legitime inferatur, [illegible]re & recta EI erit = omnibus *u*, ſeu cur-[illegible] ADE.

[illegible]I. Recta BA, hoc eſt BT+TM+MN [illegible] æquabitur omnibus *i*.

[illegible]II. Recta BI menſurabitur per omnia λ.

[illegible]V. Adeoque integra tangens BE æqualis er[illegible] omnibus, *u*+λ, & quia omnia *u* æquan-tu[illegible] curvæ ADE, erit recta BE æqualis cur-[illegible] ADE + omnibus λ ſeu rectæ BI.

[illegible]. Cum autem lineolæ A, 11; 11, 10; 10,

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9; 9, 8; 8, I; pariter ſint infiniteſimæ angulos perpetuo in 8, 9, 10, 11, facientes, notum eſt eas omnes in lineam curvam *AI* deſinere, quæ in eandem partem ac curva *ADE* concava eſt; cujuſque hæc eſt proprietas: ut quælibet recta *EI*, *D*, 8, *X*, 9, &c. curvam *ADE* contingens huic ſit ad angulos rectos, & contra.

Cum enim ex hypotheſi *D*, 8, & *D*, 9, ſint æquales, & 8, 9, infiniteſima, erunt ex lemmate 33. angulo *D*, 8, 9, & *D*, 9, 8, recti, unde conſequitur propoſitum.

VI. Quod etiam *RG*, *NF*, *K*4, *ZS* &c. curvæ ſint modo dictam habentes relationem ad curvam *ADE*, eodem modo demonſtratur.

§. 7. Unde hoc tandem oritur theorema ſi ab eodem principio *A* duæ egrediantur Curvæ *AI* & *AE*, in eandem partem concavæ, ita ad ſe invicem relatæ, ut, quæ interiorem *AE* tangit recta, verb. *D*, 8, ſit exteriori *AI* ſemper ad angulos rectos, erit *D*, 8, pars tangentis inter duas curvas intercepta perpetuo æqualis curvæ *AXD* inter punctum contingentiæ *D*, & principium commune *A* interjectæ.

Eſt enim *DX* = *D* 1, tum *XZ* = 3, 1, ac *Z*, 4 = 3, 7, &c. ex modo aſſumtis; unde liquet intentum.

§. 8. Hoc notetur, (quod inter præcedentia jam attigimus) ſi curva *ADE* ſit verſus

axem

axem *AQ* concava, *Fig. LXXVI.* rectas *TD* & *BE* curvam in punctis ſucceſſivè ab axe remotioribus tangentes, utroque ſui extremo creſcere; altero quidem per infiniteſimam *OE* ſeu *u*, altero per *BC* ſeu λ; unde facta $TD = s$, erit tangens proxima $BE = s + u + \lambda$; eſt enim $CD = TD$ per ſuperiora. Si vero fuerit curva *ADE* (*Fig. LXXIX.*) verſus axem convexa, tangentes, altero ſui extremo, quo curvam tangunt, creſcere, per *u* ſeu *OE*; altero vero, quo axem interſecant, decreſcere per *TC* ſeu λ. Hoc eſt, ſi *TD* ſit $= s$; erit $BE = s + u - \lambda$, etenim $BQ = CD$ per anteriora.

Nec opus eſt addere, ſi rectæ curvas in punctis ad axem ſucceſſivè appropinquantibus, tangant, futurum ut, poſitâ $BE = s$, ſit $TD = s - u - \lambda$ in priori, ſed $s - u + \lambda$ in poſteriori caſu.

§. 9. Cum autem omnis curvarum in rectas mutatio in eo ſita ſit, ut inveniatur ſumma omnium *u*, ſeu longitudo rectæ *IE*; hinc ſeſe offerunt methodi, à prioribus, quas §. 1, 2, &c. recenſuimus, diverſæ: verb. gr. Sit *Fig. LXXVI.* *DQ* applicata: *y*, ac eadem, quæ in ſuperioribus, ſymbola; eſt, uti notum eſt, $TD : DQ :: OE : EH$, ſeu $s : y :: u : a$, unde ſemper $yu = sa$.

Ut jam hinc longitudo curvæ *ADE* indagetur: deſcribatur, *Fig. LXXVII.* curva ſecunda *ANI*, cujus applicata *NO* ſit $= DQ = y$,

infiniteſima $OE = u$, erit $GI = EH = a$; adeoque $EI = y + a$, & curvilineum $MIE =$ omnia yu; cum autem ſit ex hypotheſi, $a : u :: y : s$. ſeu $IG : GN :: NO : OC$, erit OC hujus curvæ tangentem determinans æqualis ipſi $OC = DT$ in *LXXVI* Figurâ, ſeu s, & $BC = \lambda$: adeoque BE, *Fig. LXXVI* & *LXXVII.* quæ tangens eſt in curvâ ADE, erit ſubtangens in curvâ ſecundâ MNI, ac $ME =$ omnia $u =$ curva ADE; & $BM =$ omnia λ; unde, ſi poſſet inveniri relatio curvæ MNI ad axem ME, ex datâ applicatâ $NO = y$, & ſubtangenti $CO = s$, quæ lineæ TD curvam ADE tangenti æqualis eſt; inventa eſſet recta ME æqualis curvæ ADE.

Quare omnis curvarum menſura ad methodum tangentium inverſam, ſeu ad hoc problema reducitur: datâ *Fig. LXXVII.* curvæ applicatâ NO & ſubtangenti CO, invenire æquationem, quâ curva MNI mediante applicata ad axem ſuum MO refertur.

Schol. I. Quomodo autem tangentium inverſa methodus cum fructu in geometriâ tractari queat, ſi CO in terminis proximis data ſit, cap. de tang. §. 93. & ſeqq. oſtenſum eſt; unde hoc ſolum ad curvarum rectificationem reſtat, ut valor lineæ CO, datus in terminis applicatam y reſpicientibus, ſeu remotis, ad terminos

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minos proximos ad interceptam MO juxta ac pplicatam NO relativos reducatur.

Schol. II. Datâ vero curvilineorum quadra-urâ. quo pacto methodus tangentium inverſa nter cognita redigi poſſit, hic in tyronum gra-iam oſtendam, licet loco forſan minus oppor-uno.

Quæritur ergo *Fig.* XXXIX. æquatio curvæ IDE, cujus applicata $DQ = IO$ ſit y, & Q ſubtangens ſeu t ſit $= \frac{2y^3}{3rr}$.

Vocatis, QP ſeu $DH{:}\,e$, & $EH{:}\,a$, ut emper, fiat. $t:y::e:a::z:r$, ac ſit r determi-ata, erit $za = re$; quare applicata conſtan-er z, in IM ad a, ut fiat curvilineum IMO quale omnibus za, applicetur r ad e in QZ, t fiat rectangulum $AGZQ$ æquale omnibus e. Hinc ſi AQ vocetur x, erunt omnia $e = rx$ ex Cap. II. §. 13. adeoque $rx =$ om-a $za =$ ſpatium IMO, ſed t ſeu $\frac{2y^3}{3rr}$: $y:z:r$, unde $3rz = 2yy$, & $z = \frac{2yy}{3r}$, ad omplementum parabolæ FOK; quod juxta præcedentia quadrabile eſt & $= \frac{1}{3}zy$; ſed, per modo aſſumta, omnia za ſeu $\frac{1}{3}zy$ æquatur om-omnibus re ſeu rx, quare $z = \frac{3rx}{y} = \frac{2yy}{3r}$,

Q 3 unde

unde tandem $9rrx = 2y^3$, quæ curvæ *ADE* æquatio quæſita eſt.

Quod autem ex hoc æquatione rurſum prodeat *TQ* ſeu $t = \frac{2y^3}{3rr}$ jam abunde notum eſt. Neque hanc methodum ſemper obtinere, modo modo omnia za ſeu ſpatium *FKO* quadrabile ſit, ulteriori demonſtrationum apparatu indiget.

§. 10. Alia curvarum longitudines explorandi methodus hoc nititur fundamento; quod ſcilicet, *Fig. LXXVIII.* integra tangens *BE* ſit æqualis omnibus u + omnibus λ, hoc eſt curvæ *ADE* + omnibus λ, uti §. 6. *N*°. *IV.* demonſtratum fuit. Unde, quia datâ curvâ *ADE* datur tangens *BE*, ſi darentur omnia λ, darentur omnia u ſeu longitudo curvæ.

Ut ergo indagentur omnia λ (vocatis, $TD:s$, $TQ:t$, $DQ:y$, infiniteſimis $QP:e$, $EO = ED:u$, $BT:i$, $BC:\lambda$,) quia *TC* eſt normalis ipſi *BE*, eſt $BE:BP::BT:BC$; hoc eſt $s+u+\lambda : t+e+i :: i:\lambda$; quare, rejectis rejiciendis, fiet $s\lambda = ti$; unde $s:t::i:\lambda$, ſit hoc ut determinata r ad z, erit $r\lambda = zi$.

Unde poſita z in *TG*, donec formetur curvilineum *AFB*; erit hoc æquale omnibus zi, quod æquale eſt omnibus $r\lambda$, unde $\frac{Omn.\ zi}{r} = Spat.$

$= \frac{Spat.\ AFB}{r} =$ omnia λ.

Quæ ſi ſubtrahantur ex tangenti BE reliquentur juxta modo tradita omnia u, ſeu longitudo curvæ ADE.

Coroll. I. Unde, ſi omnia zi ſeu ſpatium AFB ſit quadrabile, erit curva ADE in rectam mutabilis. Quare denuo ex curvilineorum omnium quadraturâ curvarum omnium longitudo data erit: exempla quilibet indagâre poteſt.

Coroll. II. Hoc addo, modum hinc non ita vulgarem derivari, quo ex dato quolibet curvilineo menſurabili AFB inveniri poteſt curva ADE, quæ longitudinis ſuæ menſuram patitur. Sit $AT: x$, $GT: z$, ac AGF quælibet curva, cujus ſpatium AFB menſurabile eſt, verb. gr. parabola, unde erit $rx = zz$, & juxta præcedentia ſpatium $AF\text{B} = \sqrt{\tfrac{4}{9} r x^3} =$ omnia zi; quocirca omnia $\lambda = \frac{ſpat.\ AFB}{r} = \sqrt{\frac{4x^3}{9r}} =$ omnia $r\lambda$.

Jam vero, ut curvæ ADE æquatio ex hiſce datis inveſtigetur, & longitudo tangentis DT; cum ſit $s:t::r:z$, hoc eſt DT: $Q::r:GT$, eò deductum eſt hoc problema, ut invenienda reſtet curva ADE, cujus tangens

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gens *DT* ad ſubtangentem T*Q* eam ſemper habeat rationem, quam r ad z.

Hoc autem, juxta tradita §. 101. Cap. de tang: effici poſſe ſequentia palam facient.

Sit igitur, uti loco mox allegato, $AQ:n$. atque, ductâ *AK* ipſi *A*B normali, ad eundem axem *A*T deſcribatur curva *AIN*, cujus applicata T$I = AL:f$, T$L:g$, hanc autem tangat recta *NIS*, ac ſit ſubtangens T$S:b$; erunt, (veluti §. 101. Cap. de tang.) ductis diagonalibus B*K* & T*L*, donec ſibi mutuo occurrant in E, puncta *D* & *E* in curvâ quæſitâ *ADE*; adeoque $g:x::s:t::i:\lambda$; propter triangulorum B*C*T, T*AL*, & T*QD* ſimilitudinem: ſed ex hypotheſi fuit $s:t::r:z$, quare & $g:x::r:z$, ergo $rx = gz$; jam verò ex ſuppoſito *AFG* eſt parabola, ideoque $rx = zz$, quo circa $g = z$, & $rx = gg$.

Sed, ob triangulum T*AL* in *A* rectangulum, eſt $gg = ff + xx$, unde prodit $rx - xx = ff$ æquatio curvæ *NIA*, quæ circulus igitur eſt; cujus ſubtangens T*S* eſt $b = \frac{2rx - 2xx}{r - 2x}$, unde *AS* ſeu $b - x = \frac{rx}{r - 2x}$.

Jam autem, ex §. 101. Cap. I. de tang: & ſubſequenti ſcholio, eſt $b - x : b :: g : s :: x : t :: f : y$; quare, in locum b & $b - x$ ſuffectis valoribus,

erit

it $\frac{rx}{r-2r} : \frac{2rx-2xx}{r-2x} :: g : s :: x : t :: f : y$; unde

nerget $t = \frac{2rx-2xx}{r}$, & $\frac{2rf-2xf}{r} = y$,

at, loco f ſurrogato ipſius modo invento va-

ore, cub. $\overline{r-x}$ in $4x = rryy$.

Tandem, cum ſit TA+AQ= QT, ſeu

+$n = t$, erit, valore ipſius t repoſito,

$x-2xx = rn$, quare per inventas duas haſce

quationes $rx-2xx = rn$ & cub. $\overline{r-x}$ in

$x = rryy$ ejecto x, emerget (ſalvo calculo) æ-

uatio ſequens $n^4 + 2rn^3 - 8rrnn + 2yynn$

-$6ryyn + y^4 = 0$. Quæ curvæ ADE rela-

onem ad axem AQ ſeu n, mediante ap-

licata QD ſeu y, exprimit.

Hinc jam quomodo tangens BE inveniatur

lus quam cognitum eſt: ut & hanc curvam

ADE æquari tangenti BE — $\sqrt{\frac{4x^3}{9r}}$, ſeu tan-

enti BE — omnia λ.

Quo pacto vero in aliis curvarum ADE &

AIN poſitionibus res hæc tractanda ſit, hoc per-

epto quilibet intelliget.

Coroll. I. quâ ergo ratione ex quovis curvi-

ineo quadrabili curva inveniri queat, cui recta

equalis exhiberi poteſt, ex corollario prægreſſo

atet, ſed ex §. 18. Cap. II. aliiſque, ex quâvis

urvâ geometricâ innumera curvilinea quadratu-

 ram

ram admittentia deducuntur, quamobrem ex quâvis curvâ geometricâ infinitas curvas in rectas mutabiles derivari poſſe hinc inferri poteſt.

Scholium. Cap. I. §. 101. methodum tradidimus, ex dato curvilineo *ABN* curvam (*Fig. LXXVIII.*) *ADE* deſcribendi, quæ eam inter ſe habent relationem, ut ſit perpetuo $AT : AL :: TQ : QD$, tum $AT : TL :: QT : TD$. Quod, cum illic loci multarum ſatis infiniteſimarum ope præſtitum ſit, quomodo expeditius paullo effici queat, hic oſtendiſſe forſan tyronibus non inutile fuerit, retentis ergo § ſuperioris ſymbolis, ſint infiniteſimæ $NM = KL = o$, $RL = \varphi$; eſt $b : f :: i : o$, unde $fi = bo$; tum $BA : AK :: RL : KL$, ſeu $x + i : f + o :: \varphi : o$, unde $xo = f\varphi$; tandem $ET : BT :: EL : RL$. Hoc eſt $s + u : i :: s - g + u : \varphi$; unde $s\varphi = si - gi$; ex quibus tribus æquationibus eliſis infiniteſimis orietur $s = \frac{bg}{b - x}$, quod, licet lemmatis in modum §. 10. præmitti potuiſſet, ne propoſitionum tamen ſe mutuo inſequentium filum abrumperetur, hoc potius inſequenti ſcholio comprehendere volui, inſimulque monere, me præcedenti propoſitione methodi magis univerſalitatem ſpectaſſe, quam poſitionis rectarum, flexuræque curvarum rationem habuiſſe; cum in ſingulis fere hujus problematis caſibus, ac juxta variam curvilinei dati *ABF* naturam, variæ ſint.

§. 11. Sit porro *Fig. LXXIX.* Curva *ADE* convexa verſus axem *AQ*, quam rectæ T*D* & B*E* tangant in *D* & *E*; vocenturque $TD : s$, $DQ : y$, $TQ : t$, $AQ : x$, ductâque B*C*, ut *CE* ſit æqualis B*E*, hoc eſt normali in *C* juxta lemma 33. dicantur infiniteſimæ $TB : i$, $TC : \lambda$, $DE = OE : u$; $DH = QP : e$, $EH : a$, $CB : \kappa$, erit

erit B$E = s+u-\lambda$, unde tangentium TD & BE differentia continua eſt: $u-\lambda$; ſi vero eædem tangentes TD, & BE conſtanter producantur in F & I, ita ut TF appellatâ h, linea BI ſit perpetuo $h+\lambda$; erit $DF = s+h$ et $EI =$ BE + B$I = s+u-\lambda+h+\lambda = s+h+u$; quare linearum DF & IE differentia erit = D$E = u$, lineæque EF & EI æquales; unde clauſo per FI triangulo EFI, erit FI utrique perpendicularis, ex lemmati 33, & parallela infiniteſimæ CB, ac ex T ducta TS itidem et ipſi CB æquidiſtans faciet BS = CT = λ; remanebitq. SI = TF = h; cum autem FI ſit quoque infiniteſima, quoniam CB talis eſt, continuatâ, ut in §. 6. tangentium ſerie, rurſus omnia FI abibunt in curvam AFI, cui omnes rectæ curvam ADE tangentes normales erunt; unde, ſi ambæ hæ curvæ in eodem puncto A deſinant, ſemper recta DF æquabitur curvæ AD, tum juxta §. 7. tum etiam, quia EF æquatur omnibus u, quoniam BE æquatur omnibus, $u-\lambda$, & BI omnibus λ, uti tangentes continuando apparebit.

Quare, ſi datâ curvæ ADE relatione, adeoque omnibus eam tangentibus, inveniri poſſet curvæ AFI natura, ſeu reſpectus ad axem AQ, inventa foret hac unica operâ omnium geometricarum curvarum longitudo.

Coroll.

Coroll. I. Hoc ſaltem hinc notum eſt, datâ quâvis curvâ *A* F I aliam *A D E* inveniri poſſe, cui recta æqualis dari poteſt.

Sit enim in curvâ *A* F I applicata F*G*: z, & ſubtangens *N G*: k, ipſa *G* T normalem curvæ *F* T definiens: l, intercepta *A G*: f, infiniteſima F*K* = *G V*: o, *K* I: φ, F I: μ; erit ob triangulorum *E C* B & *E* F I ſimilitudinem, *F* I: *F E*:: *C* B: *C E*, ſeu $\mu : h+s+u :: \varkappa : s-\lambda+u$, unde, rejectis rejiciendis per lemm. 10. erit $\frac{s\mu}{h+s} = \varkappa$. Sed & T*G*: *G* F:: T*C*: *C* B, hoc eſt $l : z :: \lambda : \varkappa$; unde $\frac{z\lambda}{l} = \varkappa$, ideoque $\lambda = \frac{ls\mu}{hz+sz} = \frac{li}{h}$, (nam T F, h: T*G*, l:: T B, i: *C* T: λ,) unde denuo $\mu = \frac{hzi+szi}{hs} = \frac{ho}{z}$ (nam T F: F*G*:: F I: F*K*,) quare, ſi jam alia poſſit inveniri æquatio infiniteſimas, i & o involvens, utraque earum elideretur, hunc infinem, applicatâ *G T* in *G L*, & *V* B in *V M*; donec per puncta *L* & *M* deſcribi poſſit nova curva *L M*, cujus ſubtangens *G X* vocetur g, erit rurſum *L R* = *G V* = o, & *M R* = $i-o$; jam ex paſſim notis eſt *X G*: *G L*:: *L R*: *R* M; ſeu $g : l :: o : i-o$, unde $lo = gi - go$; quæ æquatio eaſdem cum ſuperiori infiniteſimas continet, quare

quare ex utrâque emergit tandem $o = \frac{gi}{l+g} = \frac{bzi + szzi}{bbs}$, & eliſa i tandem $s = \frac{blzz + bgzz}{gbb - lzz - gzz}$, quia autem $bb - zz = ll$ fit $s = \frac{blzz + bgzz}{gll - lzz} = TD$.

Hâc autem inventâ notum erit punctum D in curvâ ADE, unde cæteræ ejus proprietates, & ratio ad axem AQ deduci poſſunt.

Hoc addo, cum rectam DF curvæ AD æquari jam ſæpe oſtenſum ſit, erit curva $AD = s + b = \frac{b^3 g}{gll - lzz}$; ut vero valor ipſius s in aliis terminis reperiatur, quia $NG : GF :: GF : FT$ ſeu $k : z :: z : l$, erit $kl = zz$; quo valore in locum ipſius zz ſurrogato prodibit $s = \frac{bgk + blk}{gll - kll}$.

Schol. I. Atque hinc ad ingenioſſimam *Nob. Hugenii* curvarum evolutionem manu quaſi ducimur; ſi enim recta FE inſtar fili flexilis eſſe concipiatur, ac curvæ EDA circumpoſita, deinde extremitate ſua F à puncto A exire, erit linea ADE evoluta, ac AFI ex evolutione deſcripta; illa data, retentiſque ſymbolis modo aſſumtis, invenietur s ſeu TD eadem cum eâ, quam ſtatim exhibuimus.

§. 12. Ut vero pateat tyronibus modus, quo curvæ ADE relatio ad axem AQ inveniri poteſt;

teſt; hoc eſt inter $DQ:y$, & $AQ:x$; erit primo $TF:FG::TD:DQ$, ſeu, $h:z::s:y$, unde $zs = hy$, deinde $FT:TG::TD:TQ$ ſeu $h:l::s:\frac{ls}{h} = TQ$, cui ſi addas $TG+GA = l+f$, erit igitur $QT+TG+GA = \frac{ls}{h}+l+f = x = AQ$, unde repoſito in utraque æquatione $s = \frac{hgk+hlk}{gl-kl}$ emergent pro priori æquatione $gly - kly = gkz + klz$; & pro ſecundâ $gx + fk = gk + fg + gl + kx$.

Cum autem, datâ relatione curvæ AFI, omnes litteræ exceptis, x & y poſſint ad AG ſeu f reduci, hâc eliſâ remanebit æquatio inter x & y, ſeu curvæ ADE, quæ quærebatur.

Poſſent circa methodum evolutionum inverſam, ſeu ex evolutâ deſcriptam inveniendi, hoc loco quædam adjungi: cum autem (*Fig. LXXVI*) datâ evolutâ ADE, ejus deſcriptæ AI proprietates non niſi rectis IE, $8D$, &c. determinari queant, neque hæ, niſi cognitâ curvæ ADE longitudine, cognoſcantur; ſolutionem hujus problematis curvarnm rectificationem, methodum ſaltem tangentium inverſam, involvere, eo ipſo manifeſtum eſt; quæ, cum non exiguis adhuc difficultatibus premantur, pluribus hiſce addendis ſuperſedeo.

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CAPUT. VI.

De *Menſura Curvilineorum ex Centris Gravitatum.*

§ 1. CElebre, tum hoc, tum antiquis temporibus, menſurationis fundamentum fuit, *Centri gravitatis conſideratio*; quæ eodem cum ſuperioribus fonte manans, niſi materiæ dignitas particularem ſibi diſcuſſionem depoſcere videretur, jam inter tertii capitis corollaria locum invenire potuiſſet.

Reſumtis enim Cap. III. §. 4. &c. tum figurâ *V. N°.* I. tum ſymbolis, erat cylindrici plano AOF inſiſtentis truncus inferior, poſito angulo *R*B*V* ſemirecto, æqualis $d+\frac{1}{2}y$ in ye; cum autem ye ſit rectangulum Q*DHP*, erit (factâ $\zeta\theta = \frac{1}{2}y$ & $\beta\zeta = d$.) juxta mechanicæ regulas $d+\frac{1}{2}y$ in ye momentum rectanguli Q*DPH* ad axem motus *C*B appenſi; ſi quidem momenta gravium exprimuntur per rectangula ex mole corporum in reſpectivam ab axe diſtantiam ductâ. Jam autem omnia ye conſtituunt figuram AOF, unde momentum omnium ye in ſuis reſpectivè diſtantiis ad axem *C*B appenſorum, æquatur momento totius figuræ

AOF

A O F in diſtantia $\xi\omega$ ad eundem axem *C B* appenſæ, ſi punctum ω pro centro gravitatis totius figuræ *A O* F aſſumatur. Quod mechanicæ principia non ignorantibus ſatis notum eſt.

Unde ſi figura *A O* F vocetur f, & $\xi\omega$ diſtantia centri gravitatis ω ab axe motûs dicatur: k, erit $kf =$ omnia $d+\frac{1}{2}y$ in ye.

Quare expoſitis trunco reſecto, ſeu omnib. $dye+\frac{1}{2}yye$, figura A*OF* ſeu f, & diſtantia centri gravitatis ejuſdem k, ex datis duobus datur tertium.

§. 2. Quomodo autem hæc ungulis vel ſeminormaliter, vel alio quovis angulo per A O reſectis applicari queant, quilibet ſuo marte indagabit: cum nihil hic diverſum occurrat.

§. 3. Hoc addo, cum ſit $fk =$ omnia $dye+\frac{1}{2}yye$, truncum inferiorem æquari cylindrico, cujus baſis figura A O *F* ſeu f, & altitudo æqualis lineæ $\xi\omega$, quâ ω centrum gravitatis figuræ ab axe motus *C B* diſtat.

§. 4. Si vero angulus *R* B *V* non ſemirectus, ſed alius quivis ſupponatur, ac cylindricus baſi A O F inſiſtens habeat altitudinem *S* T ſeu *D* δ ſeu h, ac $\beta\mu$ ſit b, erit totus cylindricus æqualis omnibus hye, ac truncus inferior $=$ omnib. $\frac{h}{b}\,\overline{dye+\frac{1}{2}yye}$, quare truncus ſuperior

æqua-

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æquabitur omnibus, $hye - \frac{h}{b} dye - \frac{h}{2b} yye$
ſeu $\frac{h}{b}$ in $b - d - \frac{1}{2}y$ in ye;

Cum autem $b - d - \frac{1}{2}y$ ſit æqualis lineæ $\theta\mu$, quâ centrum gravitatis rectanguli *QPDH* diſtat à lineâ *TV*, erit linea $\psi\omega$, ſeu diſtantia centri gravitatis totius figuræ *AOF* ab axe *TV* $= b - k$, unde juxta §. I. erunt omnia $b - d - \frac{1}{2}y$ in $ye = bf - kf$; adeoque truncus ſuperior $\frac{h}{b}$ in $bf - kf$ ad truncum inferiorem kf ut $b - k$ ad k, ſeu ut $\psi\omega$ ad $\omega\xi$, hoc eſt ut partes lineæ $\xi\psi$ reſectæ a centro gravitatis figuræ.

§. 5. Quod ſi hæc ad ſolida, tum conöidica, tum annularia transferre placet, patet ex § 20. &c. Cap. III. ſolidum tubulatum ex figura *AOF* circa axem *CB* rotatum, æquari omnibus, $\frac{2dyec + yyec}{2r}$; quod ſolidum ſi vocetur: s, quia omnia $dye + \frac{1}{2}yye$ ex §. 3. ſunt æqualia kf; erunt omnia $\frac{dyec + \frac{1}{2}yyec}{r}$ ſeu corpus hac ratione genitum $= \frac{kfc}{r} = s$.

§. 6. Unde, ad ſimilitudinem §. 3. ſolidum omne annulare ſeu s æquale eſt cylindrico, cujus baſis eſt f ſeu figura *AOF*, & altitudo

$\frac{kc}{r}$ ſeu circumferentia circuli, cujus radius eſt $\zeta\omega$ ſeu k, aut diſtantia centri gravitatis figuræ ab axe motus.

§. 7. Aſſumatur jam ſecundum ſolidum annulare $= p$; & figura, cujus circumvolutione genitum eſt, ſit $= g$; diſtantia centri gravitatis figuræ ab axe rotationis $= h$; erit ſolidum hoc ſecundum ad ſolidum primum, hoc eſt, p ad s uti $\frac{hgc}{r}$ ad $\frac{fkc}{r}$, quare $shg = pfk$; unde ſi tres exponantur rationes ſolidi ad ſolidum ſeu, s ad p, figuræ ad figuram ſeu f ad g, & diſtantiæ centri gravitatis in primâ figurâ ab axe motus, ad ſimilem diſtantiam in ſecundâ ſeu k ad h, ex cognitis duabus cognoſcitur tertia.

§. 8. Hinc ſi truncus quilibet aut ungulæ aut alteri trunco æqualis ſupponatur, multum operæ in quibuſdam curvilineis quadrandis conferet, ſi centrum gravitatis cognitum ſit. Applicatis enim (*Fig. LX.*) ad eundem axem AO figuris AFO, AGO; vocatiſq. $DQ:h$, & $RD:f$, $QP:e$, erit truncus inferior cylindrici baſi AFG inſiſtentis, ac per AO ſeminormaliter reſecti, æqualis omnibus $\frac{2hfe + ffe}{2}$ qui ſupponatur æqualis omnibus $\frac{yye}{2}$, ſeu ungulæ ſeminormaliter per AO reſecti cylindrici

in

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i basi AIO, ſi nimirum QI ſit continuo $= y$, erit perpetuo $h + \frac{1}{2}f$ in $fe = \frac{1}{2}y$ in ye; hoc eſt juxta mechanicæ theoremata.

Omnium fe ſeu trapeziorum $SRED$ momenta ad axem AO æqualia momentis omnium ye ſeu trapeziorum $PHIQ$ ad eundem axem.

Sit autem planum curvilineum $AFG = k =$ omnia fe, ejuſq. centri gravitatis Y diſtantia ZY ab axe $= l$; nec non planum curvilineum $AIO = g =$ omnia ye, ejuſq. centri gravitatis ab axe AO diſtantia $NV = n$; erunt, juxta hujus, omnia $\frac{2hfe + ffe}{2} = kl$, & omnia $\frac{yye}{2}$ $= gn$, adeoq. $gn = kl$.

Unde cognitâ figuræ AGF menſurâ, ejusque centro gravitatis; & centro gravitatis plani AIO cognoſcitur plani AIO magnitudo.

Quod ſi præterea ad eundem axem applicetur figura AMO, ſitque $IK = q$, & truncus in plano AFG jam æqualis ſtatuatur non ungulæ in AIO, uti mox factum fuit, ſed trunco in baſi $AHOM$ per AO reſecto ſeminormaliter, hoc eſt, ſi fiat ſemper $\frac{2hfe + ffe}{2} = \frac{2yye + qqe}{2}$, erit ergo $h + \frac{1}{2}f$ in $fe = y + \frac{1}{2}q$ in q; & TX diſtantiâ centri gravitatis X figuræ $AHOM$ vocatâ n, ac ipſâ figurâ $AHOM$

 ſeu

ſeu omnia $qe = g$; erit denuo $gn = kl$, ut ſupra.

Ac ſi QR ſeu $h+f$ vocetur: z; nec non QK ſeu $q+y$ dicatur: s; erit in primo caſu $rr - hh = yy$ & in ſecundo $rr - hh = ss - yy$.

Unde primum & ſecundum *Cl. Sluſii* lemma ortum vel duxiſſe, vel ducere ſaltem poſſe videntur. Particularia ſatis egregia & multa hinc in appendice *Meſolabi* derivavit Eruditus Author.

§. 9. Cum autem ex §. 9. & 24. Cap. III. generis alîus tum truncorum, tnm ſolidorum infiniteſimæ deducantur, eadem patet ad inveſtigandam curvilineorum quadraturam ſemita, poſitâ enim (*Fig. LXII.*) infiniteſimâ $QN: e$, applicata $AN: y$, intercepta $RN: x$, ac determinata $BR: b$, erit truncus cylindrici baſi RAF inſiſtentis ac per HB ſeminormaliter reſecti æqualis omnibus $bye + xye$, juxta Cap. III. §. 10. hoc eſt omnibus, $b+x$ in ye, ſeu omnibus, $b+x+\frac{1}{2}e$ in ye, per lemm. 10. cum autem ye ſit rectangulum $PQNA$, ac KC ab ipſius centro gravitatis C ad axem HB ducta ſit $= b+x+\frac{1}{2}e$, erit $b+x+\frac{1}{2}e$ in ye ſeu $bye+xye$ momentum parallogrammi $PQNA$ ad axem motus BH.

Jam autem rurſum ex Mechanicâ momentum omnium ye in reſpectivis ſuis diſtantiis KC æquantur momento totius figuræ RAF in diſtantia FL ad axem motus appenſæ, ſi L to-

tius

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us figuræ centrum gravitatis ſupponatur. Fiat rgo figura $RAF = f$, & $FL = d$, erunt omia, $bye + xye = fd$, hoc eſt truncus modo ictus æqualis cylindrico in Baſi RAF & altiudine KC; unde rurſum eadem ac modo recenta deduci poſſunt ac hiſce ad figuras §. 33. & 34. ap. III. applicatis, lemmata *Cl. Sluſii*, terum & quartum, in curvilneorum menſurâ ſas magni momenti ortum ſuum debere videnr, ſed nequid nimis.

§. 10. Notetur hinc demonſtrari, (reſum *Fig. LX.*) momenta figurarum ad axem AO iter ſe eſſe (retentis §. 8. ſymbolis) uti omnia $\frac{2bfe + ffe}{2}$ ad omnia $\frac{2qye + qqe}{2}$, ſeu diviſone factâ per e conſtantem, ut omnia $b + \frac{1}{2}f$ f ad omnia $y + \frac{1}{2}q$ in q, hoc eſt ut momen omnium linearum RD in una figura AFG, l momenta omnium linearum KI in alia figu $AHOM$; quod a plurimis aſſumitur.

§. 11. Similia hiſce circa ſuperficies curvas emouſtrari poſſent, quæ cum ex modo traditis ivi operâ colligantur, breviter perſtringam, ſumtis igitur Cap. III. §. 2. & 3, 4. ſymbolis ac *ig. LXIX.* erit $du + yu$ trunci inferioris ſupercies curvæ AEO inſiſtentis, juxta ibidem trata. Hæc autem æquatur momentis omnium ctularum DE, curvam $ADEO$ compo-

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nentium, ac ad axem *B C* appenſarum; ſi enim *D E* ſeu *u* medio ſui puncto *F* ſeu centro gravitatis appendatur ad axem *B C* in diſtantia *F L*, quæ æqualis eſt *C Q* + *Q D* + *Y F* ſeu $d + y + \frac{1}{2} a$, erit ipſius *D E* momentum ex mechanicis ſemper $dy + uy + \frac{1}{2} au$, hoc eſt, per lemm. 10, $= dy + uy$; cum autem momenta omnium *D E* curvam *A D E O* conſtituentium æquentur momento totius curvæ in diſtantia *X Z* ad eundem axem appenſæ, ſi quidem *X* pro gravitatis centro totius curvæ ſupponatur, erunt vocatis, curva *A D E O*: k, & *X Z*: l, omnia $du + yu = kl$.

Adeoque trunci ſuperficies curvæ inſiſtens æqualis ſuperficiei cylindricæ, ſeu rectangulo kl ex curva k & diſtantia centri gravitatis curvæ ab axe ſeu l, & poſitâ $d = 0$, erit ungulæ quadrantalis baſi *A E O* inſiſtentis ſuperficies = omnia yu æqualis momentis omnium *D E* ad axem *A O*, ſeu omnium $y + \frac{1}{2} a$ in u, ſeu *Q D* + *F Y* in *D E*; unde poſita curvæ integræ longitudine, k, & ejus centri gravitatis ab axe *A O* diſtantiâ l, ſequitur rurſum omnia yu eſſe $= kl$.

§. 12. Quod ſi hæc ad ſuperficies rotatione genitas extendere placeat; erunt omn. $\frac{dyc + yuc}{r}$ æqualia ſuperficiei ex curvæ *A D E O* rotatione circa axem *B C* genitæ, ex Cap. IV.

§. 14.

§. 13. Hæc, juxta præcedentem, æquatur $\frac{kc}{}$; hoc eſt ſuperficiei cylindricæ, cujus baſis e curva k, & altitudo eſt circumferentia circli radio ZX ſeu l deſcripta.

§. 14. Quod ſi ſecunda curva exponatur, cujus longitudo $= g$, diſtantia centri gravitatis a axe motus $= n$; erit ſuperficies ex hujus curvæ circa hunc axem rotatione genita per præcedentem $= \frac{gnc}{r}$; & hæc ad ſuperficiem primam $\frac{klc}{r}$ ut gn ad kl; unde ex tribus rationibus, ſuperficiei ad ſuperficiem, curvæ ad curvam, & diſtantiæ centri gravitatis in prima curvæ ad eandem diſtantiam in ſecunda, datis duabus dabitur tertia.

Plura hinc nec exiguæ notæ particularia ſubſequentur, centriq. gravitatis inveniendi multifaria methodus, ſi quis hæc penitius proſecutus fuerit, hæc autem apud eruditiſſimos authores *Walliſium*, *Steph. ob Angelis*, aliosque in compendio reperient, quibus doctiſſima eorum monumenta evolvere volupe fuerit.

Appendicula.

QUo vero hæc facilius in uſum deducantur, non inutile forſan fuerit planorum, ſolidorumq., nec non ſuperficierum, ac linearum curvarum exponentia algebraica in unam quaſi tabellam, contraxiſſe.

Sit ergo, Figurâ LXXX. curvilineum AOF, eique

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circumſcriptum rectangulum $AOFM$, fiatque BC ipſi AO ac NI ipſi AM parallela, vocatiſq. ut ſemper, $QD:y$, $QA:x$, $CQ:d$, $NA:b$, $OF:p$; infiniteſimiſque $DH:e$, $HE:a$, $ED:u$, ſit CD ſeu $d+y = z$; r ſit radius circuli, cujus circumferentia: c.

Exprimentur:

Rectangulum $AFOM$ per omnia pe.

Spatium Curvilineum AFO per omnia ye, §. 4. Cap. II.

Columna aut priſma in baſi AOF, cujus altitudo CQ, per omnia dye §. 2. Cap. III.

Truncus inferior hujus columnæ, ſi per BC ſeminormaliter ſecetur per omnia $\frac{2dye+yye}{2}$, §. 4. Cap. III. aut per omnia $\frac{zz-dd \text{ in } e}{2}$, § 4. Cap. III.

Momentum Figuræ AOF ex centro ſuæ gravitatis ad axem BC appenſæ per eadem, §. 1. Cap. VI.

Truncus inferior ejuſdem columnæ per NI ſeminormaliter reſectæ per omnia $by+xy$ in e §. 10. Cap. III.

Momentumq. Figuræ AOF ex centro ſuæ gravitatis ad axem NI appenſæ per eadem, §. 9. Cap. VI.

Trunci inferiores dictæ columnæ alio quolibet angulo per BC aut NI reſectæ, in primo caſu per omnia $\frac{2dy+yy}{2}$ in $\frac{eh}{b}$ §. 4. Cap. III. aut per omnia $\frac{zz-dd}{2}$ in $\frac{eh}{b}$ §. 4. Cap. III. in ſecundo caſu per omnia $by+xy$ in $\frac{eh}{b}$ §. 11. Cap. III. pro variâ ratione ipſius b ad h.

Ungula ſemiquadrantalis ejuſdem columnæ per AO ſeminormaliter reſectæ per omnia $\frac{yye}{2}$, §. 7. Cap. III.

Momentumq. figuræ AOF ad axem AO ex centro gravitatis ſuæ appenſæ per eadem, §. 2. Cap. VI.

Truncus ſeu ungula ſemiquadrantalis ejuſdem columnæ per AM reſectæ per omnia yxe, §. 9. Cap. III.

Mo-

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Momentumq. figuræ AOF ad axem *AM* ex centro gravitatis ſuæ appenſæ per eadem, juxta §. 9. Cap. VI.

Ungulæ truncique ſemiquadrantales ad alium angulnm reſectæ per omnia $\frac{byye}{2b}$, §. 6. Cap. III. aut per omnia $\frac{byxe}{b}$ §. 11. Cap. III.

Solidum rotatione genitum ex rectangulo AFOM circa AO ſeu cylindrus per omnia $\frac{ppec}{2r}$, §. 14. Cap. III.

circa *AM* per omnia $\frac{pxec}{r}$ §. 24. Cap. III.

Ex curvilineo *AO*F circa AO ſeu conoides, per omnia $\frac{yyce}{2r}$ §. 17. Cap. III.

Ex eodem curvilineo circa A *M* per omnia $\frac{yxec}{r}$ §. 24. Cap. III.

Ex AOF circa B*C* ſen annulare per omnia $\frac{2dy+yy}{2}$ in $\frac{ce}{r}$ §. 20. Cap. III. ſeu omnia $\frac{zz-dd}{2}$ in $\frac{ec}{r}$ §. 20. Cap. III.

Ex AOF circa N*L* ſeu tubulatum per omnia $by+xy$ in $\frac{ec}{r}$ §. 27. Cap. III.

Superficies Columnæ in altitudine C*Q* curvilineo AOF inſiſtentis, qua parte curvæ AF ſuperſtat per omnia *du* §. 1. Cap. IV. qua parte rectæ AO per omnia *de* §. 1. Cap. IV. qua parte rectæ OF omnia *da* §. 1. Cap. IV. Hinc *integra columnæ* ſuperficies per omnia $du+de+da$ §. 1. Cap. IV.

Superficies Ungulæ ſemiquadrantalis per AO reſectæ per omnia *yu* §. 4. Cap. IV.

Momentnm curvæ AEF ex centro ſuæ gravitatis ad axem AO appenſæ per eadem §. 11. Cap. IV.

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Superficies trunci inferioris curvæ inſiſtens per BC ſeminormaliter reſecti, quâ parte curvæ AEF inſiſtit per omnia $du + yu$, ſeu zu §. 3. Cap. IV.

Ac *momentum Curvæ* AEF ex centro gravitatis ſuæ ad axem BC appenſæ per eadem §. 11. Cap. VI.

Superficies rotatione genita exceptis baſibus, ex rectangulo AMFO circa AO ſeu cylindri per omnia $\frac{pec}{r}$ §. 9. Cap. IV.

Ex curva AEF circa AO ſeu conoidis per omnia $\frac{yuc}{r}$ §. 11. Cap. IV.

Ex Curva AEF circa BC ſeu pars ſuperficiei ſolidi annularis per omnia $du + yu$ in $\frac{c}{r}$ ſeu $\frac{zuc}{r}$ §. 12. Cap. IV.

Ex totâ figurâ AOF ſeu integra ſuperficies ſolidi annularis, exceptâ baſi, per omnia, $\frac{zuc}{r} + \frac{dec}{r}$ §. 13. Cap. IV.

Ipſa Curva ADEF per omnia u §. 1. Cap. V.

C A-

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CAPUT. VII.

De Maximis & Minimis.

Cum omnis æquatio unam pluresve indeterminatas continens per curvæ unius pluriumve locum geometricè deſignari poſſit, ita ut ſingulæ occurrentes indeterminatæ vel tanquam applicatæ diverſarum linearum ad eundem axem poſitæ conſiderari poſſint, vel etiam una ex illis interceptam quandoque repræſentare; plurimæque oriantur curvæ (*Fig. LXXXII.*) ADEZ, quarum, ſi ad rectam AZ referantur, applicatæ ex A in *I* perpetuo creſcunt, ex *I* in *K* per infiniteſimum ſpatium diſtantes ſubſiſtunt, & æquales ſunt, ac ex *K* in Z denuo, manente eâdem curvæ æquatione, decreſcunt; patet inter varios applicatarum hiſce in curvis valores *extremum* unum dari, cum nimirum nec creſcunt nec decreſcunt; & hoc quidem in caſu *maximum*, cæteris omnibus hinc inde ab hoc *extremo* ſeu maximo deficientibus.

Simile quid obſervatur in aliis curvis, aut etiam in hac eâdem, ſi non ad axem AZ. verum ad rectam YX applicetur, quo quidem caſu applicatæ ex *Y* in *V* decreſcunt, in *V* & *S* per infiniteſimam diſtantiam disjunctæ ſibi mutuo æquantur & ſubſiſtunt, ac ex *S* in *X* rurſum augentur; unde applicatarum punctis *V* & *S* inſiſtentium valores omnium ad hanc curvam pertinentium *minimi* ſunt.

Ex quibus conſequitur, omnem æquationem indeterminatas quotcunque continentem, ſi unica ex illis ad valorem *extremum* determinari debeat, ad quendam ex modo dictis curvarum locis, & caſum quo nec creſcunt nec decreſcunt, adeoque ſibi invicem æquales ſunt duæ ſeſe proxime excipientes applicatæ, referri poſſe. Ad curvam ſcilicet verſus axem ſuum concavam ſi *maximum*, convexam vero ſi *minimum* expetatur.

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Ex qua *extremi* & linearum, quæ curvas in reſpectivis punctis tangunt, proprietate ſequens in *maximi* & *minimi* determinatione emergit fundamentalis canon.

§. 1. Si curvæ cujuſdam *ACDEHZ* ſubtangens *BT* ſit omni aſſignabili major aut infinita; erunt applicatæ B*C* & *ID* (quæ mediante tangenti *TC* ad ſubtangentem expoſitam *TB* relativâ, ſeu per ipſi coincidentem curvæ infiniteſimam *DC*, conjunguntur) ſibi mutuo æquales, adeoque inter reliquas curvæ applicatas *maximæ* aut *minimæ*.

Demonſtratio ſit *BT*:*t*, *BC*:*y* applicata, *CR*:*e* infiniteſima, *DR*:*a* applicatarum incrementum aut decrementum; erit, juxta proprietatem omnibus curvis communem, $a = \frac{ye}{t}$; poſitâ vero *t* infinitâ $= km$ & *e* infiniteſimâ explicatâ per $\frac{r}{m}$ erit $a = \frac{ry}{kmm}$, quod per lemm. 10. nihilo æquivalet.

Quare, cum hoc in caſu differentia applicatarum nulla ſit, eo ipſo nec creſcunt nec decreſcunt, ſed ſibi mutuo æquantur, adeoque juxta modo præmiſſa *maxima* ſunt, aut *minima*. *q. e. d.*

§. 2. Unde tandem manifeſtum eſt; ſi expoſitæ æquationis indeterminata quælibet ad *maximum* aut *minimum* determinanda veniat, æquationem hanc, tanquam ex curvæ convenientis

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entis (in qua nimirum determinanda ad extremum quantitas applicatæ vicem ſuſtinet) naturâ profluentem conſiderandam eſſe; cujus curvæ ſubtangentialis æquatio ex ſupra traditis regulis eſt inquirenda, ac ipſa ſubtangens ſupponenda æqualis cuidam infinito: unde juxta Schol. General. *N°. XII. extremi* quæſiti ſeu *maximi* aut *minimi* determinatio ſequetur.

§. 3. Sint, ut exemplis hoc illuſtremus, x intercepta, tum z, y, f, ad eandem applicatæ; curvarumque hinc genitarum ſubtangentes reſpectivæ, l, t, n. ita ut l ad z, t ad y, n ad f referatur; d, b, c, r, s, ſint quantitates determinatæ; *extremum* per applicatam y perpetuo denotetur.

I. Caſus, *cum termini algebräici extremum aliquod deſignantes non niſi unam nicognitam quantitatem continent.*

Sit $rrx^3 - x^5 + bbccx =$ *alicui extremo*, vocetur hoc y^5 vel rry^3 aut $bbccy$ ſimilive modo, quo poſſit æquationem curvæ repræſentare; habebitur curvam hanc exprimens æquatio, $rrx^3 - x^5 + bbccx = y^5$, cujus æquario ſubtangentialis juxta præcedentia, eſt, $3rrxxt - 5x^4t + bbcct = 5y^5$; in quâ ſi t ponatur omniaſſignabili major ſeu infinita, juxta Scholii General. *N°. XII.* omnes termini infinito vacui, ut $5y^5$, evaneſcent: unde, cæteris per t diviſis,

ſis, emerget $3rrxx - 5x^4 + bbcc = 0$, quæ eſt æquatio quæſita *maximum* aut *minimum* determinans.

II. Caſus, *cum æquatio duas continet in eodem termino indeterminatas* x *&* y, *quarum altera* y *ad extremum aliquod eſt determinanda.*

Sit æquatio expoſita, $r^4 - x^3y + bbxx - cxyy = 0$; erit curvæ hancce æquationem conſtruentis æquatio ſubtangentialis $-3xxyt + 2bbxt - cyyt = 2cxyy + x^3y$; & poſitâ t infinitâ, termini infinito vacui ex Schol. General. *N°. XII.* nihilorum ad inſtar rejici poſſunt, unde, reliquis per t diviſis, oritur æquatio ad extremum determinata $-3xxy + 2bbx - cyy = 0$; ex quâ, ſi ad expoſitam applicetur, tum ipſius x, tum ipſius y valor, uti notum eſt, indagari poteſt.

§. 4. Quoniam jam in utroque hoc caſu æquationem ad *extremum* aliquod determinatam, non niſi termini ſubtangentem t continentes, cæteris evaneſcentibus, ingrediuntur: patet in expoſitâ qualibet æquatione, omiſſâ indeterminatâ y, quæ *extremum* ex hypotheſi deſignat, ſingulos terminos reliquam indeterminatam x continentes per numerum poteſtatem ipſius x in quolibet termino exponentem, multiplicandos eſſe, & hoc facto, ex iis terminis hoc modo mul-

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multiplicatis componi æquationem ad valorem *extremum* ipſius y determinatam.

Quod exhibita modo exempla hoc ritu tractando innoteſcit. Demonſtratio ex Cap. I. §. 17. n. 5. haurienda.

Schol. Cum autem Nob. *Huddenii extremorum* ſeu *maximi* & *minimi* methodus, tum hâc, tum aliis hactenus poſt ipſam inventis, non paucis in caſibus, latius pateat ac magis comprehenſiva exiſtat, eam quidem ob cauſam, quod in indeterminatarum poteſtatibus non adſtricta, aſſumtâ quâlibet progreſſione arithmeticâ, intentum aſſequatur; tum primario, quod hujus ope terminus quicunque per cyphram multiplicari, adeoque ex datâ æquatione tolli poſſit; unde hinc promanantes problematum ſolutiones generaliores multo ac infinitis quandoque caſibus ſufficientes evadunt.

Sit Verb. Gr. æquatio ad circulum $2rx - xx = yy$, cujus applicatarum maxima expetitur; habebitur ex præcedenti $2rx - 2xx = 0$ & $r = x$; quod & ex methodo Huddenianâ, uti notum, conſequitur: ſi vero alia juxta eandem methodum aſſumatur progreſſio, ac terminus $2rx$ per cyphram multiplicetur, obtinebitur mox $yy = xx$, ſeu $y = x$.

Quod maximum in circulo ſine relatione ad ipſius diametrum, adeoque & locum, ad quem futuræ ſunt infinitorum circulorum eodem vertice deſcriptorum applicatæ maximæ, primâ inſtantiâ determinat. Operæ pretium mihi facturus videor, ſi methodi modo traditæ, cum admirandi tanti Viri invento convenientiam, inſimulq. ipſius hoc fundamento innitentem demonſtrationem aperiam.

§. 5. Sit in eum finem æquatio; $x^5 - bx$ [illegible] $yx^3 - ccyxx + bbccx + b^5 = 0$; oportet

valo-

valorem ipſius y ad maximum aut minimum determinare.

Aſſumtis, d, & r, quarum utraque conſtantis quidem magnitudinis ſed pro arbitrio ſumendæ ſupponitur, ac juxta calculi conditionem nunc negativos, nunc affirmativos numeros ſignificare: ducatur æquatio primò in d, ac deinde in x^r, ſeu ſimul & ſemel in dx^r, numero r ipſius x poteſtatem deſignante; obtinebitur $dx^{5+r} - bdx^{4+r} - dyyx^{3+r} - dccyx^{2+r} + bbccdx^{1+r} + db^5 x^r = 0$. in quâ, ſi juxta præcedentem, ſinguli termini x continentes per numerum poteſtatem ipſius x exponentem multiplicentur, emerget $\overline{5+r}\,dx^{5+r} - \overline{4-r}\,bdx^{4+r} - \overline{3-r}\,dyyx^{3+r} - \overline{2-r}\,dccyx^{2+r} + \overline{1+r}\,bbccdx^{1+r} + rdb^5 x^r = 0$, pro æquatione ad extremum determinatâ; quæ ideo per x^r rurſum diviſa exhibebit:

$$\overline{5+r}\,dx^5 - \overline{4-r}\,bdx^4 - \overline{3-r}\,dyyx^3 - \overline{2-r}\,dccyx^2 + \overline{1+r}\,dbbccx + rdb^5 = 0.$$

Adeoque æquationem expoſitam quâlibet progreſſione arithmeticâ juxta leges *Huddeniani* theorematis, multiplicatam.

Coroll.

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Coroll. Patet hinc I. quemlibet expoſitæ æquationis terminum, Verb. Gr. yyx^3, per ciphram poſſe multiplicari, adeoque ex æquatione ad *extremum* determinatâ tolli, ſit enim $3 - r = 0$ erit $r = -3$. tum II, quemlibet e reſiduis, Verb. Gr. bx^4, poſſe quolibet numero tum affirmativo tum negativo, ut s, multiplicari, cæteris ex eo determinationem accipientibus; ſit enim $-\overline{4-r}\,d = s$, erit (poſito loco r invento valore -3) ipſa $s = -7d$, adeoque $d = \frac{-s}{7}$. quæ quidem, ni fallor, ea ſunt, quæ methodus *Huddeniana* requirit.

§. 6. III. Caſus, *Cum æquatio plures, imo quotlibet, continet indeterminatas, quarum unica* y *ad extremum determinanda eſt.*

Sit æquatio, $yyf - byz + ccb = 0$; oportet ipſius y valorem extremum definire. Quamobrem ex §. 2. ac retentis §. 3. ſymbolis, juxta Cap. I. §. 35. reperiatur (conſideratis ſcilicet indeterminatis, tanquam curvarum ad eundem axem deſcriptarum applicatis) ipſius æquatio ſubtangentialis: $2yyfnl + yyftl - byztn - byzln = 0$; & poſitâ t infinitâ, ac cæteris ex Schol. General N°. XII. evaneſcentibus, emerget æquatio quæſita: $yyftl - byztn = 0$, ſeu $yfl - bzn = 0$.

Schol. Et hæc quidem, cum nullæ amplius ſuperſunt

æquationes, quæ inderminatas f & z ad y aut interceptam communem x limitant; quæ ſi in promtu fuerint, etiam ſubtangentes reſpectivæ n & l ad eaſdem y aut x referri poſſunt, adeoque extremum per ſuperiores caſus omnino erit determinabile. Elegans Celeberrimi *Leibnitii* exemplum, quem modo recenſuimus caſum, elucidabit; quod Vir erudititiſſimus per quantitatum differentialium proprietates reſolvit; nos autem ex ſubtangentis valore infinito, præcedentium in morem, deducemus; cum Cap. I. §. 35. methodum huic negotio inſervientem ſatis, ni fallor, expeditam ſuppeditet.

§. 7. Sit (*Fig. LXXXI. N°. 1.*) *AE* ſuperficies dirimens aerem & vitrum; *BC* radius lucis in *CD* reflexus, aut in *CF* refractus; ſi aer lucis motuï reſiſtat ut r, vitrum ut s, quæritur punctum *C*, in quod incidens radius ex puncto *B* ac punctum *D* per reflectionem, *F* per refractionem, illuſtrans omnium facillimè, ſeu vi omnium minimâ munia hæc ſua peragere poteſt?

Sit eum in finem in caſu reflectionis $DE:b$, $DC:f$; in caſu vero refractionis $EF:b$, & $CF:f$; AB dicatur: d, $AC:x$, $AE:c$, unde $CE:c-x$; quarum b, d, c, determinatæ, f, z, x indeterminatæ exiſtunt, item *BA* & *DF* ipſi *AE* normales.

Quoniam jam ex mechanicis conſtat, vires requiſitas inter ſe eſſe in longitudinum emenſarum & reſiſtentiæ mediorum ratione compoſitâ, ſeu, ut rz ad sf; vis utriuſque ſumma exponetur

tur per $rz+sf$; quæ omnium minima hoc in caſu expetitur. Sit ergo $rz+sf=yy$ æquatio repræſentans virium ſummam, in quodcunque tam ſuperficiei AE punctum radius ex B inciderit; in quâ, ut y minima reperiatur, (*Fig. LXXXI. N°. II.*) ad interceptam communem $AC: x$, ordinatæ concipiantur tres curvæ AD, AB, AF; quarum applicatæ dicantur $CB: z$, ejuſque ſubtangens $CK: l$; CD in caſu reflectionis, CF in caſu refractionis vocentur: f, earumque ſubtangentes CL & CM pro varia intentione appellentur n; tandem in curvâ POH applicata $HC: y$, ejuſque ſubtangens $CI: t$; et poſita curvarum inter ſe relatione $rz+sf=yy$, æquatio ſubtangentialis earum omnium ex Cap. I. §. 35. $rztn+sflt=2yyln$, in quâ, ſi ad extremum ſit determinanda, ſeu ad limitem ON, erit t infinita; unde ex Schol. Generalis, *N°. XII.* rejecto $2yyln$, ac diviſione per t inſtitutâ remanet $rzn+sfl=0$, pro æquatione determinatâ.

Cum autem hic ad omnimodam problematis determinationem æquationes neceſſariæ ex datorum naturâ reperiri poſſint; ac ſit I. in triangulo BAC in A rectangulo, (vide *Fig. LXXXI. N.* 1. & 2.) $dd+xx=zz$, quæ eſt ad locum curvæ BA, erit ejus ſubtangentialis $l=\frac{zz}{x}$.

II. In triangulo DEC aut EFC in E rectangulo eſt $cc - 2cx + xx = ff$, æquatio ad curvam AD aut AF, cujus ſubtangentialis $n = \frac{ff}{x - c}$.

Quarum ſubtangentium n & l inventus valor ſi in æquatione $rzn + sfl = 0$ ſurrogetur, emerget $rfx = csz - xsz$; quæ, loco f & z ſubſtitutis valoribus, ſolam x continebit, adeoque penitus determinata eſt.

Corol. I. In Caſu Dioptrico, cum ſit $x : c - x :: sz : rf$, ſi F & B (*Fig. LXXXI. N'.* 1) in ejuſdem circuli peripheria ſitæ eſſe intelligantur eſt $z = f$; adeoque $x : c - x :: s : r$, ſeu ut AC ad CE ita reſiſtentia vitri ad reſiſtentiam aëris. Adeoque in caſu minimæ obtinet regula *Carteſii* in dioptricis tradita.

Coroll. II. In caſu catoptrico, cum radius CD maneat in aere, ideoque reſiſtentia non mutetur, erit $r = s$, & $x : c - x :: z : f$, ſeu $AC : CE :: BC : CD$. Unde cum, ob laterum proportionem & angulos in E & A rectos, triangula ACB & DCE ſint ſimilia, patet hoc in caſu angulum incidentiæ BCA æquari angulo reflectionis DCE.

C A-

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CAPUT. VIII.

De Variis Calculi in Analyſi infinitorum Generibus, & methodo infiniteſimali directâ.

§. 1. CAlculi *infiniteſimalis* fundamentum conſiſtit in conſiderandis quantitatibus infiniteſimis, tanquam fractionibus; quarum denominatores numerum omni aſſignabili majorem m includunt: ſic $\frac{r}{m}$ eſt ipſius r pars infiniteſima, &c.

§. 2. Hinc, quem lemm. 8, 9, 10. &c. fuſè ſatis tradidimus, omnibus, ni fallor, hactenus adhibitis naturâ prior enaſcitur *quantitatum infiniteſimarum Algorithmus*; unde poſtea æquationum, ex quantitatibus indeterminatis, ſeu parte ſui infiniteſimâ (juxta curvæ quam ſpectant, aut ſpectare poſſunt, naturam) per totam ſui longitudinem creſcentibus aut decreſcentibus conſtantium, *redactio*, aut ſi mavis, *reductio ad infiniteſimas* juxta Cap. I. §. 1. & 2. aliaſque plurimas innoteſcit.

§. 3. Quod ſi hæ *fractiones* ſeu *quantitates*

infiniteſimæ per vocales *a* & *e*, &c. litteras, compendii gratiâ, deſignentur, oritur tangentium inventioni inſerviens *Cl. Barovii* methodus. Videatur Cap. I. §. 3. Schol. II. & §. 4, 5, 6, 7, 8.

§. 4. Si vero eædem quantitates infiniteſimæ vocalium & litterarum loco, per indeterminatarum, quas reſpiciunt, differentias (quæ infiniteſimarum naturam nihilo ſecius retinent) exprimantur, deducitur hinc *Celeberr. Leibnitii* calculus, *differentalis* nomine vocatus, cujus ſpecialem demonſtrationem, cum inter præcedentia, ſuperiorum ad inſtar, expreſſe non attingatur, hic adjungo. Sint quantitates indeterminatæ quotcunque, x, y, z, f, earumq. infiniteſimæ reſpectivæ, e, a, u, i, quæ juxta *D. Leibnitii* placita per indeterminatarum differentias, ſeu dx, dy, dz, df, denotentur.

I. Quæritur ipſius $x \mp y$, ſeu quantitatum ad ſe mutuo additarum vel á ſe invicem ſubſtractarum, differentiale?

Fiat $x \mp y = z$, quæ æquatio redacta ad infiniteſimas (poſitis nimirum $x + e$ loco x, $y + a$ loco y, $z + u$ loco z, (quod & in ſequentibus, ſemper obſervatum velim) dabit, $x + e \mp y \mp a = z + u$, & deletis æqualibus, $e \mp a = u$; ſeu reſumtis ſymbolis *Leibnitianis* $dx \mp dy = dz$.

II. Quæritur ipſius xy, ſeu indeterminatarum

um ſe mutuo multiplicantium, differentiale?

Fiat $xy = z$; quæ ſuperiori modo ad infiniteſimas reducta dabunt $xy + ye + xa + ae = z + u$, & deletis æqualibus, tum & per lemm. 10. rejectaneis, erit $ye + xa = u$; ſeu juxta D. Leibnitium $ydx + xdy = dz$.

III. Quæritur ipſius $\frac{x}{y}$, ſeu indeterminatarum ſe mutuo dividentium, differentiale?

Fiat $\frac{x}{y} = z$, erit $x = yz$, & poſt redactionem ad infiniteſimas & rejecta rejectanea $e = yu + za$; unde poſito $\frac{x}{y}$ loco z, & æquatione ad u ordinatâ, emerget $u = \frac{ye - xa}{yy}$; & ex notatione differentialium $\frac{ydx - xdy}{yy}$.

IV. Quæritur ipſius y ad poteſtatem p evectæ differentiale?

Fiat $y^p = z$, quæ ad infiniteſimas reducta, deletis juxta Cap. I. §. 10. delendis, dabit $py^{p-1}a = u$, ſeu $py^{p-1}dy = dz$.

§. 5. Sin autem quantitates indeterminatas *Fluentium* nomine, ipſas vero infiniteſimas *fluxiones*, *momenta*, *incrementa momentanea*

tanea aptiſſimo uſitatoque *Clar. Newtono* technologemate appellemus, eodem hoc fonte calculi, ab inclyto hoc Authore in profundiſſimæ eruditionis de *Phil. Nat. Princ. Math.* tractatu adhibiti, demonſtratio haurietur.

§. 6. Hiſce corollarii ad inſtar annecti, inſimulque confirmari poſſet, Scholio, §. 13. Cap. II. ſubnexo, terminos æquationum indeterminatos ad infiniteſimas redigendi inſinuata, adeoque inde petenda methodus; quæ, licet fractionibus æquationem ac irrationalitatibus liberam poſtulet, non paucis tamen in caſibus terminos differentialium ſimpliciſſimos, compendioſè magis, quam modo traditus calculi differentialis algorithmus exhibet.

§. 7. Poſſet *differentialis* hic *algorithmus* ulterius extendi, ac quantitatum, qui poteſtatum indices indeterminatos habent, differentialibus aſſignandis adhiberi.

Quæritur verb. Gr. ipſius y^{x} differentiale?

Fiat $y^{x} = z$, ac poſito $x+e$ loco x, $y+a$ loco y, $z+u$ loco z, erit $y+a$ ad poteſtatem $x+e$ evecta $= z+u$, unde multiplicatione inſtituta, ac rejectis per lemm: 10 rejiciendis, orietur $y^{x+e} + xy^{x+e-1}a = z+u$; ac denuo

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ito ipſi z ſurrogato y^{x}, fiet tandem, y^{x+e} $x y^{x+e-1}$ $a - y^{x} = u$; jam ſi loco a & e reponantur dx & dy, differentiale quæſitum, juxta deſignationem *Leibnitianam*, obtinebitur.

Coroll. Quod ſi x fiat determinata $= p$, erit $e = 0$; & prodibit, tanquam caſus hujus theorematis particularis, §. 4. *N°. IV.*

Schol. I. Poſſet poteſtas x etiam ad quamlibet aliam poteſtatem, tum determinatam, tum indeterminatam evehi; & hæc rurſus ad aliam, poteſtatumque poteſtates hoc ritu in infinitum continuari; tum & unico pluribuſque terminis efferri; ſic verb. gr. y evectâ ad poteſtatem x poteſt hæc rurſum ad poteſtatem f, ac f denuo ad poteſtatem $rxx + x^3$ evehi; unde calculi hujus, tum & curvarum, quæ hactenus non ſatis conſideratæ fuerunt, contemplatio ulterius longe promoveri earumque tangentes (ex more Cap. I. §. 7. &c.) aliaque ſymtomata inquiri poterunt. Cum autem caſus hinc originem ducentes ferme ſint innumerabiles, adeoque eorum ad certa quædam capita reductio tantum non impoſſibilis, præſtabit in hiſce omnibus methodo potius generaliori uti, cum labor alias inſumtus operæ ſuæ pretium vix compenſaturus videatur.

Schol. II. Antequam calculum differentialem deſeram, celebre ex eo deductum curvarum ſymptoma, *repetitam* ſcilicet *differentiationum ſucceſſionem* ſilentio prætermittere nequeo. Placuit nimirum quibuſdam è clariſſimis hujus ævi Mathematicis ipſa curvarum triangula characteriſtica (Fig. LXXXIII.) DHE, EqC, CLI, differentiationi denovo ſubjicere, partiumque differentialium differentias, ſeu infiniteſimarum infiniteſimas in calculo conſiderare, apparebat enim, poſitis DH, Eq, CL, ſemper æquali-

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bus, non tamen *DE*, *EC*, *CI*, fore æqualia, adeoque differentiam quandam inter illas intercedere. Quæ quidem differentiationum continuatio num ex iiſdem, quibus differentiatio prima, fundamentis deduci queat, ſequenti modo inquiſivi.

Sit (*Fig. LXXXIII.*) curvilineum *AXd*, ſumtâque *AQ* intercepta: x, *DQ* applicatâ: y, AD curva: c, ſubtangenti *TQ*:t, tangenti *TD*:s; in axe *AZ* accipiantur ſe mutuo inſequentes interceptæ *AP*, *AR*, *AS*, quarum differentiæ ſunt infiniteſimæ ſeu omni datâ minores, *QP*:e, *PR*:o, *RS*:π, *Cl. Leibnitio* dx vocitatæ; deinde ad ſingula axis diviſionum puncta ſi eleventur applicatæ, *PE*, *RC*, *SI*, erunt applicatarum differentiæ *HE*:a, ɑ*C*:μ, *IL*:ξ; curvarumque reſpectivarum, *DE*:u, *EC*:δ, *CI*:ζ; quarum illæ *D. Leibnitio*, dy, hæ dc nominantur; quæ curvarum differentiæ infiniteſimæ verſus axem in *T*, *F*, *G*, productæ generabunt infiniteſimas, *FT*:i. *GF*:φ; ac ductis *FK*, *TB5*, *D23*, *E4*, tangentes ad angulos rectos ſecantibus aliæ rurſum orientur infiniteſimæ, *GK*:ψ, *FB* = *K5*:λ, 34 = 2 *E* = *DE*:u; 4*C* = *EC* = δ.

Jam ob triangulorum *TQD* & *DHE* ſimilitudinem, eſt.

I. $t : y :: e : a$, ſeu *TQ*: *QD*:: *DH*: *HE*; unde $ta = ye$.

II. $s : y :: u : a$, ſeu *TD*: *DQ*:: *DE*: *EH*, unde $su = ya$.

III. $t : s :: e : u$. ſeu *TQ*: *TD*:: *DH*: *DE*. unde $se = tu$.

Tum ob triangulorum *FPE* & *E*ɑ*C* ſimilitudinem eſt.

IV. $t + e + i : y + a :: o : \mu$. ſeu *FP*: *PE*:: *E*ɑ: ɑ*C*, unde (per lemm. 10.) $yo = t\mu$.

V. $t + e + i : s + u + \lambda :: o : \delta$. ſeu *FP*: *FE*:: *E*ɑ: *EC*, unde (per lemm. 10.) $so = t\delta$.

VI. $s + u + \lambda : y + a :: \delta : \mu$. ſeu *FE*: *EP*:: *EC*: *C*ɑ, unde (per lemm. 10.) $y\delta = s\mu$.

Item ob triangulorum *GRC* & *CLI* ſimilitudinem eſt.

VII. $s + \psi + \lambda + u + \delta : y + a + \mu :: \zeta : \xi$. ſeu *GC*: *CR*:: *CI*: *IL*. unde (per lemma 10.) $y\zeta = s\xi$.

VIII. $s + \psi + \lambda + u + \delta : t + \varphi + i + e + o :: \zeta : \pi$. ſeu *CG*: *GR*:: *CI*: *CL*. unde (ex lemm. 10.) $t\zeta = s\pi$.

IX. $t + i + \varphi + e + o : y + a + \mu :: \pi : \xi$ ſeu *GR*: *RC*:: *CL*: *LI*, nnde (per lemm. 10.) $t\xi = y\pi$.

Hoc

Hoc autem ſimiliter eventurum patet, quotcunque (modo numero finitæ) aſſumtæ fuerunt infiniteſimæ.

Corol. I. Manifeſtum hinc eſt, quotlibet triangula ex infiniteſimis conſtantia, DHE, EqC, CLI, &c. (modo numero ſint finita) quæ ſe invicem non interruptâ ſucceſſione ſequuntur, inter ſe & triangulo aſſignabili TQD eſſe ſimilia; eſt enim $t : y :: e : a :: o : \mu :: \pi : \xi$. & ſic in cæteris. Demonſtratio ex §. præced. Num. I. IV. & IX. levi negotio colligitur.

Corol. II. Unde ſequitur, poſita in triangulis hiſce ſimilibus una laterum homologorum æqualium ſerie, & reliqua latera, homologa homologis, æqualia fore.

Idem hoc ex calculo demonſtratur, ſint enim omnia *dx* ſeu DH, Eq, CL conſtanter æqualia, erit ideo, $e = o = \pi$; adeoque juxta *N°. I. IV. IX.* $ye = ta = t\mu = t\xi$; quare tandem $a = \mu = \xi$, ſeu omnia *dy*, vel HE, qC, LI, perpetuo æqualia.

Corol. III. Quod ſi omnia *dy* aut omnia *dc* perpetuo ſumantur ejuſdem magnitudinis, eodem modo colligetur, hic omnia *dy* & *dx*, illic omnia *dc* & *dx* conſtanter fore æqualia. Calculum hoc evincentem, cum ſuperiori plane ſimilis ſit, omitto.

Corol. IV. Ex quibus tandem conſequi videtur, cum poſitâ una laterum ſeu omnium *dx* ſemper æqualium ſerie, omnia quoque *dy* ſibi ſemper æquentur, ac omnia *dc* (juxta Corol. II.) nullam inter ipſa *dy* aut *dc*, ſi numero ſint finita, interceſſuram differentiam, cum æqualium nulla ſit differentia; unde hoc ſaltem principio, quo omnia, ni fallor, reliqua calculi infiniteſimalis genera deduci poſſunt, *differentiationum ſecundarum tertiarum &c. ultra primam continuationes* demonſtrari nequeunt.

Ne autem hoc demonſtrationis genere in re tanti momenti quiſpiam offendatur, cum per modum inductionis intentum evincere ſtudeat, aliam adjungo; quem in finem ſequentia præmitto lemmatica:

Lemm. I. Si quotlibet, modo numero finitæ, infiniteſimæ ad ſe mutuo addantur, earum ſumma erit quantitas infiniteſima: evincitur ex lemm. 8. vel etiam hoc modo: ſint

ſint quantitates quotlibet aſſignabiles, ſed numero finitæ $b+c+d+g$ &c; quæ (poſito k pro numero quolibet finito quantum libet magno) ſint $= kq$. erit & q quantitas aſſignabilis; jam ſi ſumantur ſingularum partes infiniteſimæ, erit earum ſumma $\frac{b}{m} + \frac{c}{m} + \frac{d}{m} + \frac{g}{m}$ &c. $= \frac{kq}{m}$, ſeu æqualis parti infiniteſimæ ipſius kq.

Lemm. II. Sint infiniteſimæ, numero infinitæ, ac ſibi mutuo æquales, erit earum ſumma æqualis quantitati aſſignabili; ſic infiniteſima $\frac{r}{m}$ infinities ſumta eſt $= \frac{mr}{m} = r$, ſeu quantitati aſſignabili.

Lemm. III. Sint tres quantitates, A, B, C, quarum A eſt minima, C maxima, B inter utramque magnitudine media. Dico ſi A & C ſint aſſignabiles, etiam B fore aſſignabilem, hoc eſt nec infinitam nec infiniteſimam.

Demonſtr. Si neges, ſit primò B infinita, ergo C finita & aſſignabilis, erit major ipſâ B infinita, quod abſurdum. Secundò, ſit infiniteſima, ergo dabitur quantitas aſſignabilis A minor quantitate infiniteſimâ B, contra lemma 7.

Lemm. IV. Sint infiniteſimæ quælibet numero infinitæ, erit earum ſumma B æqualis quantitati aſſignabili. *Demonſtr.* ſi enim omnes ſupponantur æquales maximæ, erit earum ſumma C quantitas aſſignabilis juxta lemm. hujus II. ſin vero ſupponantur æquales minimæ, erit quoque ſumma earum A quantitas aſſignabilis juxta lemma II. hujus; unde cum ſumma B media ſit magnitudine inter A & C, & harum utraque ſit aſſignabilis, erit & juxta lem. præced. ſumma B aſſignabilis. Sed ad rem.

Reſumtis iiſdem ſymbolis, ac *Fig.* LXXXIII. ſit $VT{:}\,b$, $QZ{:}\,z$, $WA{:}\,b$, erit $ZW = y+b$, WY ſit e, & $XY{:}\,x$, erit propter triangulorum TQD & DHE ſimilitudinem æquatio prima $ye = ta$; inſuper & ob triangulorum VZW & WYX ſimilitudinem, eſt $VZ : ZW :: WY : YX$ ſeu $b+t+z : y+b :: e : x$ unde $bx+tx+zx = ye+be$, pro æquatione ſecundâ. Ex quibus ſequentia deducuntur.

Corol. I.

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Corol. I. Cum DH ac *WY* ſeu omnia *e* aut *dx* ſint æqualia, ex hypotheſi, ſi *t* ſeu TQ ſit aſſignabilis, QZ vero ſeu ſumma ipſarum *dx*, aut *z*, conſtet numero infinitesimarum finito, erit juxta lemm. hujus I. *z* infiniteſima; quam ob cauſam & *h* ac *b*, (cum eaſdem, non plures ſaltem, numero infiniteſimas, ac *z*, ex conſtructione contineant) pariter futuræ ſunt infiniteſimæ; unde per lemma 10. termini *hx*, *zx* & *be* nihilo fient æquales remanebitque $ye = tx$; quamobrem, cum ex æquatione prima ſit $ye = ta$, erit $a = x$; unde idem, quod Schol. hujus II. Corol. II. evictum fuit, denuo ſequitur: nimirum, ſi omnia *e* ſeu *dx* ſunt æqualia, omnia *a* ſeu *dy* fore æqualia, quin & cætera, illic loci per methodum inductionis tradita, hinc legitimè colligi poſſunt.

Corol. II. Si vero tum *z*, tum *h*, tum *b*, conſtent numero infiniteſimarum infinito, erunt ſingulæ harum juxta lemm. IV. hujus, ex quantitatum aſſignabilium genere; quare nullus in æquatione ſecundâ terminus evaneſcit, & hoc ſolo in caſu, poſitis, ut ſupra, *dx* æqualibus, inter ipſas *dy* intercedet differentia; aſſumtâque *a* majori ipſâ *x*, erit

$$a - x = \frac{ye}{t} - \frac{ye - be}{b + t + z}.$$

Hujus differentiæ proprietates accuratius indaganti patet ipſam eſſe natura ſuâ quantitatem *infiniteſimam*, ſeu termino Leibnitiano, *differentialem*, neutiquam vero *infinite- ſime infiniteſimam*, aut *differentio differentialem*; quod tamen, ſi calculus differentialis quantitatibus purè differentialibus juxta celeb. *Leibnitium* applicari debeat, requiri videtur. Nî alio (uti credibile eſt) prorſus fundamento ſua dederit Vir eruditiſſimus.

Schol. III. Sedulo diſcrimen hoc attendendum reſtat, num infiniteſimæ, ſi unâ plures ſimul conſiderandæ veniant, numero ſint finitæ, num vero infinitæ, cum hac quidem cautelâ accuratè adhibitâ plurimæ objectiones, quibus calculus quilibet infiniteſimalis urgeri alioquin videtur, ſponte diſpareant; neglectâ vero, non exiguam quandoque confuſionem inducant.

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CAPUT. IX.

De Æquationum in Analyſi infinitorum naturâ, & proprietatibus quibuſdam.

§. 1. ÆQuationes indeterminatas (hoc enim nomine in locorum doctrinâ paſſim notæ ſunt) vulgaris conſiderat Analyſis; quæ duplici viâ ad infiniteſimales reducuntur, vel *Primo* ponendo $x \pm e$ loco x &c. juxta modum §. 1. & 5. Cap. I. traditum; ſic æquatio indeterminata circulum ſpectans $2rx - xx = yy$ hac ratione abibit in ſequentem infiniteſimalem $re - xe = ya$. vel *ſecundo* integram æquationem per cujuslibet indeterminatæ pro arbitrio ſumtæ infiniteſimam multiplicando, ſic ducendo præcedentem in a emerget $2rxa - xxa = yya$.

Notari poteſt, quarumlibet aliarum indeterminatarum z & f infiniteſimas u & i, modo hæ ſuas indeterminatas toties ingrediantur, quoties e ipſam x aut a ipſam y, (quæ conditio in ſimili caſu perpetuo requiritur) hic adhiberi poſſe.

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§. 2. Exortæ hinc *æquationes infiniteſimales*.

I. Terminorum infiniteſimalium additionem ac ſubſtractionem patiuntur, quod ſatis notum eſt.

II. Dividi ac multiplicari inſtar aliarum poſſunt per quantitates determinatas, indeterminatas, ac infiniteſimas, neque ulla in his difficultas eſt, niſi forſan in poſteriori caſu.

Si enim $re - xe = ya$; dividatur per e, erit $r - x = \frac{ya}{e}$; adeoque tota fit indeterminata; quæ ſiquidem terminus $\frac{ya}{e}$ ex lemmate 12 aſſignabilis eſt, non vero infiniteſimus; tum infiniteſimæ ex more præcedentium per aliunde conquiſitam æquationem tolli poſſunt.

Sin autem $re - xe = ya$ per infiniteſimam a multiplicetur, fiet $rae - xae = yaa$; ex qua, licet omnes termini ex lemm. 10. evaneſcant, non pauca tamen quantitates poſitivas concernentia deduci poſſunt, mediante hoc ex ipſâ infiniteſimarum naturâ profluenti lemmate.

§. 3. Si quotlibet in ſe mutuo ductæ infiniteſimæ verb. gr. aae^3i, rurſum dividantur per alias in ſe invicem multiplicatas infiniteſimas, verb. gr. e^4i, quarum numerus præcedentium numero unitate minor eſt, quod emerget $\frac{aa}{e}$

erit

erit quantitas infiniteſima. *Demonſtratio* ex iis, quæ ſcholio generali tradita ſunt, facillime conſequitur; ſit enim $a = \frac{f}{m}$, & $e = \frac{b}{m}$, eſt $\frac{aa}{e} = \frac{ff}{bm}$, ſeu infiniteſima pars ipſius $\frac{ff}{b}$.

§. 4. Reſumta jam modo inventa æquatio $rae - xae = yaa$ dividatur per e, fiet $ra - xa = \frac{yaa}{e}$, quare *Fig. LXXXVII.* vocatis $AQ: x$, $QM: l$, $DH: e$, $HE: a$, $MN: o$, $BD: \mu$. inter infiniteſimas hoc modo quæratur lineola ipſi $\frac{aa}{e}$ reſpondens; cum ſit $BH : HE : HE : HI$, ſeu $\mu + e : a :: a : \lambda$ erit (ob ipſius μ ſeu BD juxta lemm. 41. nullitatem) $\frac{aa}{e} = \lambda = HI$; adeoque $ra - xa = y\lambda$; ſed $EH : HI :: EP : PN$. ſeu $a : \lambda : y + a :: l - e + o$, quare, rejectis per lemm. 10. rejectaneis, $y\lambda = la$, quæ ad æquationem modo inventam applicata dabit $r - x = l = QM$, ut decet. Poſſet æquatio $re - xe = ya$ per plures infiniteſimas multiplicari, unde aliarum linearum determinabilium magnitudo innoteſceret. Cum autem inter mox ſecutura, plurima huc ſpectantia reperiantur, poterunt inde peti, quæ hujus theorematis illuſtrationi exempla inſerviunt.

§. 5. III. Æquationes infiniteſimales, manen-

ante utrinque æqualitate, ad quaslibet poteſtates evehi poſſunt; ſic æquatio $re - xe = ya$ quadratè multiplicata dabit $rree - 2rxee + xxee = yyaa$, quæ diviſa per e, dabit $rre - 2rxe - xxe = \frac{yyaa}{e} = yy\lambda$; jam autem $TP : PN :: DH : HI$. ſeu $t + e : l - e + o :: e : \lambda$; unde $le = t\lambda$: quæ ad inventam æquationem applicata, ejectis infiniteſimis, dabit $t : l :: yy : rr - 2rx + xx$, adeoque rationem ſubtangentis ad ſubnormalem.

Sit retentis iiſdem ſymbolis, ac Figurâ, æquatio omnibus curvis conveniens, $le = ya$; quæ multiplicata cubicè dabit $l^3 e^3 = y^3 a^3$, ac rurſum per ae diviſa, $l^3 \overline{\frac{ee}{a}} = y^3 \overline{\frac{aa}{e}}$; jam cum HI ſeu λ ſit $= \overline{\frac{aa}{e}}$, erit $H\Lambda = \overline{\frac{ee}{a}}$, quæ vocetur π. Adeoque erit $l^3 : y^3 :: \lambda : \pi :: HI : H\Lambda$. Inventis jam λ & π inter quantitates infiniteſimas, ſi conſideremus lineas homologas inter quantitates aſſignabiles, ipſi TD ducta normali TW, donec productam DQ in M ſecet, vocetur $QW : n$, erit $HI : H\Lambda :: QM : QW$, ſeu $\lambda : \pi :: l : n :: l^3 : y^3$.

Poſſunt hæc variis caſibus applicari, plurimarumque hinc linearum rationes inveniri, ac curvilineorum ſolidorumque nondum penitus exhauſtæ proprietates aliquatenus ampliari.

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Depreſſionem æquationum infiniteſimarum per radicum extractiones, cum ſine quadam præparatione eandem tolerare non videantur, hic tranſeo.

§. 6. Æquatio infiniteſimalis, verb. gr. $re - xe = ya$, ſi ſinguli ejuſdem termini infinities, ac juxta conditionem Cap. II. §. 12. expreſſam ſumantur, mutatur in æquationem *collectivam*, ſeu appoſito termino *Leibnitiano*, *ſummatricem*, ſic omnia re — omnia xe = omnia ya, eſt æquatio *Collectiva* ſive *ſummatrix*. Harum exempla in ſuperioribus paſſim occurrunt.

Schol. Clarum hinc eſt, ex datâ qualibet æquatione infiniteſimali & collectivam ipſius legitime conſequi; licet ex collectivâ quâlibet infiniteſimalis per regreſſum deduci ſemper nequeat. Sic reſumta *Fig.* XXVI. poſitiſque $QD:y$, $QP:e$, $DI:x$, $GI:a$, idem curvilineum AFO tum per omnia ye, tum per omnia xa exprimetur, unde quidem omnia ye = omnia xa; non tamen ſequitur fore in ſingulis trapezium $DQPE$ ſeu ye ſemper æquale trapezio $DIGE$ ſeu xa, quod notetur ad errores præcavendos.

C A-

CAPUT. X.

De *Methodo Infiniteſimali inverſâ.*

SInt x, y, f, z, indeterminatæ; ac e, a, i, u, infiniteſimæ ad ipſas reſpectivè pertinentes.

§. 1. Quæritur indeterminatus terminus, ad quem terminus infiniteſimalis conſtans, ex quantitatibus determinatis & infiniteſimâ quâcunque, ut rre, ba &c. reduci poteſt? loco infiniteſimæ e aut a ſurrogetur quantitas reſpectiva indeterminata x aut y, & habebitur terminus quæſitus rrx aut by.

§. 2. Quæritur idem; ſi terminus infiniteſimalis præter determinatas, ſi occurrant, ac infiniteſimam, etiam indeterminatas, ſed ad infiniteſimam tantum relativas, contineat: ut $rrxe$, y^3a, &c? ipſi infiniteſimæ e aut a ſufficiatur indeterminata relativa x aut y; ac quod emergit, per ipſarum x aut y poteſtatis exponentem dividatur, & obtinebitur $\frac{1}{3}rrx^3$ & $\frac{1}{4}y$. *N. B.* potuiſſet uterque hic caſus, tanquam terminos infiniteſimales proprios ſpectatas, unico canone determinari.

§. 3. Quæritur terminus indeterminatus, ſi expoſitus infiniteſimalis præter determinatas & in-

infiniteſimam alias inſuper indeterminatas, ſed ad infiniteſimam neutiquam relativas, contineat, ut $rrxxy^3a$, z^3ffxxe?

Et hactenus tantum reſponderi poſſe videtur, per ſupra tradita nec has nec ullum terminum infiniteſimalem alienum, ſi ſolus hoc modo exponatur, reducibiles exiſtere; ac non niſi ipſum tot comitentur termini infiniteſimales, quot juxta Schol. Cap. II. §. 13. ſubjunctum requiruntur.

§. 4. Ut vero, expoſità terminorum infiniteſimalium multitudine, verb. Gratia, $ffxxa + 5ffxye + 2xxyfi$, dignoſci poſſint, quinam ſint reducibiles: ſumatur quilibet ex hiſce terminis $2xxyfi$, cujus pars fi, (quæ, ſi indeterminatis aliis juncta non foret, infiniteſimalem proprium conſtitueret) inſtar termini infiniteſimalis proprii juxta §. 2. hujus reducatur in $\frac{1}{2}ff$, ac loco fi ſubſtituatur, habebitur $xxyff$; qui redactus juxta præcedentia ad infiniteſimas dabit, $2xxyfi + 2ffyxe + ffxxa$; unde patet totam terminorum expoſitam multitudinem, excepto $3ffxye$, ad indeterminatum $xxyff$ eſſe reducibilem.

Demonſtratio præcedentium è locis ſat multis & Cap. II. §. 13. Scholio peti poteſt.

§. 5. Cognito jam, quinam ſint termini infiniteſimales ad congruos indeterminatos reducibiles, habebitur eo ipſo omnium æquationum in-

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ifiniteſimalium (modo termini eas conſtituen-
ts ſint vel proprii, vel alieni legem Schol. Cap.
I. §. 13. adimplentes) ad ſuas indeterminatas
dductio. Cumque ex Cap. II. §. 13. termini
ifiniteſimales collective ſeu toties ſumti, quo-
es infiniteſima quævis indeterminatam quanti-
tem, quam componit, ingreditur, ſint æqua-
s valori abſoluto termini indeterminati, unde
riginem duxerunt; variarum hinc æquatio-
um collectivarum, reductio ad indetermina-
s enaſcitur, & ex dato valore infiniteſimalium
llectivo, earundem valorem abſolutum mul-
i in caſibus reperiendi methodus.

Ex. I. Sit æquatio collectiva, omnia re —
mnia $xe =$ omnia ya; erunt, juxta §. 13.
ap. II. omnia $ya = \frac{yy}{2}$; ergo omnia re — om-
a $xe = \frac{yy}{2}$; quod æquationis genus quodam-
odo *mixtum* appellari poteſt, cum altera pars
collective infiniteſimalis, altera abſoluta &
determinata, tum & omnia re — omnia xe
$= rx - \frac{1}{2}xx$; adeoque ex modo propoſita col-
ctiva infiniteſimali oritur hæc indeterminata,
$rx - xx = yy$.

Ex. II. Sit (*Fig. LXXXVI.*) vocatis $AQ: x$,
$Q: t$, $AT: f$, $DH: e$, $EH: a$, curva ADE
ius naturæ, ut exiſtentibus p & q determina-

 tis

tis, ſit perpetuo $p:q::f:y$. unde $qf = py = qx - qt$; & ductis omnibus in a, erit $pya = qxa - qta$; ac loco ta poſito ye, erit $pya = qxa - qye$; & addito utrinque $2qye$, ac omnibus collective ſumtis, emerget poſt tranſpoſitionem omnia $2qye =$ omnia $qxa +$ omnia $qye -$ omnia pya; adeoque (cum altera hujus æquationis pars ſit reducibilis per hujus §. 2. & 4. ad $qyx - \frac{1}{2}pyy$) tandem, juxta Cap. II. §. 13. evadet $\frac{1}{2}xy - \frac{pyy}{4q} =$ omnia $ye =$ trilineum AEP, unde & hujus quadratura innoteſcit.

Potuiſſet ipſa alio modo inveniri; ſi enim inventæ æquationi $pya = qxa - qye$ addatur utrinque qxa, erit $pya + qye + qxa = 2qxa$; quibus collectivè ſumtis, ac juxta §. 2. & 4. hujus reductis orietur ex Cap. II. §. 13. $\frac{\frac{1}{2}pyy + qyx}{2q} =$ omnia $xa =$ trilineum KAE.

Scholium Notetur reductionem hanc ſupponere indeterminatas x & y &c. nullâ ſui parte determinatas eſſe, ſed meris infiniteſimis ab initio ad finem per totam ſui longitudinem componi, & eodem quidem modo, quod circa Cap. I. §. 12. expreſſe monuimus; tum & æquationes ex reductione reſultantes terminum nullum omnino determinatum admittere. Si enim infiniteſimæ ſine ullâ ad indeterminatas ſuas limitatione accipiantur, una eademque infiniteſimalis æquatio varias quandoque curvarum ſpecies reſpiciet.

Quod ut breviter oſtendam, ſit æquatio infiniteſimalis $2ya = 2xe$, quæ collective ſumta dabit omnia $2ya =$ omnia

nia $2xc$; & juxta præcedentia reducta exhibebit $yy = xx$, ac extractâ radice $y = x$; unde primo hæc æquatio infiniteſimalis *Fig. LXXXIV.* ad triangulum verb. gr. *RPW* (in quo *RQ*:x, & *QS*:*y*;) reducitur.

Si vero inventæ æquationi $yy = xx$, determinata quantitas rr ſub quolibet ſigno addatur, erit, ex. gr. $yy = xx - rr$ ad hyperbolam, ad quam eadem hæc infiniteſimalis ſecundo referri poteſt, poſitis enim in eadem figurâ *RA*:*r*, *RQ*:*x*, *QD*:*y*, *QP*:*e*, *EH*:*a*, ſit curva *ADE* Hyperbola, unde ipſius æquatio $xx - rr = yy$, quæ reducta ad infiniteſimas dabit $2xe = 2ya$.

Unde poſitis *RP* in *PW*, *RQ* in *QS* &c, tum *EP* in *EF* ac *DQ* in *HK* &c. erit *EPF* triangulum = omnia $ya = \frac{1}{2}yy$; tum trapezium *AGWP* = omnia x*e*, adeoque hic reductio omnium x*e* ad $\frac{1}{2}$ xx locum non habet, hâc quidem de cauſâ, quod *RP* ſeu x non in totum, ſed parte ſolummodo ſui *AP* ex infiniteſimis *PQ*, *QL*, *LC*, &c. ſeu infinitis *e* componatur; reſiduâ nimirum portione *RA* determinatâ manente, quare hoc in caſu omnia x*e* ſeu trapezium *AGWP* æquatur $\frac{1}{2}xx - \frac{1}{2}rr$.

Tertio, Sit jam *QP*:x, eaque ſupponatur creſcere per QL &c. ſeu *e*; ac Q*D*:*y* decreſcere per *DZ*:*a*; exponaturq. quadrantis circuli æquatio $rr - xx = yy$, dabit hæc infiniteſimalem $-2xe = -2ya$ &. mutatis ſignis, $2xe = 2ya$; ſunt autem (poſitâ QP in QX &c. omnia x*e* = triangulo *PCV*, ac his æqualia omnia *ya* (ſi quidem *y* perpetuo in *EF*, *HK*, *NO* ponatur) conſtituent trapezium E34F ſeu $\frac{1}{2}rr - \frac{1}{2}yy$.

Unde patet quamlibet æquationem infiniteſimalem, ſimpliciter ſine reſtrictione propoſitam, unam determinatam curvam non reſpicere: cum manente eadem infiniteſimali, poſt reductionem quantitates quælibet determinatæ multis modis æquationi adjungi queant, quorum tractationem magis accuratam, cum plus requirat temporis, quam mihi mea impræſentiarum permittunt negotia, aliorum indagini ulteriori relinquo, hoc addo:

§. 6. Sit æquatio quælibet infiniteſimalis, $rxe - yye = 3yya$; quæ collective ſumta, ſi altera pars $3yya$ ex §. 2. hujus reducatur, abit in collectivè mixtam (vid §. 5. exempl. I.) omnia rxe — omnia $yye = y^3$. quæruntur hinc termini infiniteſimales collectivè ſumti, qui valorem ipſius y^3 per quaſlibet indeterminatas verb. gr. f^3 multiplicati aut diviſi, ſeu ipſum $y^3 f^{\mp 3}$ adæquent. Quod quomodo obtineatur in caſu multiplicationis, ſequens docebit theorema: terminis expoſitis infiniteſimalibus $rxe - yye$, per f^3 ſeu quantitates indeterminatas prius multiplicatis, pars æquationis expoſitæ abſoluta y^3 per terminum infiniteſimalem $3ffi$ ex f^3 oriundum multiplicata, cum ſuo ſigno legitimo addatur; & ſumma hæc infiniteſimalium collective ſumta æquabitur quæſito valori abſoluto $y^3 f^3$.

Sic, ut in eodem maneamus exemplo, ex inventa æquatione omnia rxe — omnia $yye = y^3$, ſequitur omnia $rxf^3 e$ — omnia $yyf^3 e$ + omnia $3y^3 ffi$ fore æqualia valori abſoluto $y^3 f^3$.

Demonſtratio. Cum enim (ex Cap. II. §. 13) ſit $y^3 f^3 =$ omnia $3yyf^3 a$ + omnia $3y^3 ffi =$ omnia $rxf^3 e$ — omnia $yyf^3 e$ + omnia $3y^3 ffi$; ſi collectiva hæc ad infiniteſimalem deprimarur (fieri

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(fieri enim hoc poſſe ſupponimus) ac per f'
vidatur, redibit æquatio expoſita.

Coroll. Quomodo hoc diviſionibus, per ip-
ſius f exponentem negativum, accommode-
tur; tum & in radicum extractionibus ac pote-
ſtatum ad altiores gradus evectionibus (quæ
multiplicationis ac diviſionis ſpecies quædam
ſunt) uſui eſſe queat, quilibet hinc eruet.

CAPUT. XI.

Miſcellanea.

PRæparatio Communis. Sit, *Fig. LXXXV. LXXXVI.* &c. curva *ADE*, cujus applicata *DQ*: *y* = *TI*, intercepta *AQ*: *x*, ſubtangens *TQ*: *t*, cujus abſciſſa *TA*: *f*, tangens *TD*: *s*, infiniteſimæ *QP* = *DH*: *e*, *HE*: *a*, *DE*: *u*, *FT*: *i*.

§. 1. Satis jam notum eſt, ſi *Fig. LXXXV.* & *LXXXVI. HM* ſit = *TQ* = *t*, trilineum *PEL* ſeu omnia *ta*, æquari trilineo *AEP*, ſeu omnibus *ye*; ſpatium vero *PLO*, ſeu complementum trilinei *PEL*, ſemper æquari ſummæ vel differentiæ trilineorum *APE* & *AFG*, neſcio an adhuc animadverſum ſit; illud obtinet, ſi curva *ADE* (*Fig. LXXXV.*) verſus axem concava ſit; hoc ſi verſus axem (*Fig. LXXXVI.*) convexa ſit.

Demonſtr. Cum ex hypotheſi ſit perpetuo *HM* = *QT*, & *EL* = *FP*, erit *ZL* (*Fig. LXXXV*) = *NO* = *e*+*i*; ſed in *Fig. LXXXVI.* = *e* − *i*; adeoque cum in utroque caſu ſit *MN* = *y*, erit curvilineum *POL* in primo = omnia *ye*+*yi*, in ſecundo = omnia *ye* − *yi*; unde cum *APE* ſit = omnia *ye*, ac *AGF* = om-

omnia yi, patet propoſitum. Poſſet idem more vulgari ex figurarum æquivalentia demonſtrari.

Schol. Hinc, ſi curva in A cum axe AQ concurrat, poſſet erui curvaturæ diagnoſticum; cum ſi curva exiſtat verſus axem concava ſit $= x+f$; ſi vero ſit convexa, evadat $t = x-f$. verum hæc Capite I. ex profeſſo tractavimus.

§. 2. Sit r determinata, ac $t:y::e:a::z:r$, erit $za = re$; adeoque, ſi (*Fig. LXXXV.*) in BQ applicetur ad e, ac z in HM ad a, erit ſpat. $HPM =$ omnia $za =$ omnibus $re =$ rectangulo $BQSA$, $= rx$, quod ab aliis jam demonſtratum eſt; aſt ſpatium PMN ſemper adæquari rectangulo $ASCT$ nondum obſervatum memini.

Demonſtratio facilis eſt, cum enim ex hypotheſi ſit $zy = rt$ ſeu rectangulum $HMNP =$ rectangulo $QBCT$, ac ſpatium $PHM =$ rectangulo $QBSA$, erit ſpat. $PMN =$ rectangulo $ASCT$. *q. e. d.*

Idem ex infiniteſimali calculo evincitur, ſit enim NO ipſius z infiniteſima $= o$; erit, æquatione $zy = rt$ ad infiniteſimas redactâ, $za+yo = re+ri$; unde cum ſit $re = za$ remanebit $yo = ri$, quare omnia $yo = PMN =$ omnia $ri = rf =$ rectangulo $ASCT$.

§. 3. Sit $t:y::e:a::r:z$, erit $ze = ra$, tum $zt = ry$, quæ redacta ad infiniteſimas, (ſi curva,

va, ut in *Fig. LXXXVI.* ſit verſus axem convexa) dabit $ra = to + ze - zi$; unde quia $ra = ze$, erit $to = zi$; quare ſi z jugiter in TS ad FT ſeu i applicetur, erit $SV = i$, & $VW = o$; jam, cum ſit $to = zi$, erit $o : i :: z : t$, ſeu $VW : VS :: TS : TR$, quare curvæ ASW hoc modo generatæ ſubtangens TR ſemper æquabitur curvæ expoſitæ ADE ſubtangenti $TQ = t$.

Quod notabile, videtur cum omnibus curvis, habitâ curvitatis & ſigni ipſa e & i copulantis ratione, competat.

Schol. Qua jam ratione variæ curvarum proprietates, omnibus curvis communes, per aliarum rectarum curvas in genere ſpectantium (tangentium, ſubtangentium, normalium, ſubnormalium &c.) infiniteſimas erui, ac adhibitâ, quam capite I. tradidimus, tangentium methodo, non paucis theorematum generalium acceſſionibus, quæ particulari cuidam curvæ alligatæ non ſunt, Geometria locupletari valeat, ſatis hinc manifeſtum eſt; ſuffecerit digitum intendiſſe.

§. 4. Sit Figura LXXXVI. curva ADE ejus proprietatis, ut ſubtangentis abſciſſa $AT : f$, ad tangentem $TD : s$, conſtantem habeat rationem, ut p ad q; erit $qf = ps$, ſed propter triangulum TQD rectangulum eſt $tt + yy = ss$;

in

In quâ æquatione poſita $s = \frac{qf}{p}$ erit $pptt + ppyy = qqff$; quæ redacta ad infiniteſimas ac per 2 diviſa dabit $ppte - ppti + ppya = qqfi$; factaq. $ta = ye$ juxta ſæpius oſtenſa, $pptte - pptti + ppyye = qqfti$; rurſumque poſito $ss = tt + yy$ ac $qqff = ppss$ fiet tandem $qqffe = qqfti + pptti$, quæ æquatio cum ipſarum f & t infiniteſimas e & i contineat, ſi applicetur TQ ſeu t in TS erit $SV = i$ & $VW = e - i$, curvæque ASW hinc emergentis ſubtangenti RT vocatâ n, erit $RT : TS :: SV : VW$ ſeu $n : t :: i : e - i$; unde $te - ti = ni$; ex quâ, mediante inventa modo æquatione rejectis infiniteſimis, invenietur $RT = n = \frac{qqfft}{qqft + pptt - qqff}$; unde datâ methodo tangentium inverſâ dabuntur curvarum AS & AD conſtructiones.

§. 5. Sit, *Fig. LXXXVII.* curva ADE, cujus ſubnormalis QM vocetur l, normalis $DM : k$, recta DR ex puncto curvæ D juxta datam quamlibet conditionem ducta, axemque AQ productum in R ſecans producatur verſus V, dicaturque $DR : p$, ac $MR : h$; erit $QR = l + h$, quæ compendii gratia appelletur: s; quod ſi, ductâ FE curvam in puncto E ipſi D proximo tangenti, idem fiat reſpectu appli-

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catæ EP ; ac ſit ES axem in S ſecans linea eandem ad punctum E lineaſque ad ipſum reſpectivas relationem habens, quam habet DR ad punctum D, lineaſque hoc ipſum ſimiliter ſpectantes ; ſuppoſito jam lineas DR & DS protractas in V ſibi mutuo occurrere, idemque fieri ductis ſimilibus lineis ex omnibus curvæ ADE punctis, erit punctum V ad novam hinc ortam curvam VW; cujus natura quæritur & conſtructio.

Quem in finem, productâ DH utrinque in B & O, dicantur $HI: \lambda$, $HO: \varphi$, $MN: o$, $RS: \varkappa$, quarum ſingulæ juxta lemma 41. ſunt infiniteſimæ, BD vero appelletur: μ. tandem ſit $DV: z$ erit $RV = z - p$. Jam ob triangulorum DOV & SRV ſimilitudinem eſt $OD: DV :: SR: RV$. ſeu $e + \varphi : z :: \varkappa : z - p$, unde $ze + z\varphi - pe - p\varphi = z\varkappa$, (ſignetur X.) tum &, ob rectarum BO & FS paralleliſmum, eſt $BH: HO :: FP: PS$. ſeu $\mu + e : \varphi :: t + e + i : b - e + \varkappa$, unde cum BD ſeu μ ex lemm. 41. nihilo æquivaleat, amotis juxta lemm. 10. rejiciendis, fiet $be = t\varphi$; quæ cum inventâ modo æquatione litterâ X notatâ ſi ad φ determinetur, ex utraque orietur $bze - bpe - pte + tze = tz\varkappa$, (ſignetur Y) hæc autem cum infiniteſimas e & $\varkappa$ adhucdum contineat, alia reperienda eſt æquatio, cujus ope tolli poſſint, quæ juxta

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juxta præcedentia ſequenti modo acquiritur; ſtatuatur b ſeu QR in QG, & PS in PL, donec per earum extremitates nova curva AGL deſcribi poſſit, erit $GK = e$ & $KL = PS - QR = x - e$; quare QC ipſius curvæ AGL ſubtangenti vocatâ : n, fiet $QC : QG :: GK : KL$. ſeu $n : b :: e : x - e$. adeoque $nx = be + ne$; quæ ad æquationem littera Y ſignatam applicata, exhibebit $z = \frac{pbn + ptn}{bn - bt}$.

Unde cognitâ DV ſeu z tum longitudine tum poſitione, datur punctum V in curva VW, adeoque hujus conſtructio; & ſi placuerit, etiam relatio ad axem AS, juxta methodum Cap. V. §. 12. circa evolutas inſinuatam.

Generali hoc theoremate plurimarum curvarum examen inſtitui ac conſtructio effici poteſt.

Corol. I. Poſita enim ipſa h æquali nihilo, erit $b = l$, & $p = k$, rectæque DR & DM cum ES & EN coincident; eritque VW ipſius AE evoluta ac $z = \frac{kln + ktn}{ln - lt}$, uti Cap. V. §. 11. Cor. I. inventum.

Corol. II. Si vero DR juxta aliam quamlibet conditionem ducta ſit, & repræſentet verb. gr. viam radii, poſt refractionem in ſuperficie curvæ ADE verſus R tendentis, erit VW curva á radiis poſt refractionem concurrentibus formata.

Corol. III.

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Corol. III. Quod ſi *D R* cum recta *D M* faciat angulum *R D M* = *M D Q*, erit V *w* curva ex radiis Q*D* & *P E* ſibi mutuo parallelis, ac ad ſpeculum concavum *A D E* reflexis, oriunda.

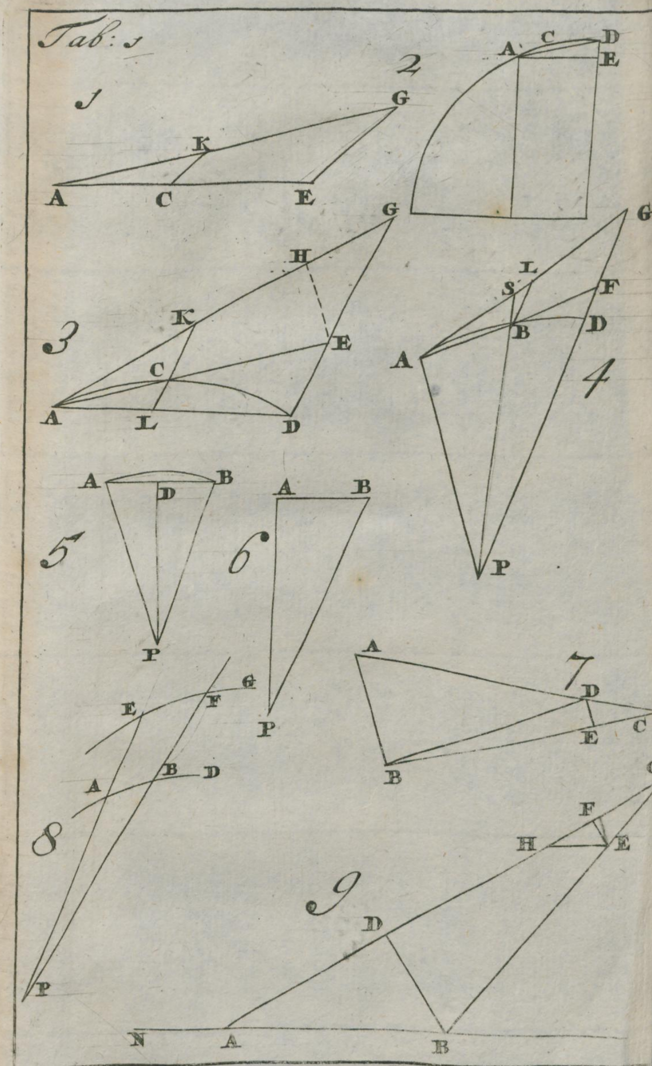
Corol. IV. Unde cum conditio hæc infinitis modis varia eſſe poſſit, juxta cujus leges linea *D R* duci poteſt; infinitarum, quæ hiſce modis generantur curvarum V *w* conſtructiones evolutarum, catacauſticarum, diacauſticarum &c. unico hoc theoremate expediuntur. Particularia, cum poſt inventum hunc canonem plus requirant laboris, quam induſtriæ, tranſilio; uti & caſum, quo interſectio V cadit ab altera axis parte.

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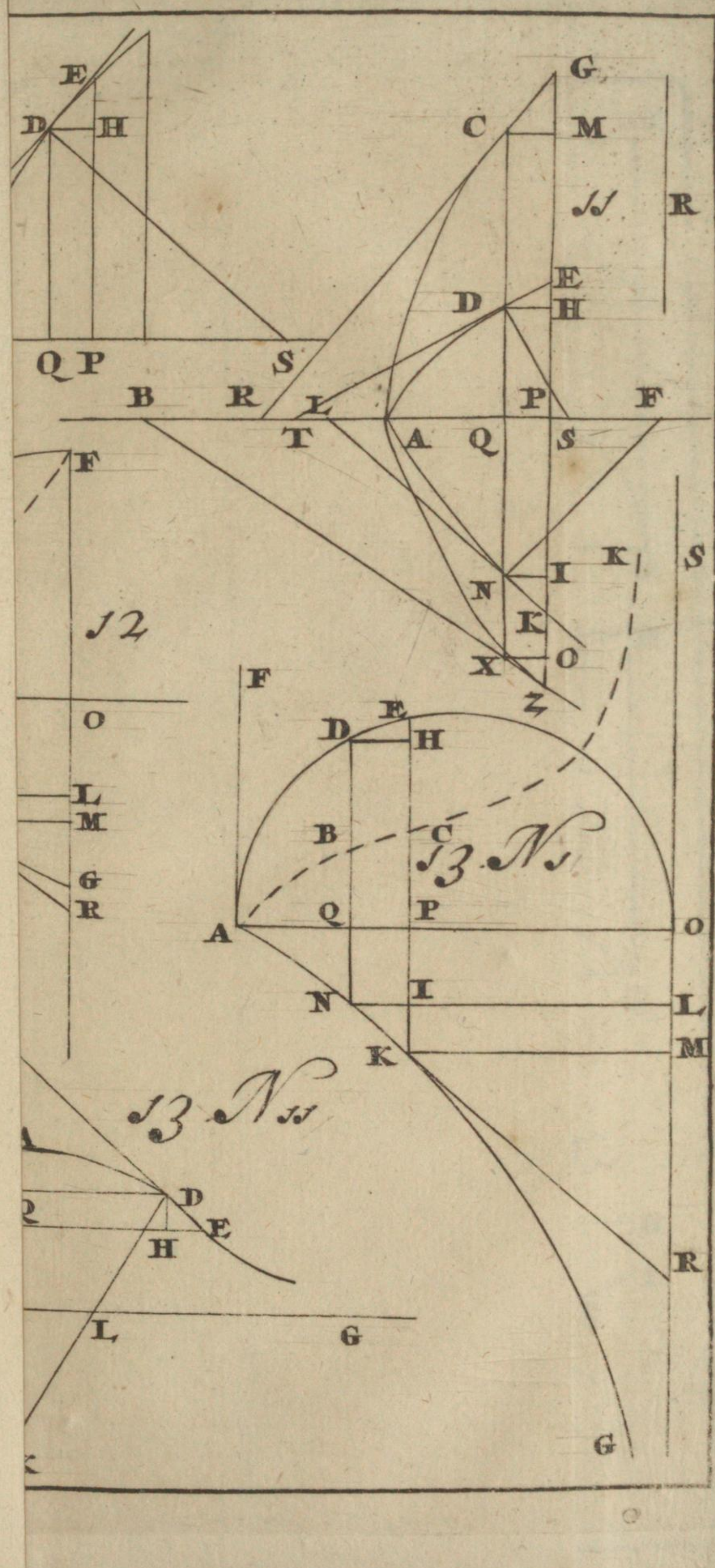
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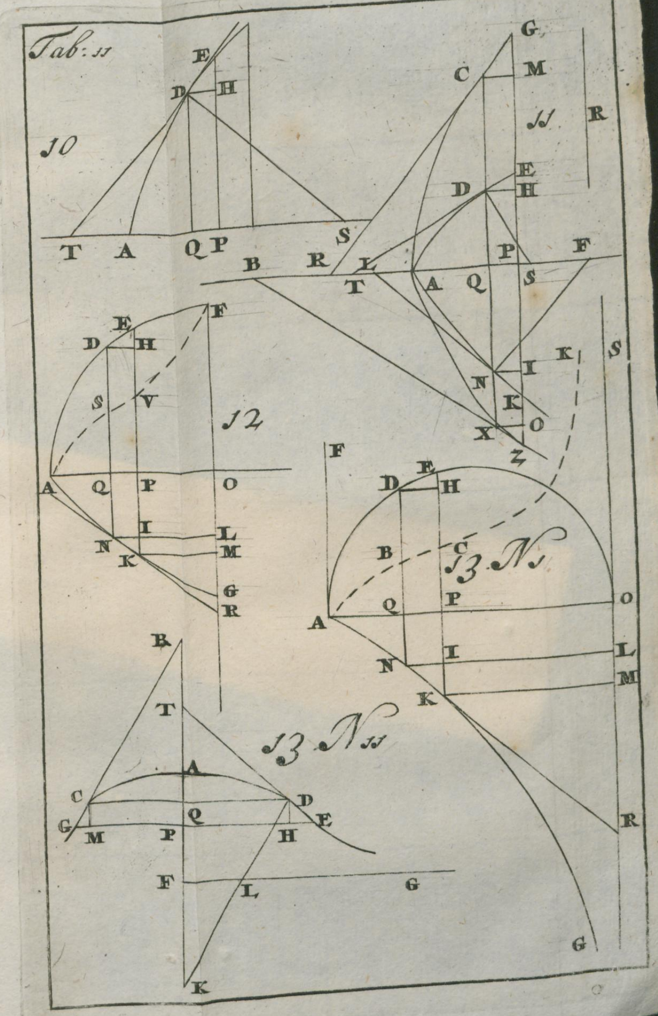
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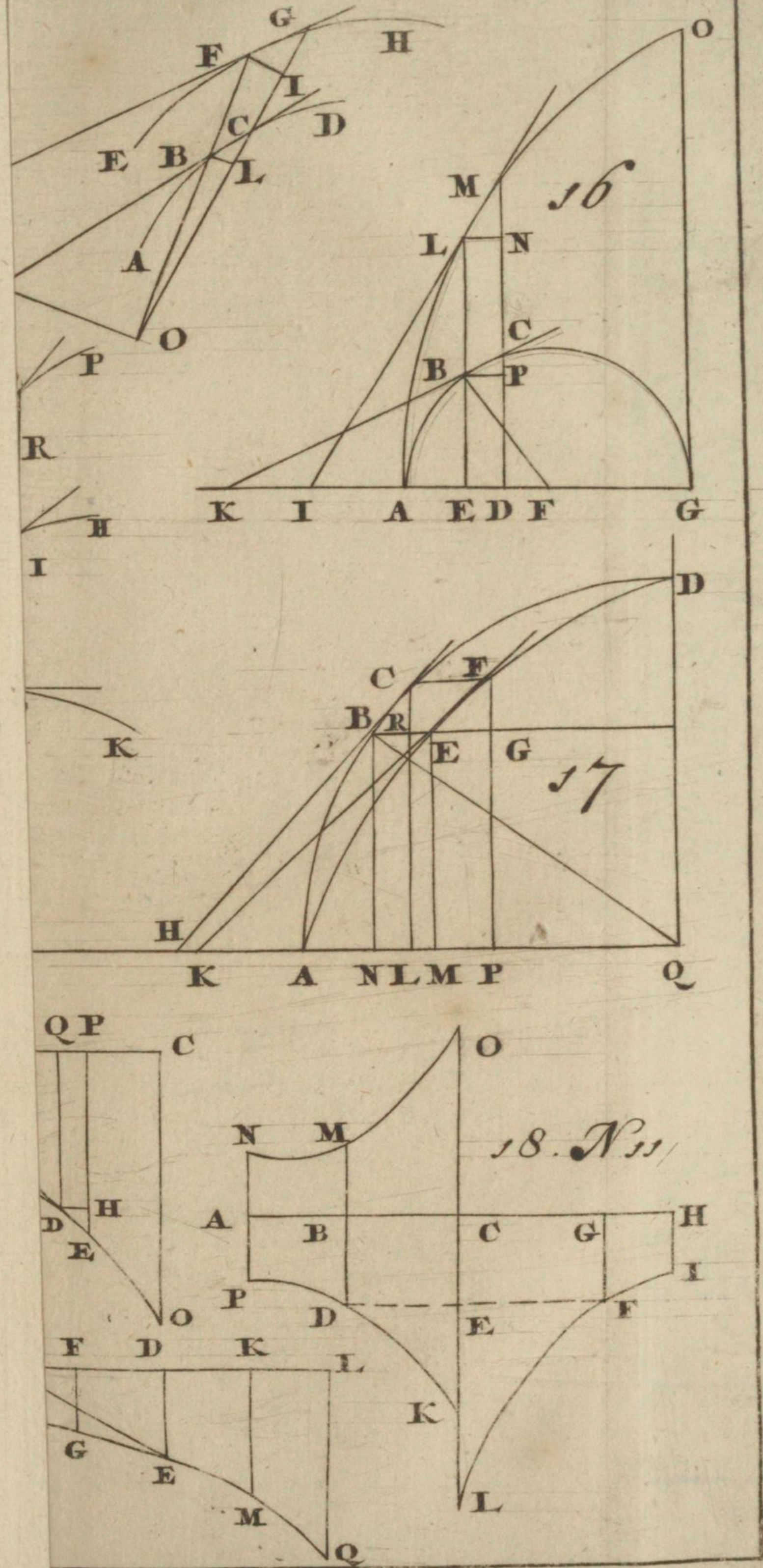
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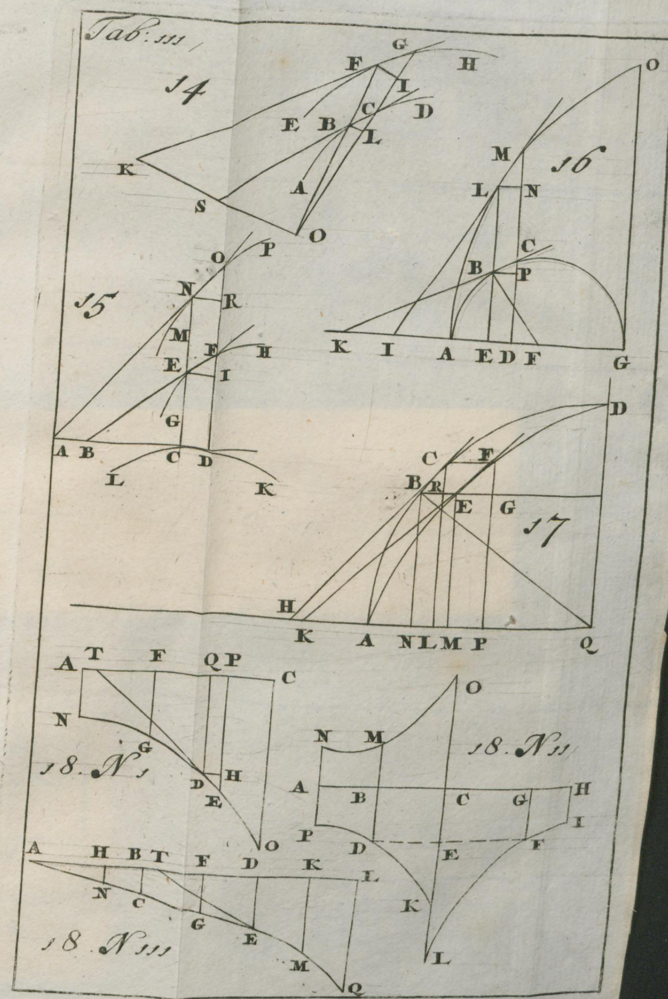
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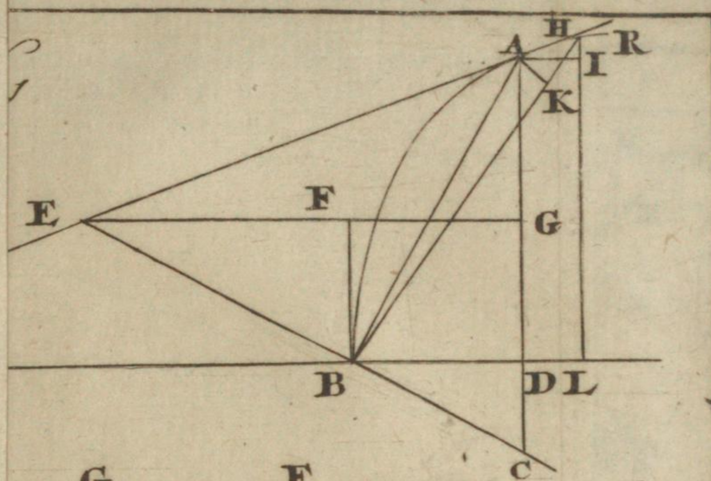
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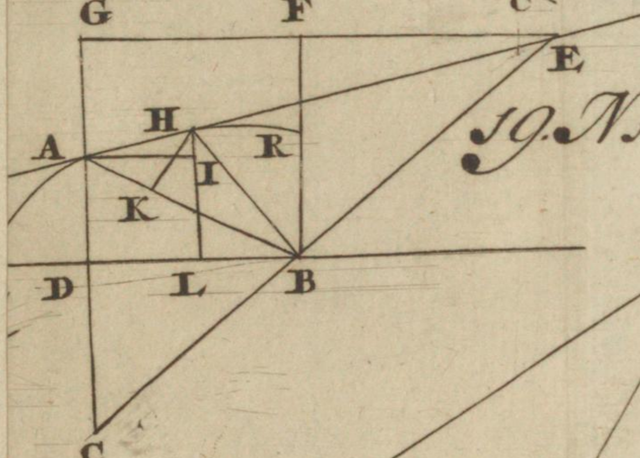
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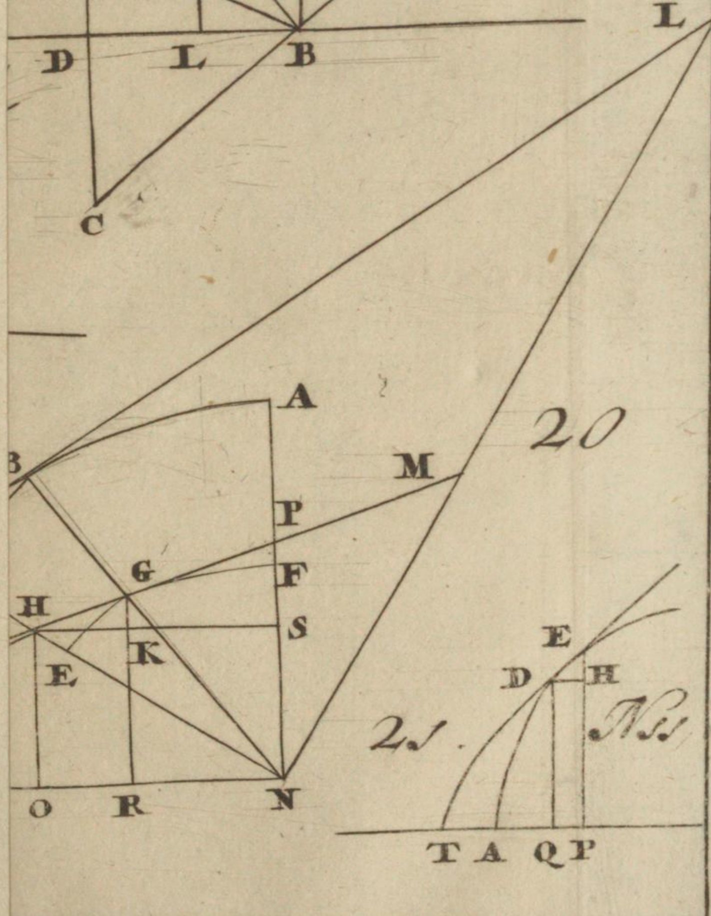
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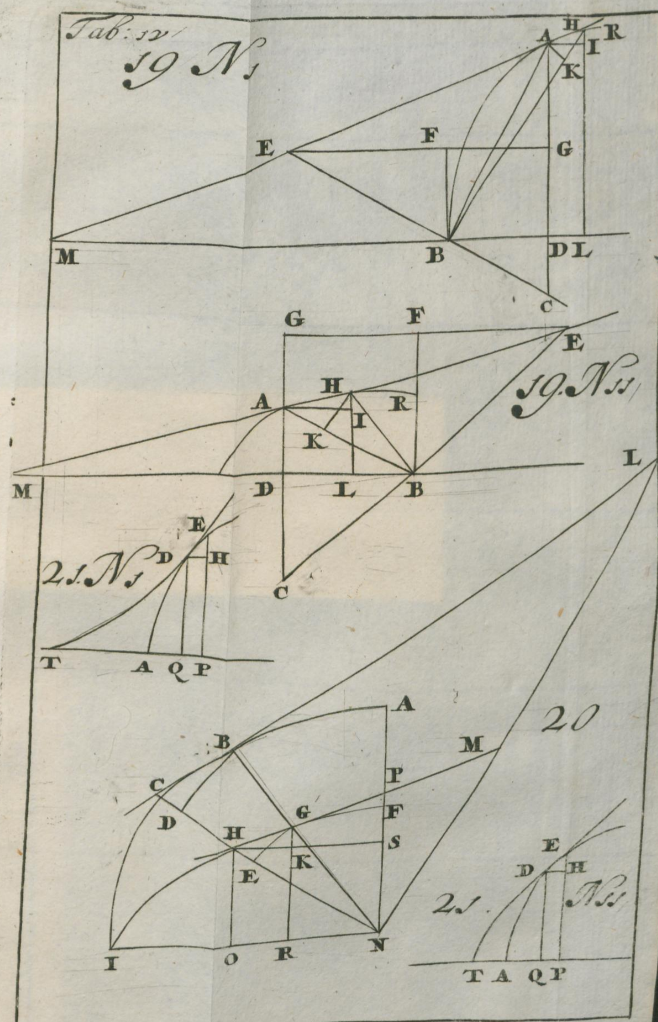
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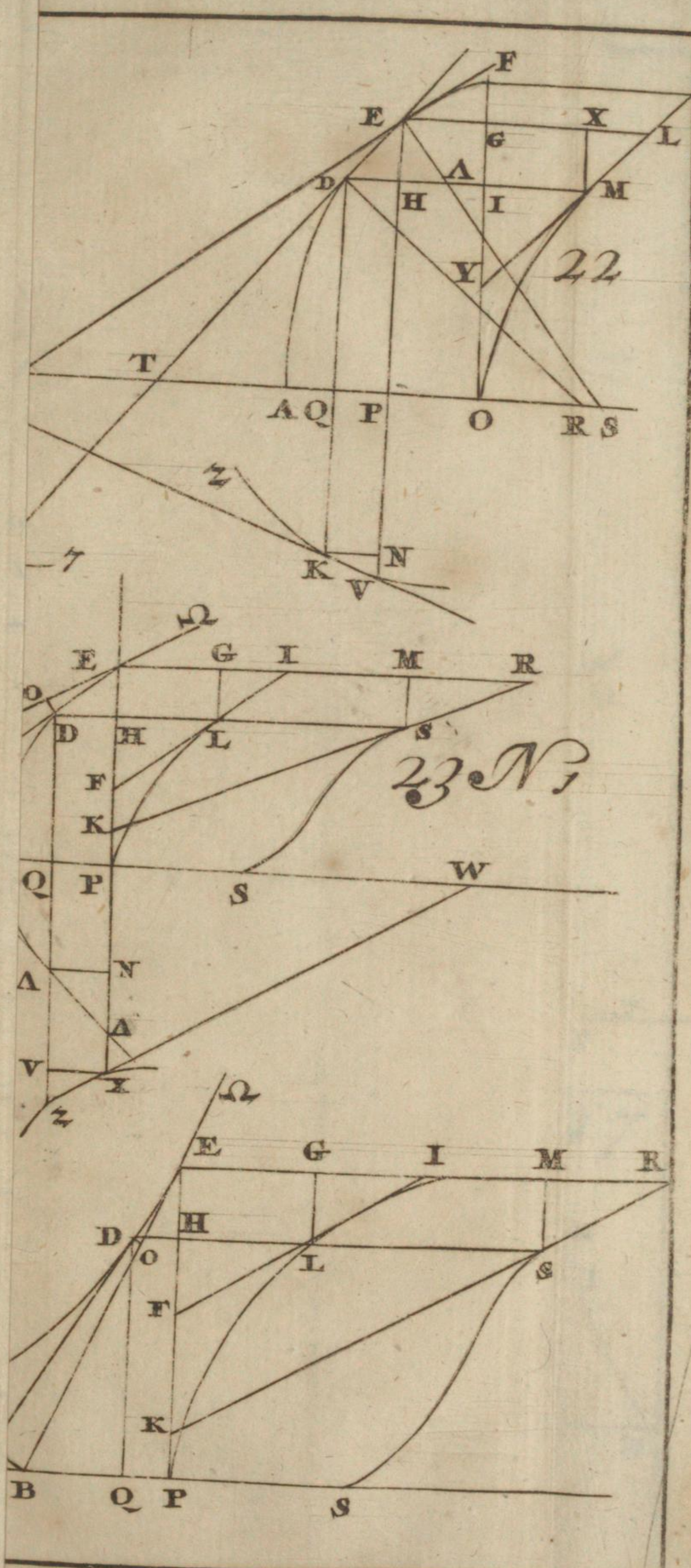
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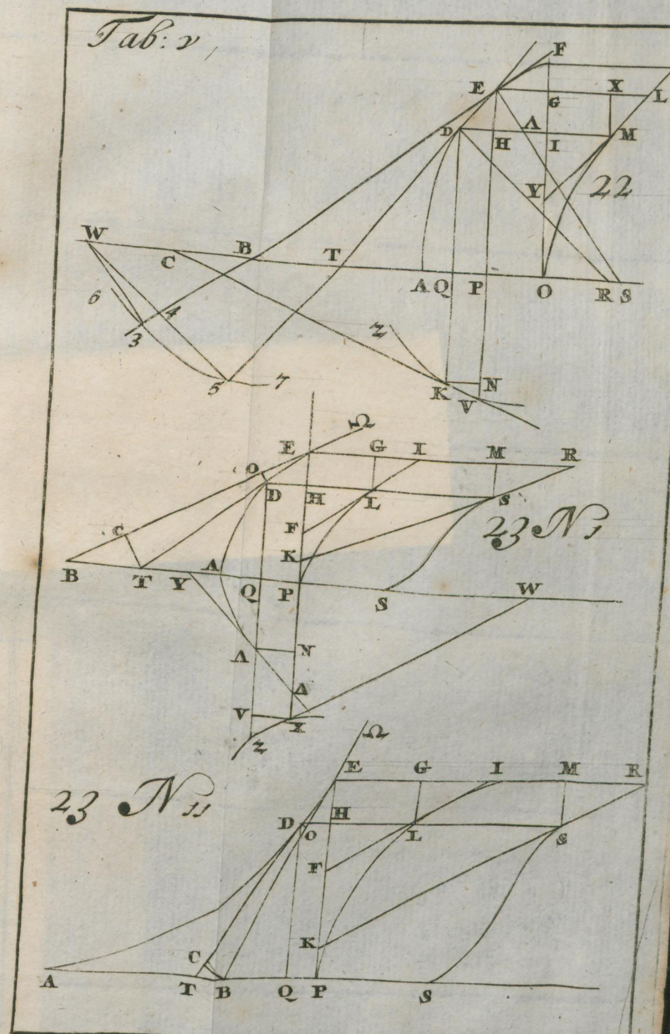
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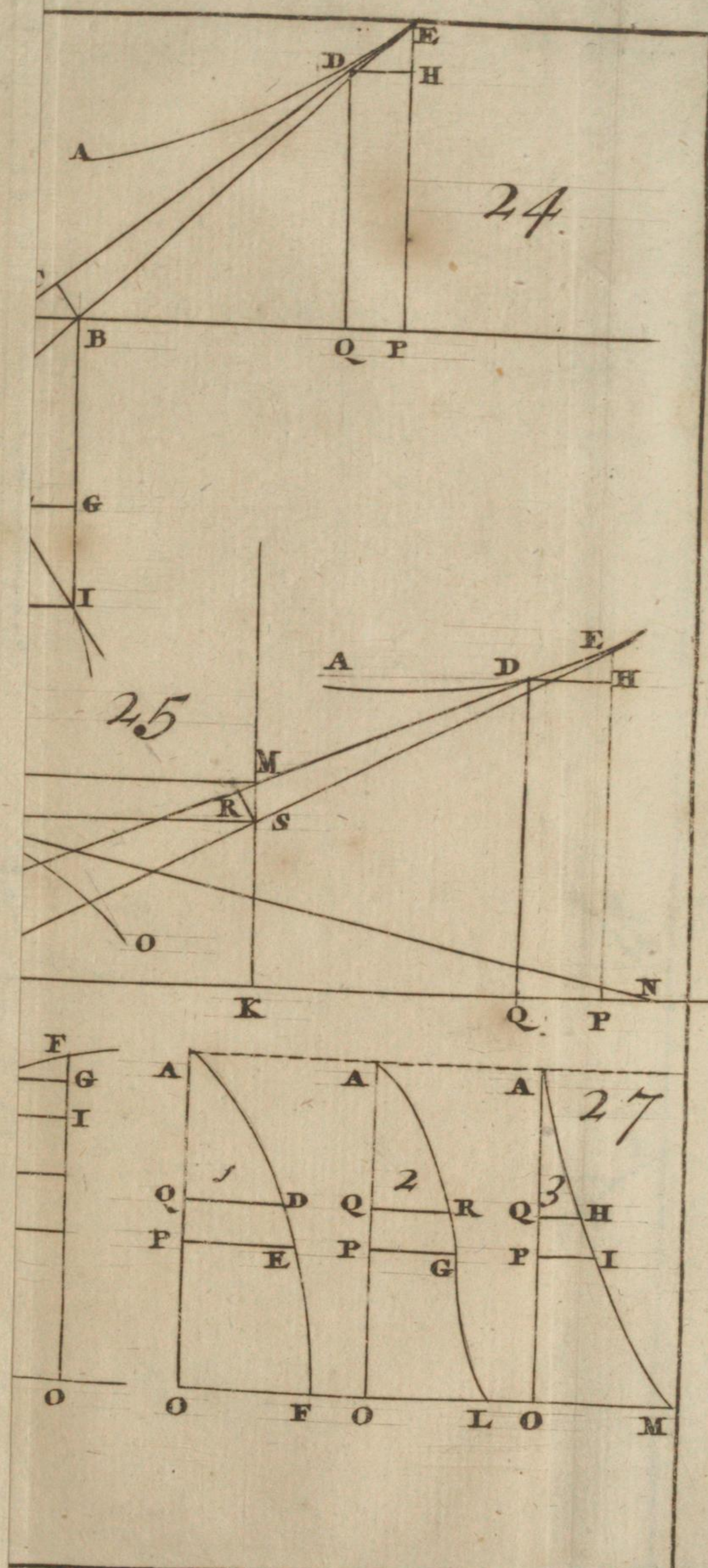
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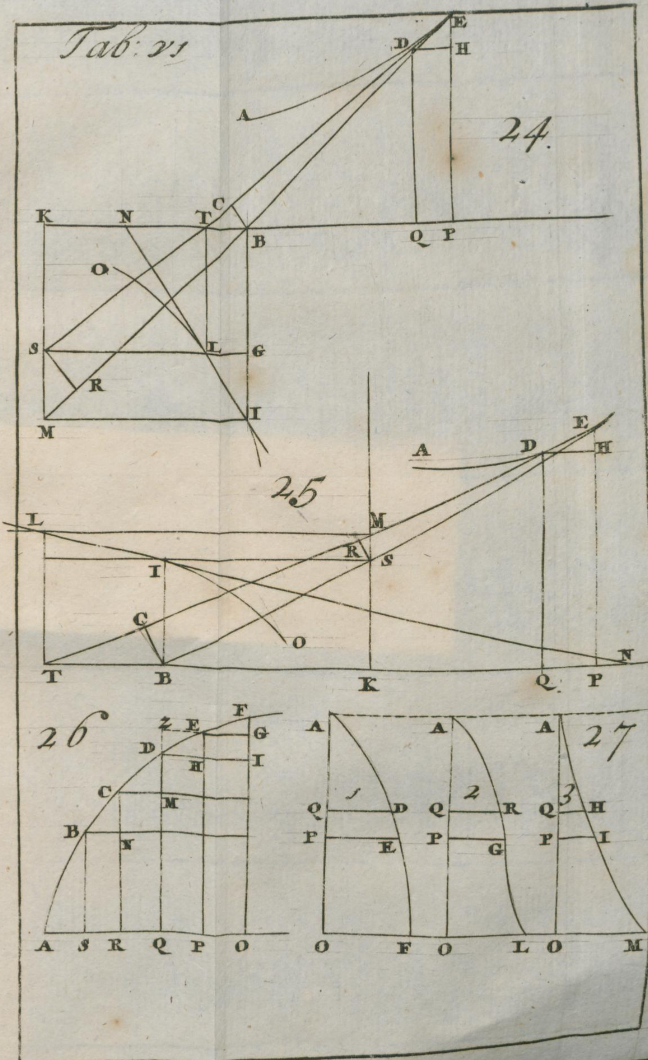
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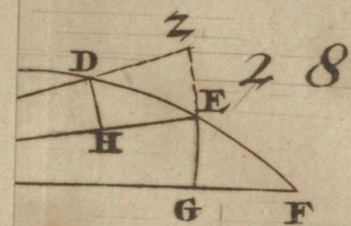
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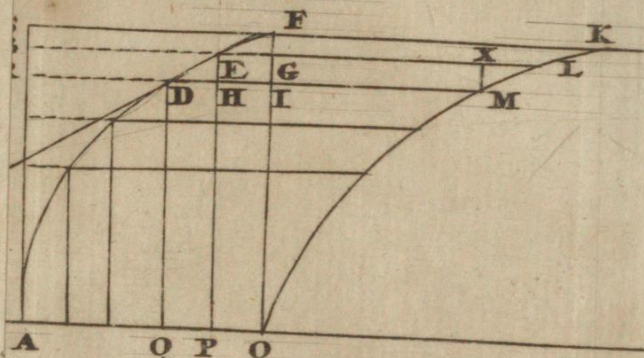
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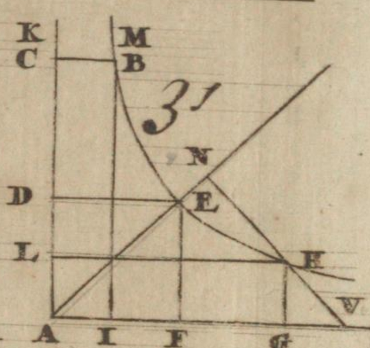


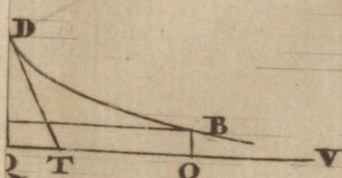
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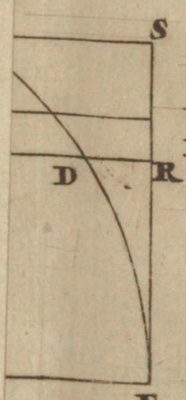
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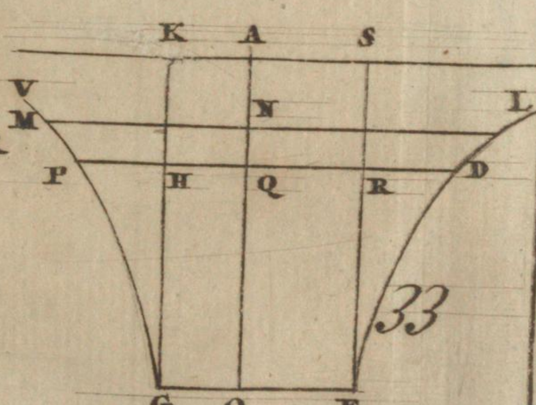

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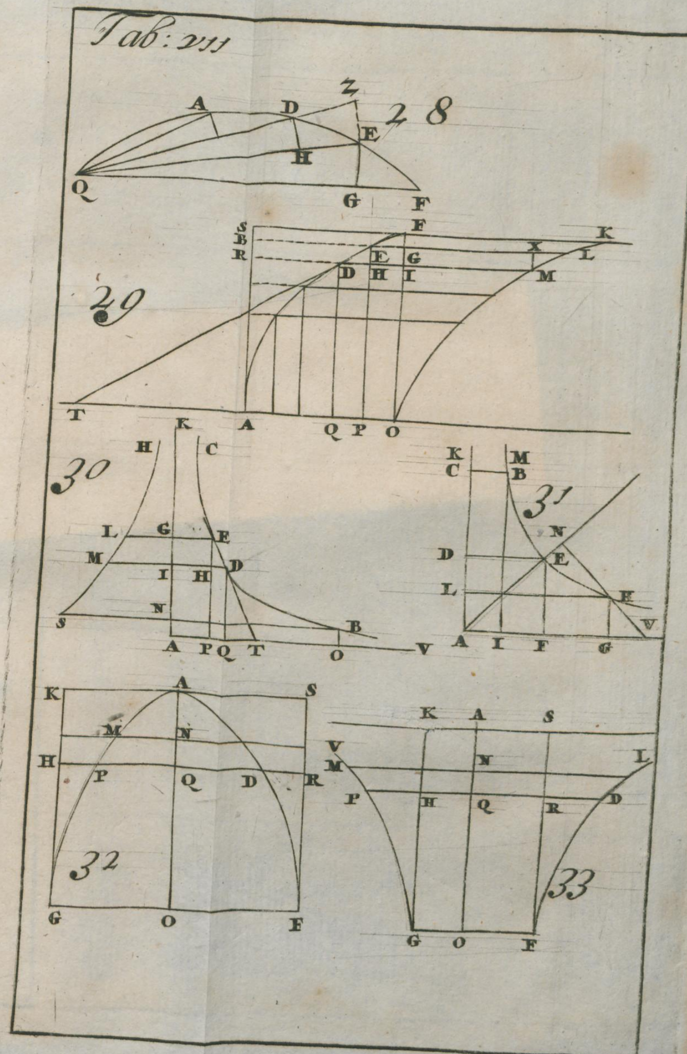

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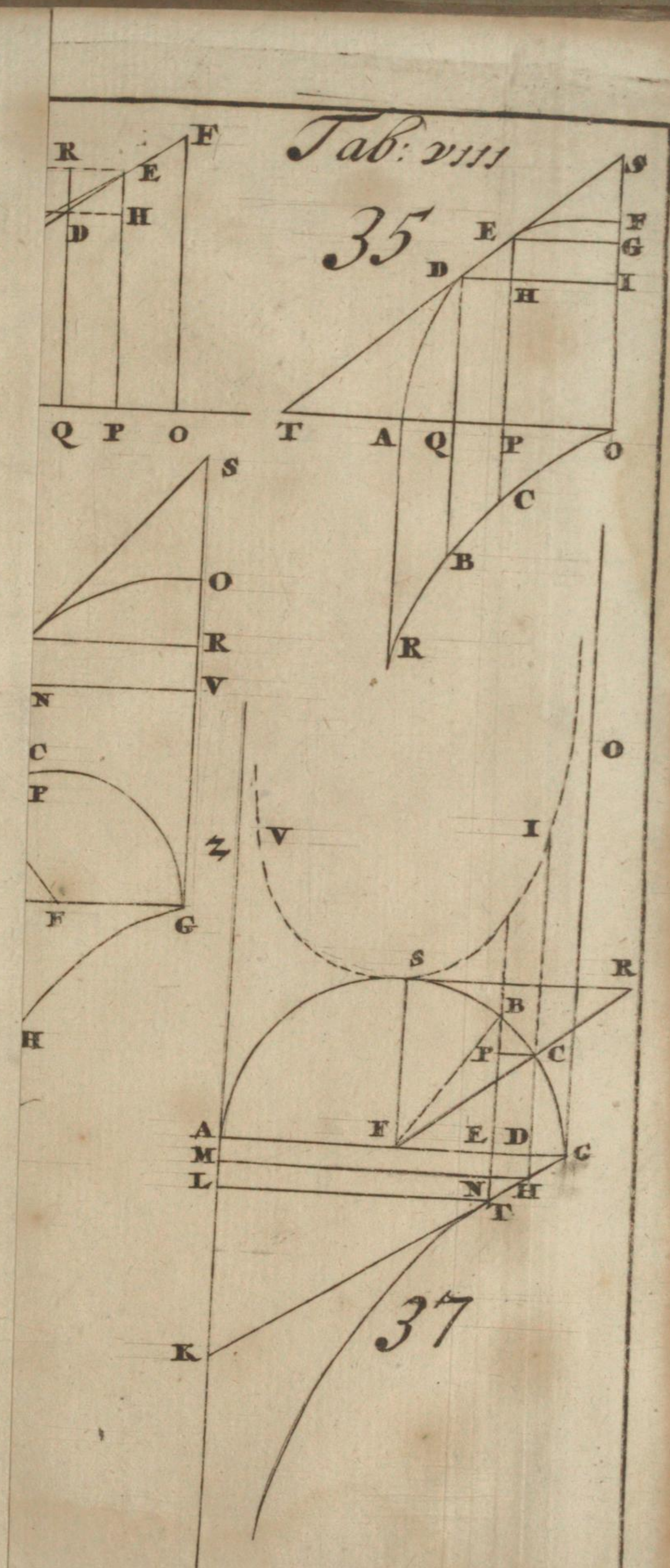

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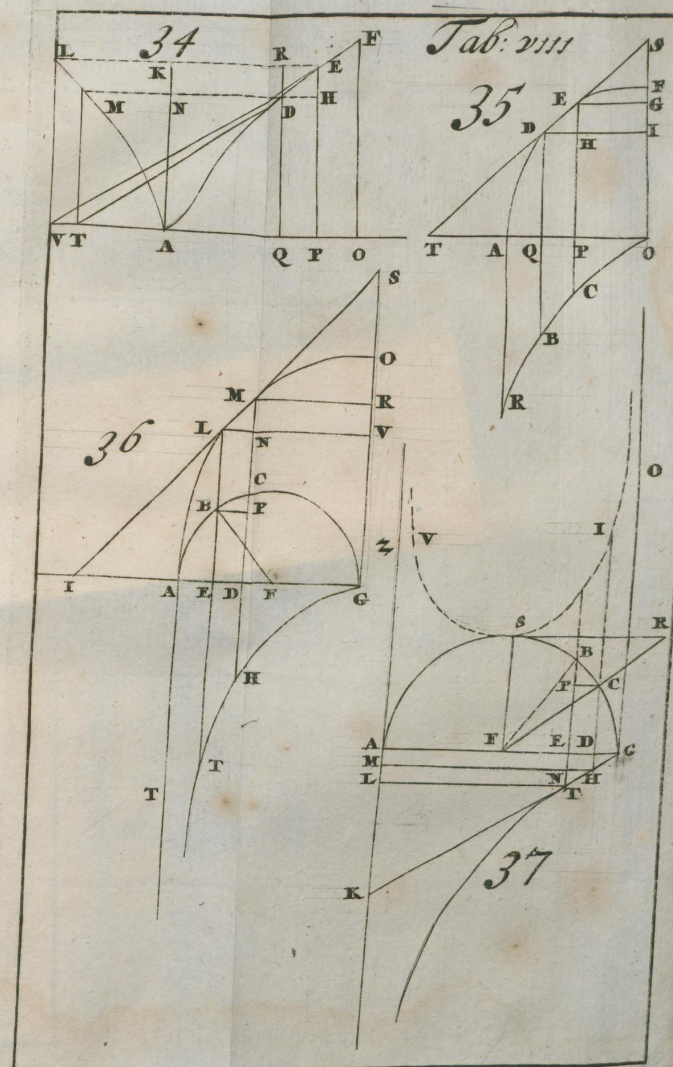
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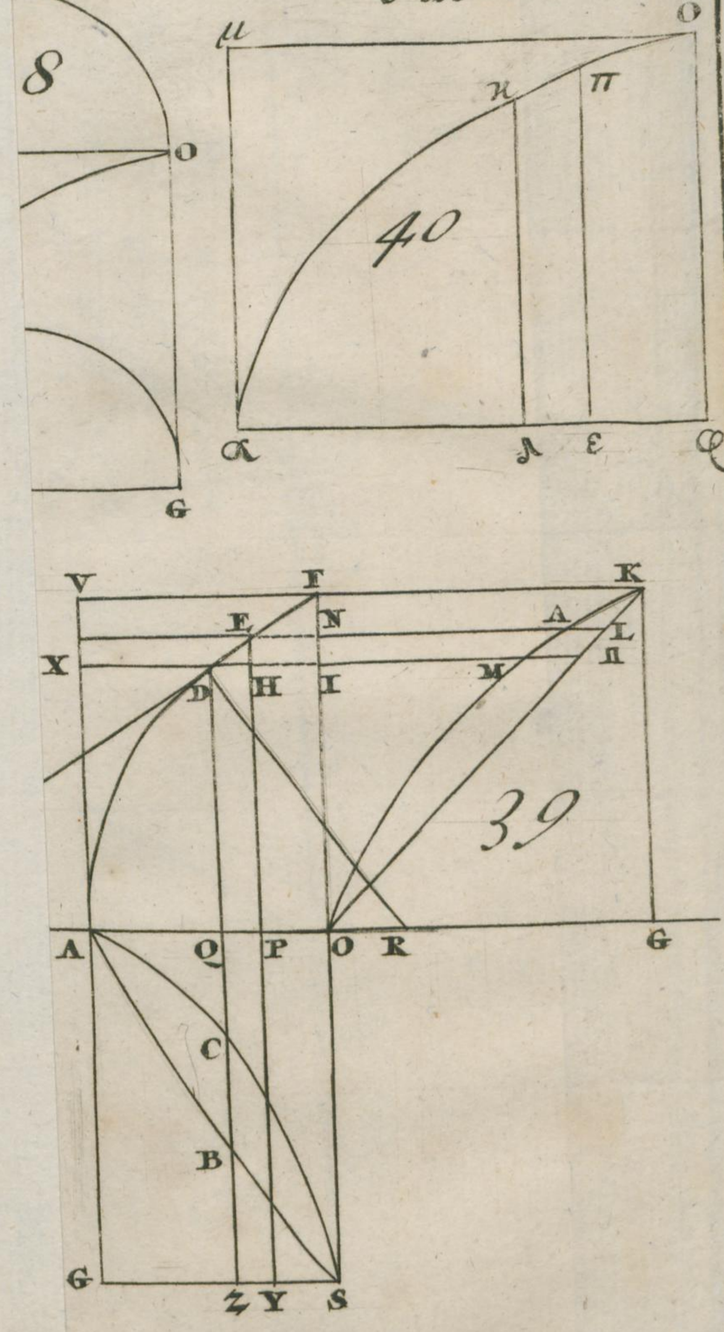
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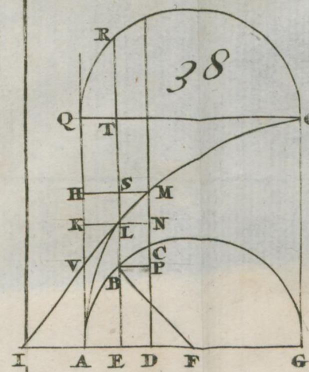
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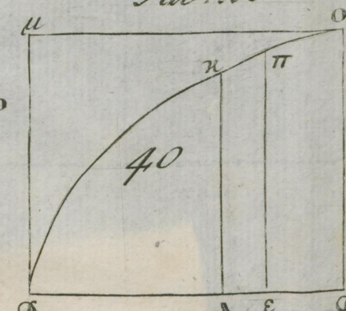


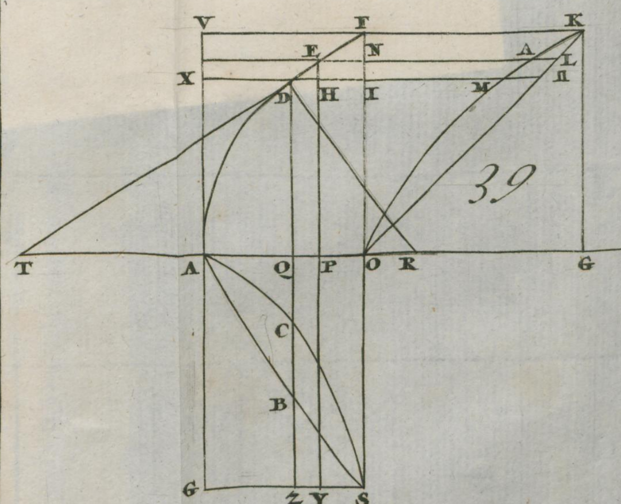
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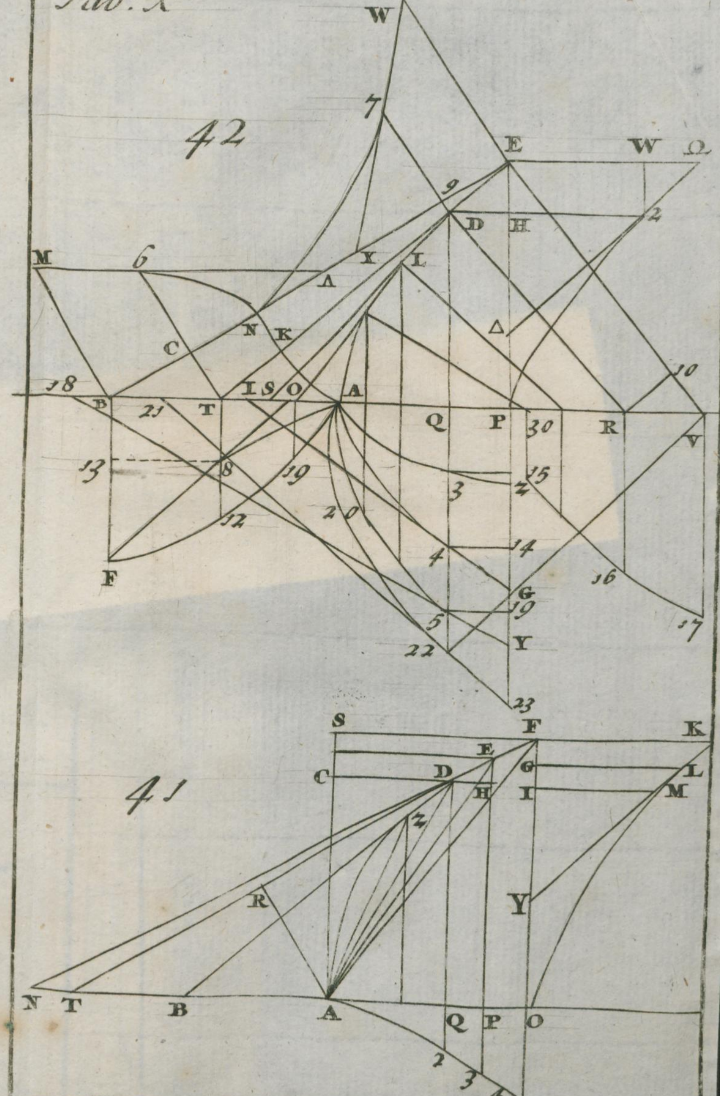
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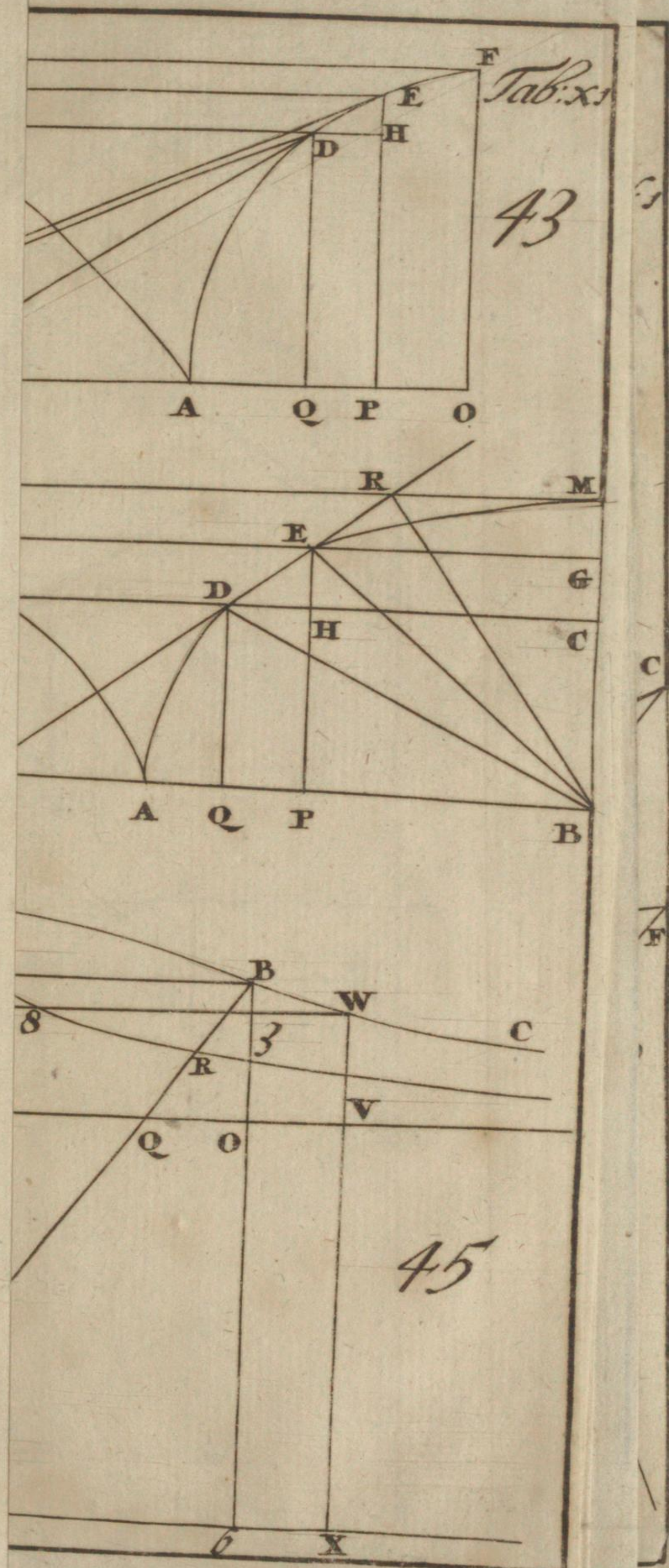
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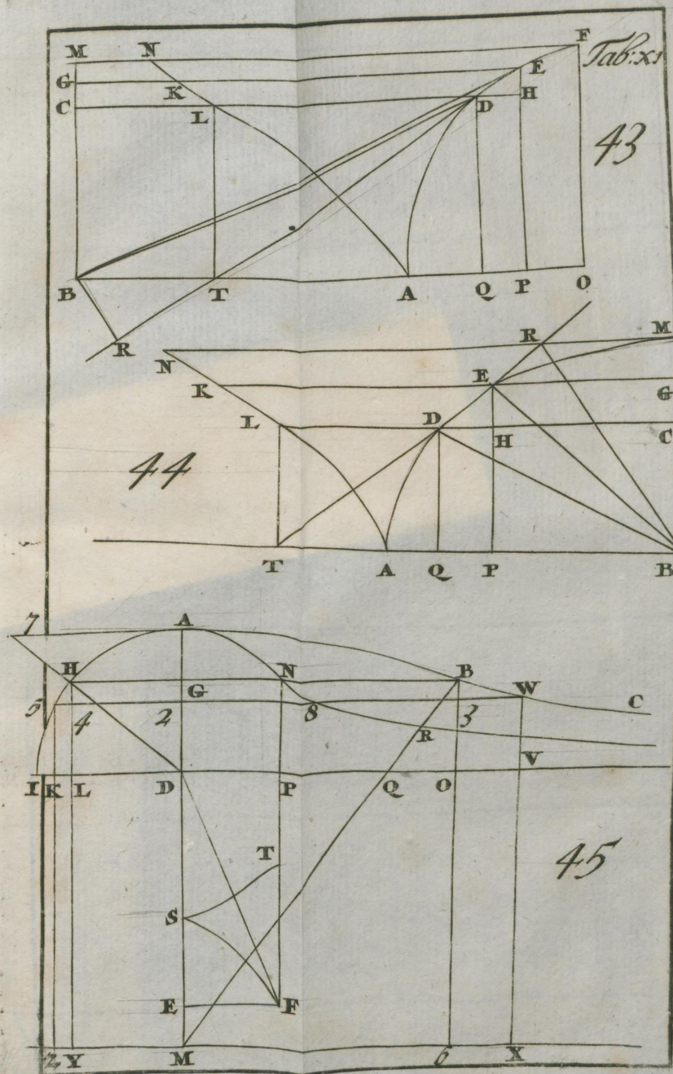
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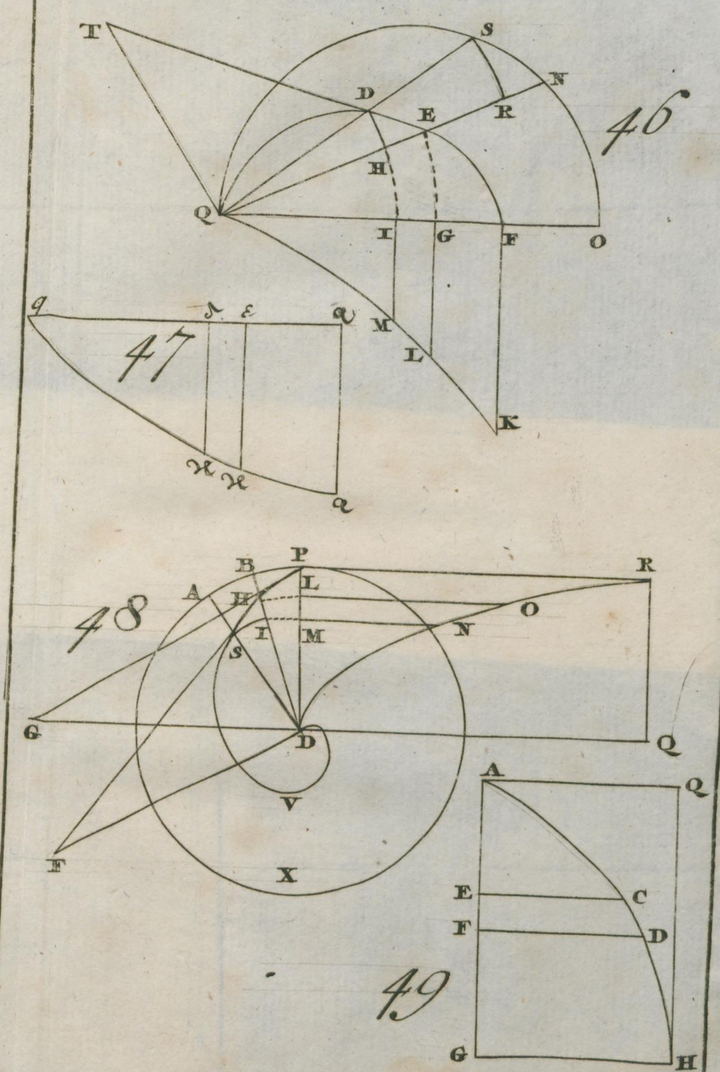
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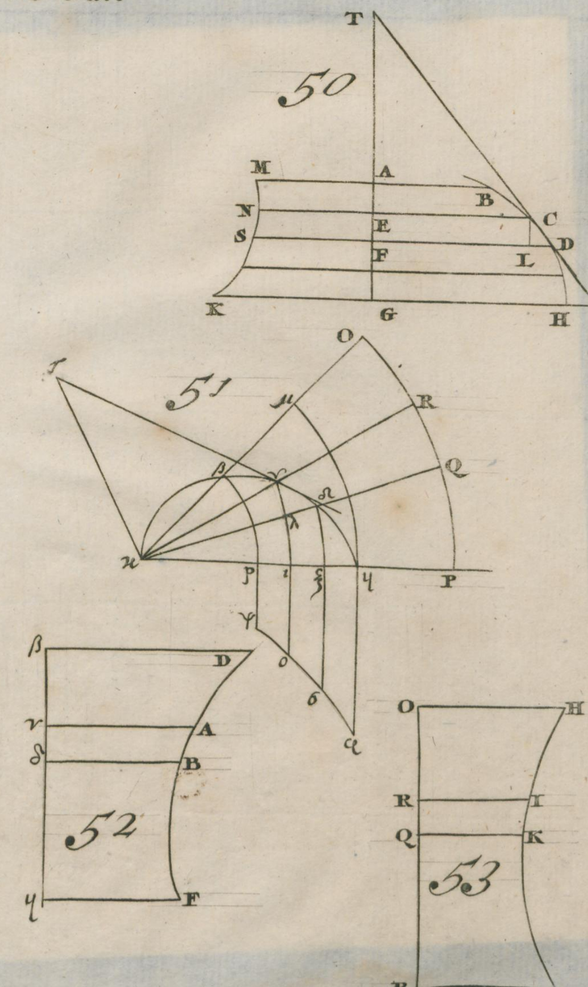
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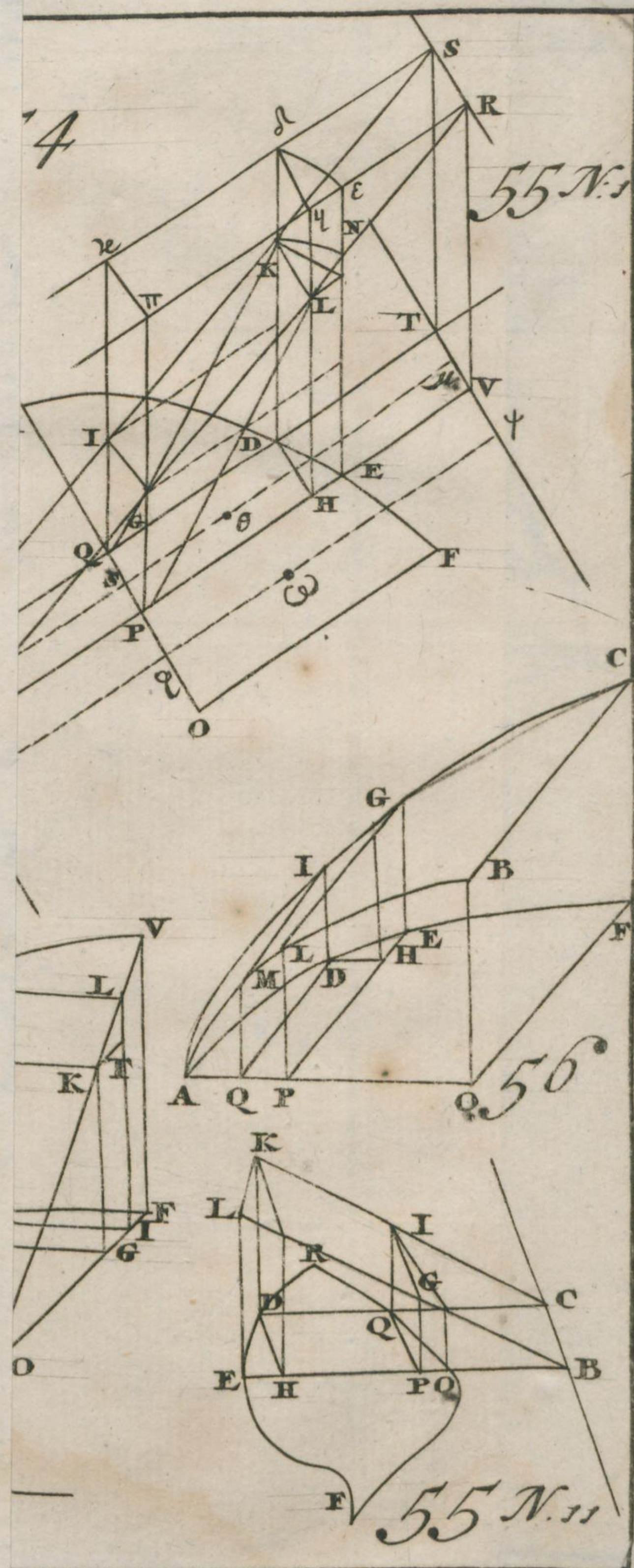
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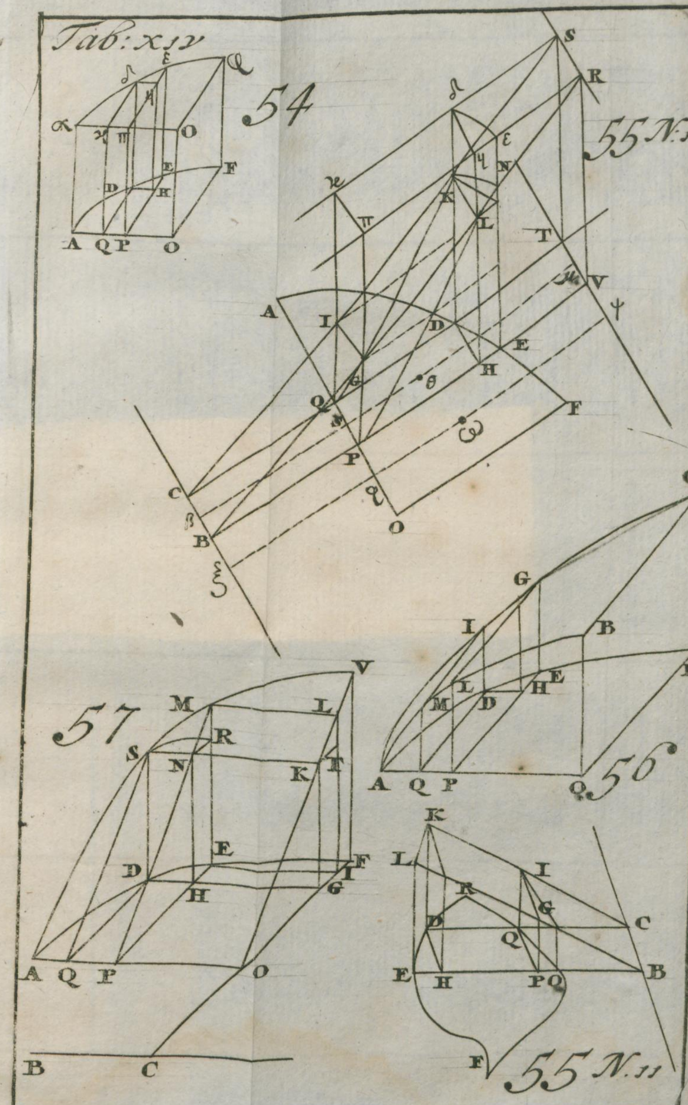


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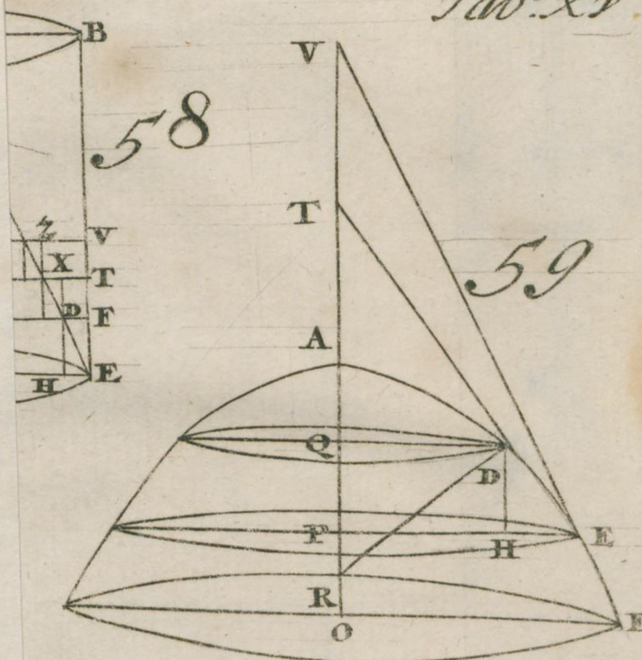
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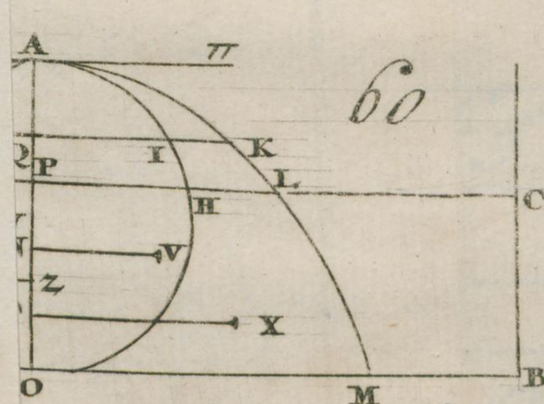


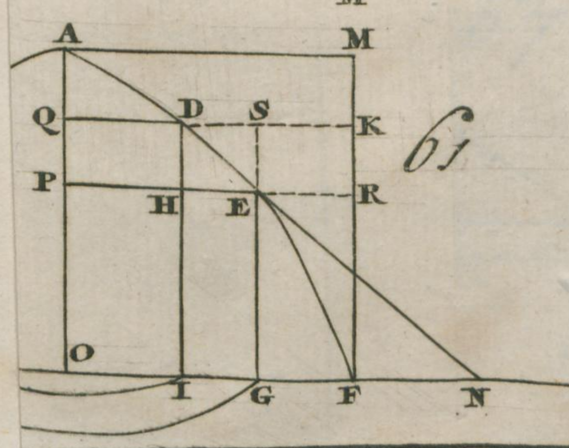
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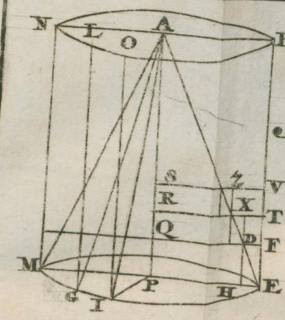




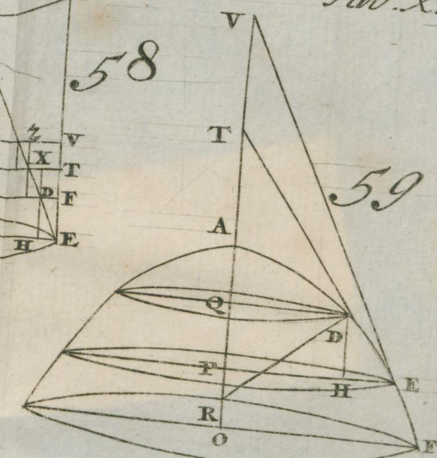
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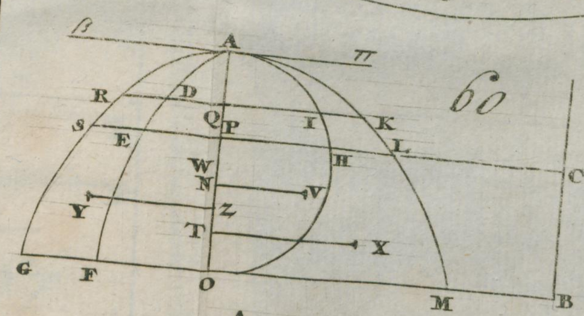
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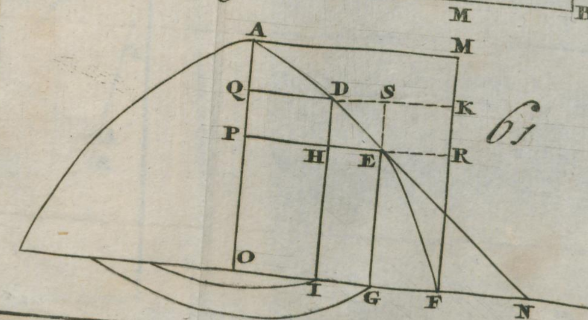
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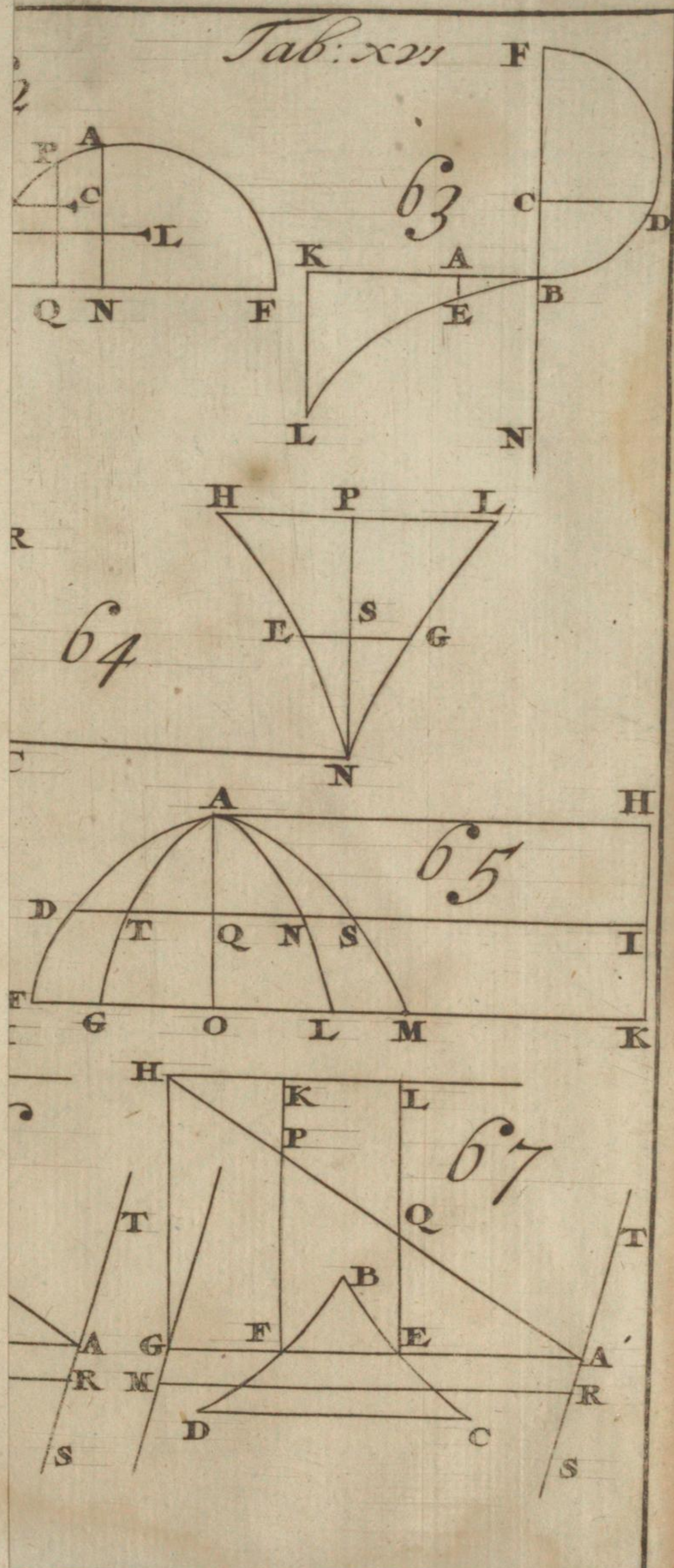
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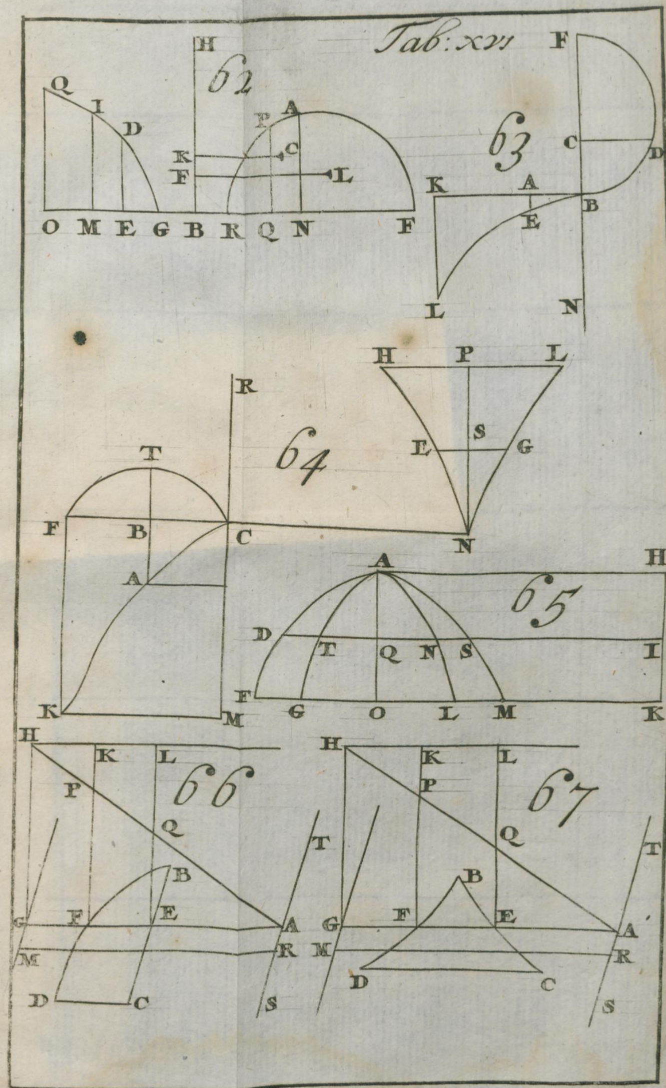
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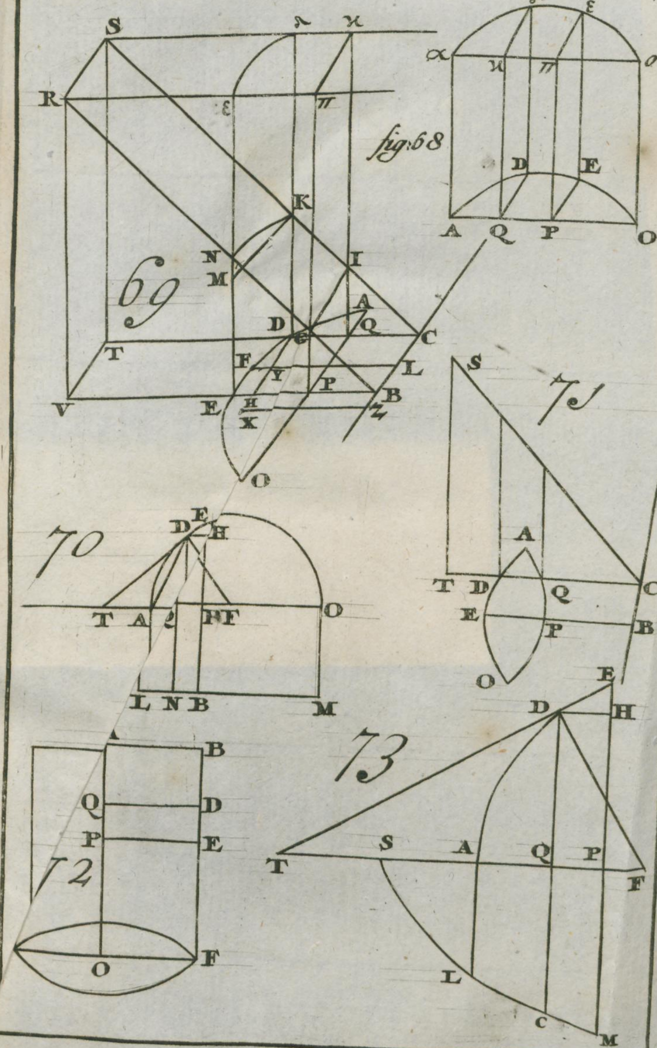


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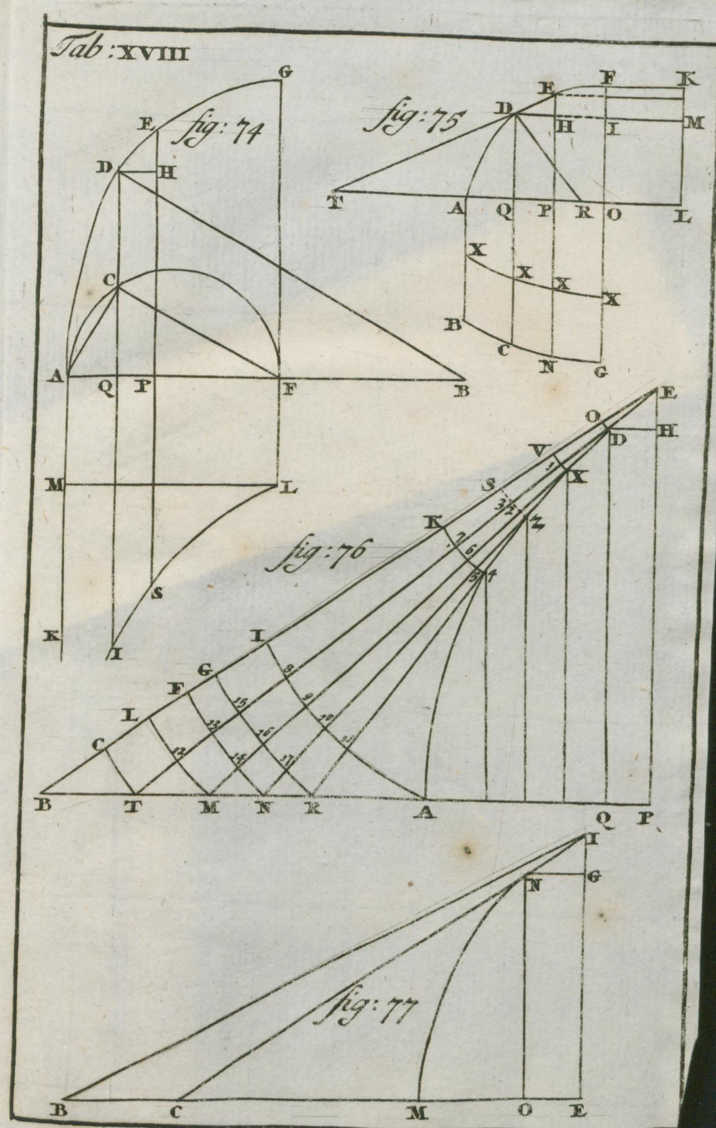


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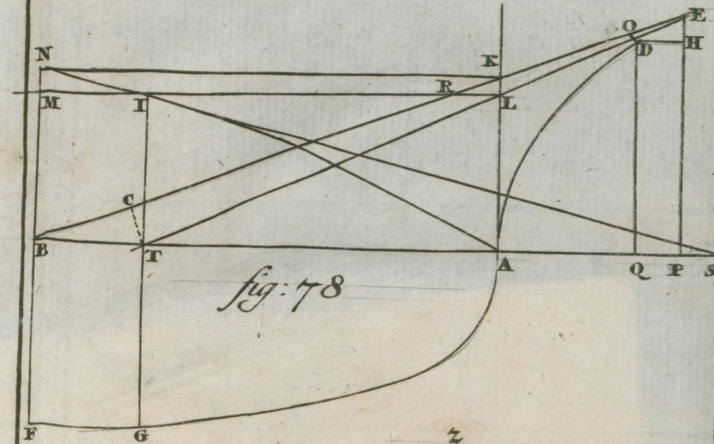


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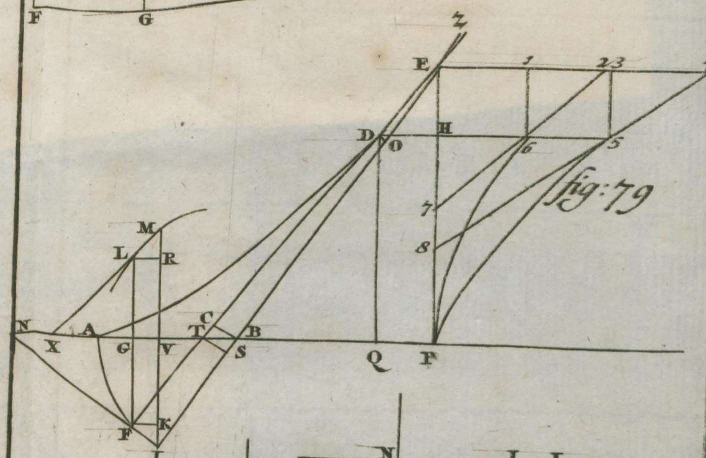


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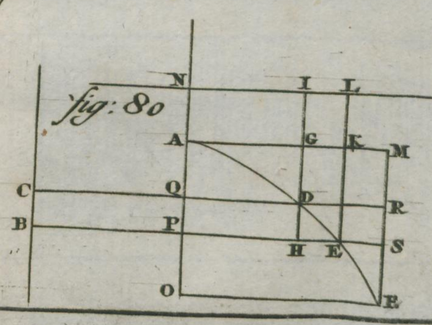
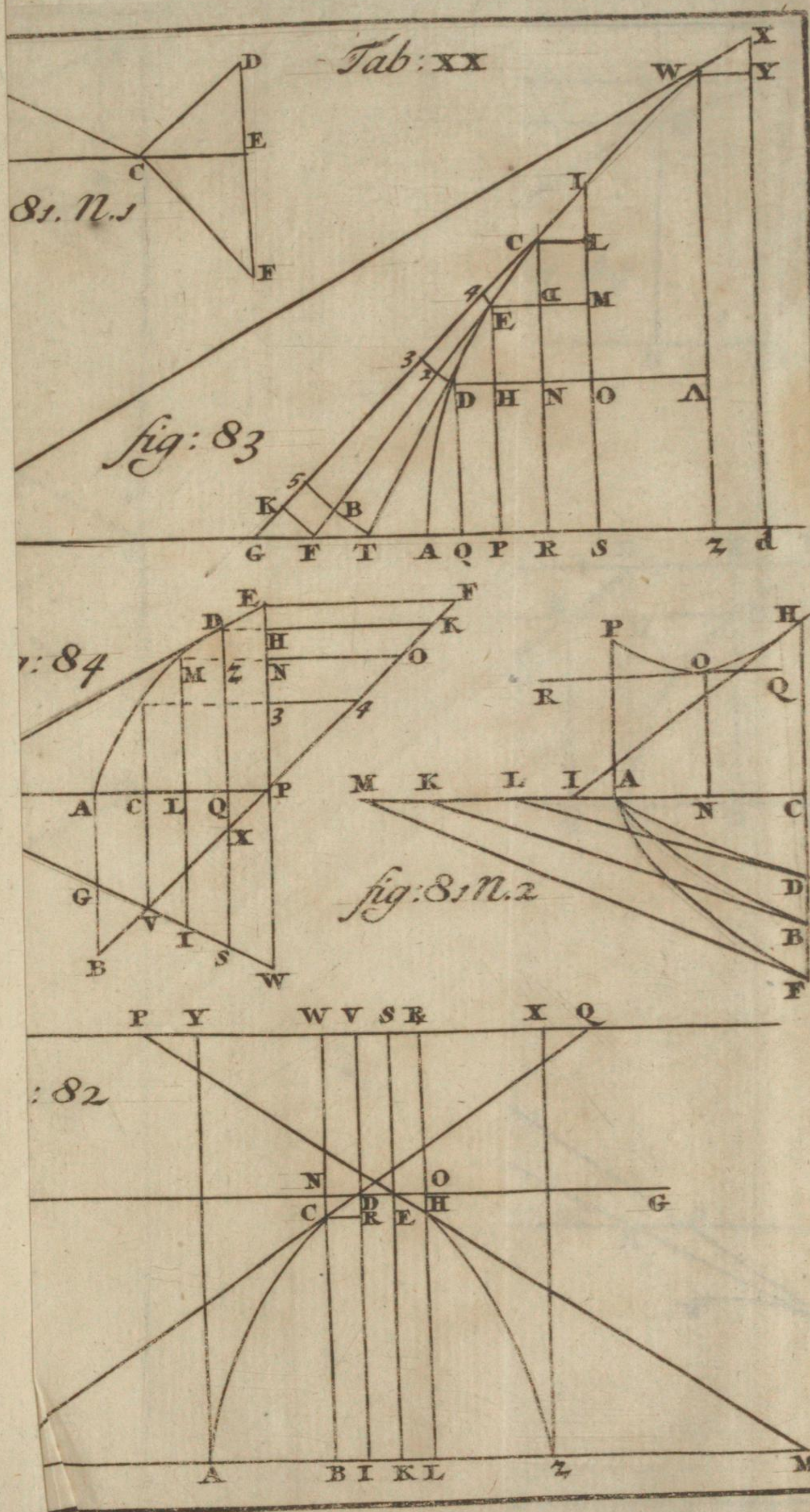


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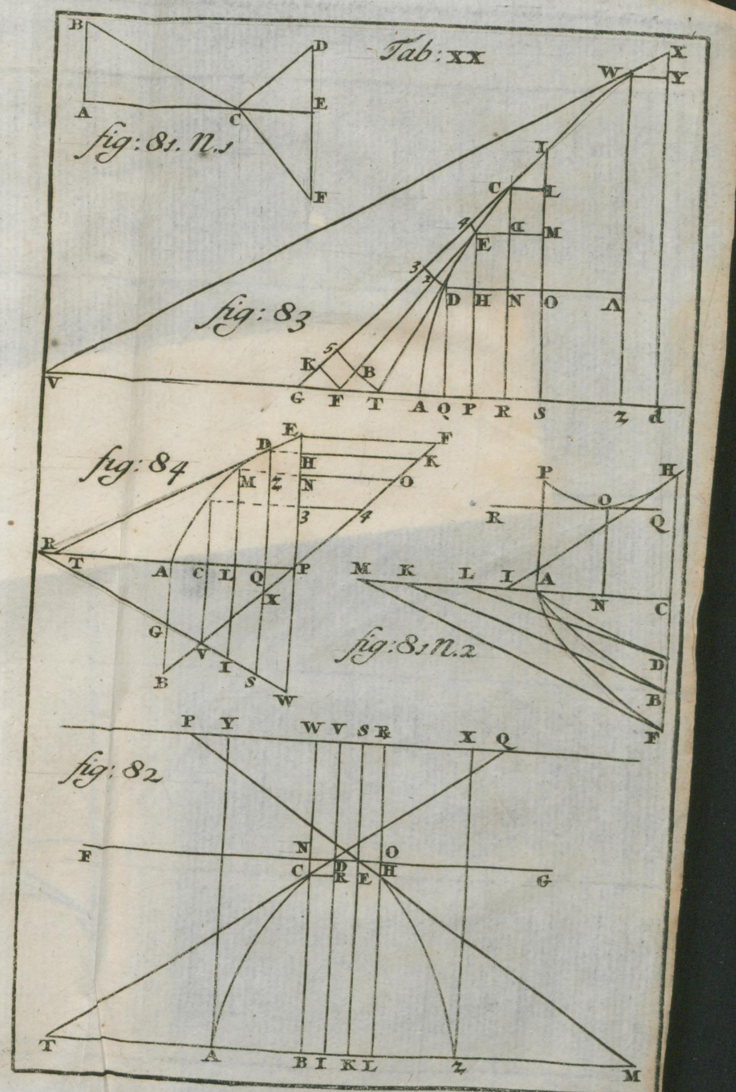
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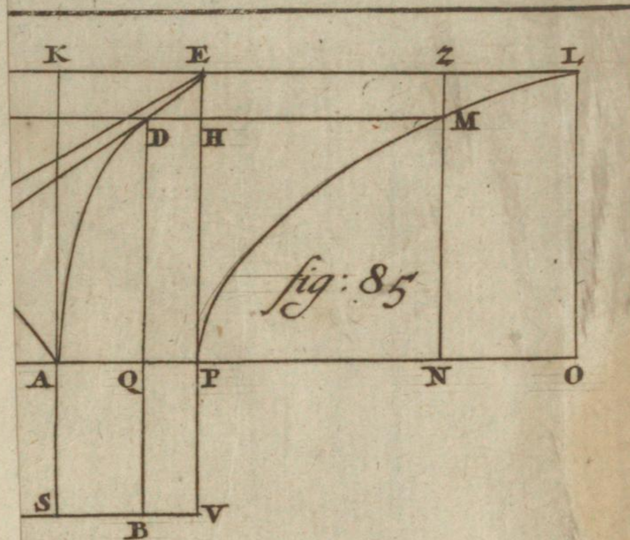
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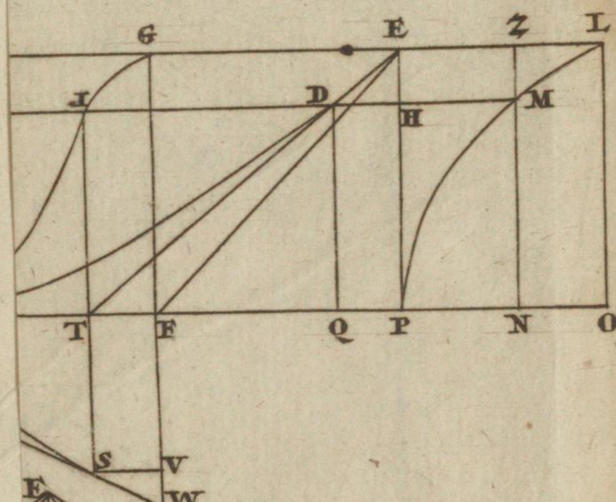
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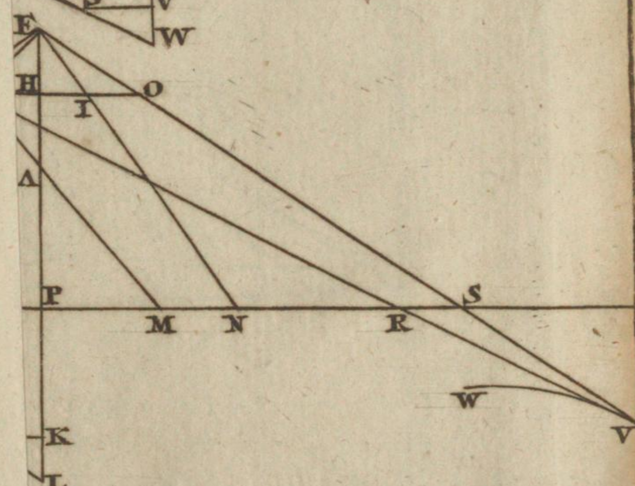


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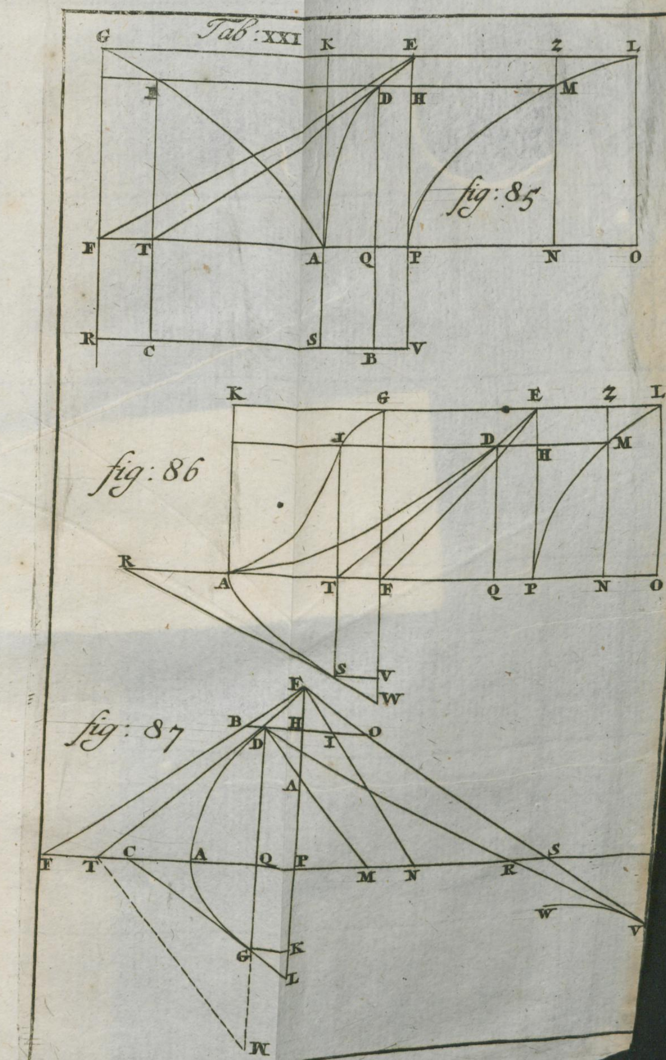
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K E Z L
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A Q P N O
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G E Z L
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